Summary of Analysis

Changjae Lee

June 30, 2024

Contents

Part I

Abstract Meausre and Integration

Chapter 1

Systems of Sets

1.1 Nets and Filters

1.2 Topology

1.3 σ-Algebras

Definition 1. Consider a collection $\mathcal{M} \subset \mathcal{P}X$ s.t.

- $X \in \mathcal{M}$
- $E \in \mathcal{M} \Rightarrow X \setminus E \in \mathcal{M}$
- $(E_\alpha)_{\alpha \in I} \in \mathcal{M} \Rightarrow \bigcup_{\alpha \in I} E_\alpha \in \Sigma$

$\mathcal{M}$ is called an ***algebra*** on X if I is finite, and $\mathcal{M}$ is called ***σ-algebra*** on X if I is countably infinite.

1.4 π-System and λ-System

Definition 2. A ***λ-system*** is a collection $L \subset \mathcal{P}X$ s.t.

1. $X \in L$
2. $x \subseteq y \in L \Rightarrow y \setminus x \in L$
3. $(x_n)_1^\infty$ and $x_n \subseteq x_{n+1} \Rightarrow \bigcup_{n=1}^\infty x_n \in L$

Chapter 2

Measures

2.1 Abstract Measures

Definition 3. A ***measure*** on X is a function $\mu : \mathcal{M} \to [0, \infty]$ s.t.

- $\mu(\varnothing) = 0$
- $(E_j)_1^\infty$ disjoint sets of $\mathcal{M} \Rightarrow \mu(\bigcup_{j=1}^\infty E_j) = \sum_{j=1}^\infty \mu(E_j)$

2.2 Borel Measures

2.3 Outer Measures

Definition 4. An ***outer measure*** on a nonempty set X is a function $\mu^* : \mathcal{P}X \to [0, \infty]$ s.t.

- $\mu^*(\varnothing) = 0$
- $A \subset B \Rightarrow \mu^*(A) \leq \mu^*(B)$
- $\mu^*(\bigcup_{j=1}^\infty A_j) \leq \sum_1^\infty \mu^*(A_j)$

2.4 Complete Measures

Chapter 3

Abstract Integration

3.1 Measurable Functions

3.2 Integration of Nonnegative Functions

3.3 Integration of Complex Functions

3.4 Product Measures

Chapter 4

Signed Measure and Differentiation

4.1 Signed Measures

4.2 Complex Measures

4.3 Differentiation on Euclidean Space

4.4 Bounded Variations

References

Index