Summary of Analysis

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June 27, 2024

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Part I

Meausre and Integration

Chapter 1

Measures

1.1 Abstract Measures

1.1.1 σ-algebras

Definition 1. Consider a collection $\mathcal{M} \subset \mathcal{P}X$ s.t.

- $X \in \mathcal{M}$
- $E \in \mathcal{M} \Rightarrow X \setminus E \in \mathcal{M}$
- $(E_\alpha)_{\alpha \in I} \in \mathcal{M} \Rightarrow \bigcup_{\alpha \in I} E_\alpha \in \Sigma$

$\mathcal{M}$ is called an ***algebra*** on X if I is finite, and $\mathcal{M}$ is called σ***-algebra*** on X if I is countably infinite.

1.1.2 measurable spaces

Definition 2. A ***measure*** on X is a function $\mu : \mathcal{M} \to [0, \infty]$ s.t.

- $\mu(\varnothing) = 0$
- $(E_j)_1^\infty$ disjoint sets of $\mathcal{M} \Rightarrow \mu(\bigcup_{j=1}^\infty E_j) = \sum_{j=1}^\infty \mu(E_j)$

1.2 Outer Measures

Definition 3. An ***outer measure*** on a nonempty set X is a function $\mu^* : \mathcal{P}X \to [0, \infty]$ s.t.

- $\mu^*(\varnothing) = 0$

- $A \subset B \Rightarrow \mu^*(A) \leq \mu^*(B)$
- $\mu^*(\bigcup_{j=1}^{\infty} A_j) \leq \sum_1^{\infty} \mu^*(A_j)$

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