Introduction

blah blah

Chapter 1

Preliminaries

1.1 von Neumann Algebras

Definition 1. Let H be a Hilbert space. For any subset $\mathcal{M}$ of $\mathcal{B}(H)$,

1. $\mathcal{M}'$ denotes the set of all bounded operators on H commutes with any element of $\mathcal{M}$.
2. The set $Z(M) = M \cap \mathcal{M}'$ is called the *center* of $\mathcal{M}$

Definition 2. A *von Neumann algebra* on a Hilbert space H is a subset $\mathcal{A}$ of $\mathcal{B}(H)$ such that

1. $I \in \mathcal{A}$,
2. $\alpha A + \beta B \in \mathcal{A}$ whenever $A, B \in \mathcal{A}$, $\alpha, \beta \in \mathbb{D}$,
3. if $A \in \mathcal{A}$, then $A^* \in \mathcal{A}$,
4. $\mathcal{A}$ is closed in the weak operator topology.

1.2 Elements of Quantum Logic

Let L be a poset. For $a, b \in L$, the operations $a \wedge b = \inf\{a, b\}$ and $a \vee b = \sup\{a, b\}$ are called *meet* and *join* respectively. In general,

$$\bigwedge_{i \in I} a_i = \inf\{a_i : i \in I\}$$

and

$$\bigvee_{i \in I} a_i = \sup\{a_i : i \in I\}$$

for any family $\{a\}_{i \in I}$ of elements of a poset L.

Definition 3 (Lattice)**.** Let L be a poset. $\mathcal{L} = (L, \wedge, \vee)$ is sad to be a *lattice* if both $a \wedge b$ and $a \vee b$ exist in L for any $a, b \in L$. $\mathcal{L}$ is said to be a σ*-lattice* or a *complete lattice* if both $\bigwedge_{i \in I} a_i$ and $\bigvee_{i \in I} a_i$ exist in L for any family $\{a_i\}_{i \in I}$ of elements of L with a countable or abitrary index set I.

Definition 4. We say that a mapping $\perp\colon L \to L$ is said to be an *orthocomplementation* on a poset L with 0 and 1 if

1. $(a^\perp)^\perp = a$ for any $a \in L$,
2. if $a \leq b$, then $b^\perp \leq a^\perp$,
3. $a \vee a^\perp = 1$ for any $a \in L$.

Definition 5 (Orthomodular poset; OMP)**.** An *orthomodular poset* is the lattice $\mathcal{L} = (L, \wedge, \vee)$ satisfying the othormodular condition,

$$b = a \vee (b \wedge a^\perp)$$