W4451 – Financial Econometrics and Quantitative Risk Management
Summer 2021

Applied Project No. 2

Financial Time Series and Risk Management

Lecturer: Prof. Yuanhua Feng
Project beginning: 03.08.2021
Project due: 13.09.2021

Question No.	1	2	3	4	Total
Points awarded					
Points max.	12	12	18	8	50

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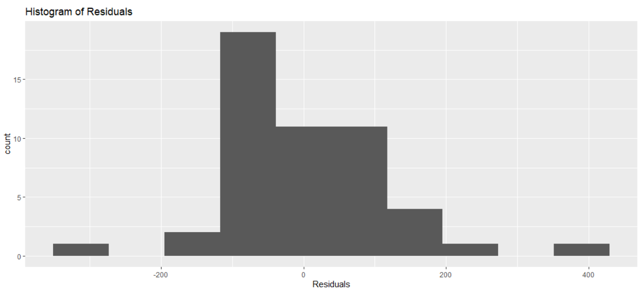
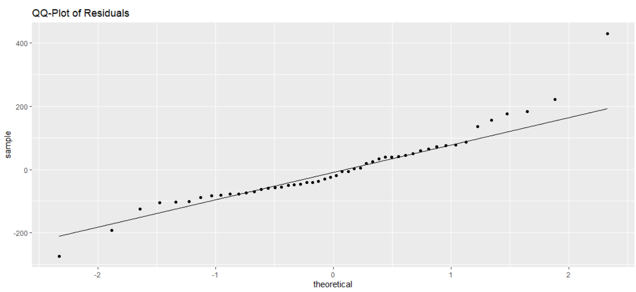
Task 1 – Introduction to Time-Series Analysis

1.1)

The lecture begins with an introduction in its first chapter, where the main parts for the module are defined and some basic assumptions are made. Really important is the definition of “time series” to get the same understanding about what is meant with this phrase. A time series is a special stochastic process X_t at time t, which is equal to a sequence of observations x_t. So, it is a collection of random variables. Time series in economic context can e.g., be interpreted as different types of prices, maybe for stocks or commodities. Also, it can show the income or expenditure in a certain time. Nearly everything that has a time dependent development of its financial values can be expressed as a time series.

Afterwards two major properties of time series are explained: autocovariance and stationarity. The autocovariance measures the linear dependence between two points on the same series observed at different times[1]. A time series is weakly stationary if the mean of an observation x_t and the covariance between x_t and x_{t-1} are time-invariant[2]. One basic operator of calculating in connection with time series data is the backshift operator “B”. B shifts the point in time one unit back, while B^2 shifts the point two units back at the time horizon. Another important operator is the differencing operator “Δ” which is the difference between x_1 and x_{t-1}. To visualise the distribution of the observations one can, use different plots e.g., histograms, correlograms or QQ-normal plots. These diagrams are very helpful in case of making a first diagnosis concerning time series.





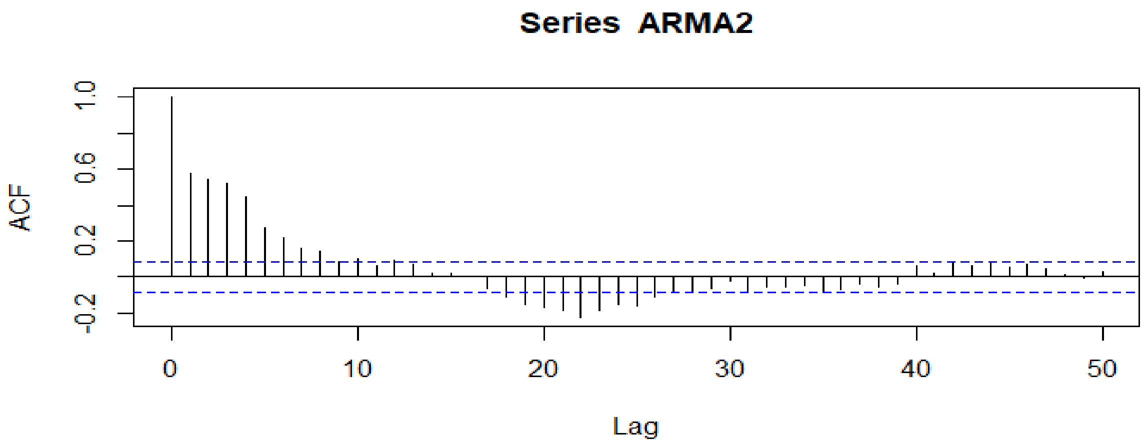
The second chapter of the lecture gives attention to the linear process. At first the moving average (MA) models are discussed. They are always causal and stationary. The white noise process is very simple but also a crucial building stone of a linear time series, hence for most of the time series which are introduced in this lecture. Thereby the focus of our module is the analysis of dependency structures in financial data sets. A q-th order moving average process is given by $X_t = \alpha_1 \epsilon_{t-1} + \cdots + \alpha_q \epsilon_{t-q} + \epsilon_t$, where ϵ_t are the innovations of the time series and are unobservable. Then autoregressive (AR) processes are introduced, which are always invertible. A p-th order autoregressive process is given by $X_t = \beta_1 X_{t-1} + \cdots + \beta_p X_{t-p} + \epsilon_t$. As one of the conclusions we can obtain that a stationary MA process is invertible, if it can be written as an AR model.

The combination of an AR and an MA process is an ARMA process which is given by $X_t = \beta_1 X_{t-1} + \cdots + \beta_p X_{t-p} + \alpha_1 \epsilon_{t-1} + \cdots + \alpha_q \epsilon_{t-q} + \epsilon_t$. As a combination of these two parts the stationarity of an ARMA process is determined by its AR part and the invertibility of an ARMA process is determined by its MA part. However, these two properties are independent of each other and should be checked separately. Furthermore, the quantity of the autocovariances is a very important part of the analysis of

[1] Time Series Analysis and Its Application (Page 16)
[2] Analysis of Financial Time Series (Page 23)

ARMA models. Although the autocovariances are difficult to obtain, the sum of the autocovariances can be obtained much more easily.

Nonstationary linear processes are discussed next and provide the next extension of the known models. The ARIMA model stands for integrated ARMA, and the order of integration is displayed by the “d “. For d = 0, ARIMA reduces simply to ARMA and for d = 1, ARIMA is integrated and has a so called “unit root”. A random walk can be used as the simplest model for a financial time series when the mean of the returns is about zero. Also, a random walk model is nonstationary in variance and covariance. When adding a drift, it can be used as the simplest model for a financial time series when the mean of the returns is significantly nonzero and nonstationary.
Estimation for linear processes is the topic of the next chapter. Therefore, one must know that μ is non-estimable from only one realization. The correlogram can show if the data is uncorrelated, when less than 5% of the estimates are outside of the bounds, or the data is correlated and hence not independent, when more than 5% of the estimates are outside of the bounds.



When it comes to estimating of an ARMA model one must consider that AR processes are very easy to estimate, because autoregression is a modified regression, but it is not so easy to estimate MA processes. Also, an AR(p) process is determined by the values of its first p autocorrelations. For estimating linear processes Yule-Walker estimators can be a very useful tool, when it tends to big sample sizes. Then these estimators are approximately normally distributed and essentially are least squares estimators[3]. For selecting the best possible ARMA (p, q) model, one must select $\hat{p}$ and $\hat{q}$ by minimising the Akaike´s information criterion (AIC) or the Bayesian´s information criterion (BIC). However, the BIC is preferable to AIC.

Forecasting of linear processes is the last part of this summary and the most important purposes of time series analysis. Here forecasting is about predicting the future values of a time series, based on the collected data from the past up to the present. The first way of predicting the values in the future is the point forecasting for ARMA models. Here we use the collected data until the present value (x_n) and use this to forecast the next value in x_{n+1}. Then we must repeat this procedure once again and add the first forecasted value to the dataset, to get the next value in x_{n+2}. When using this method step by step, one can forecast points in the future very easily however, the sample mean must be zero. If we assume that the distribution is normal, then we can also calculate the interval forecast.

[3] Time Series Analysis and Its Application (Page 115)

1.2 b)

	AR 0	AR 1	AR 2	AR 3	AR 4	AR 5
BIC Values	917.8078	900.3382	684.4529	683.6941	682.1785	682.0758

	AR 6	AR 7	AR 8	AR 9	AR 10
BIC Values	680.3119	680.1244	680.0610	679.3625	679.1155

$$Y_t = X_t + 1.4871$$

s.e. (0.0156)

$$X_t = 0.2784X_{t-1} - 0.5491X_{t-2} - 0.0535X_{t-3} + 0.0214X_{t-4} + 0.0436X_{t-5} - 0.0618X_{t-6} - 0.0044X_{t-7} + 0.0102X_{t-8} - 0.0320X_{t-9} - 0.0228X_{t-10} + \epsilon_t$$

s.e. (0.0448) (0.0466) (0.0527) (0.00527) (0.0530) (0.0530)

(0.0531) (0.0532) (0.0475) (0.0458)

$$var(\varepsilon_t) = \sigma^2 = 0.2273$$

According to the BIC criterion, the AR (10) is the optimal order since the value here is the lowest of all 11 possibilities.

Task 2 – Financial Time Series

2.1)

In Chapter 5, we learn about ARCH and GARCH models along with some of the important properties of financial time series. The ARCH (Auto-Regressive Conditional Heteroskedasticity) model is an approach for modelling two variances over a specified observation time. The ARCH model is denoted by:

$$\boldsymbol{Y_t \,|\, F_{t-1} \sim N(0, h_t)}$$

$$\boldsymbol{h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 + \cdots + \alpha_p Y_{t-p}^2}$$

In this model, we assume that $\boldsymbol{E(Y_t) = 0}$, and the conditional mean is limited to a specific period. h_t refers to conditional variance, and it is assumed to be non-negative. Moreover, the conditional distribution is always assumed to be a normal distribution, such as a t-distribution or other asymmetric distributions. The conditional variance in the ARCH model depends on the squared returns, but not on the heteroskedasticity. However, in the GARCH (Generalized Auto-Regressive Conditional Heteroskedasticity) model, heteroskedasticity of the variance is considered. Other than that, the GARCH model can be rewritten as an ARMA model of centralized square of returns, whereas the ARCH model is not directly associated with the ARMA model.

The conditional distribution is assumed to be normal, and the mean is assumed to be zero in the GARCH model. The model is denoted by:

$$\boldsymbol{Y_t \mid F_{t-1} \sim N(0, h_t)}$$

$$\boldsymbol{h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 + \cdots + \alpha_p Y_{t-p}^2 + \beta_1 h_{t-1} + \cdots + \beta_q h_{t-q}}$$

Although the conditional distributions for both ARCH and GARCH models are normal, the unconditional distribution for both models are non-normal, and the observations are correlated but not independent.

In Chapter-6, some of the extensions of the GARCH model are briefly discussed. Some of the extensions include:

- GARCH-t models:

  GARCH-t model is also a symmetric distribution, but its moment exists if the order is smaller than the degree of freedom of t-distribution. For example, a $\boldsymbol{t_5}$ distribution only has finite moments until order 4, whereas $\boldsymbol{t_7}$ distribution only has finite moments until order 6. The GARCH-t model is denoted by:

$$\boldsymbol{Y_t \sim GARCH(p,q), Y_t = n_t \sqrt{h_t}}$$

$$\boldsymbol{h_t = \alpha_0 + \Sigma_{i=1}^{p} \alpha_i Y_{t-i}^2 + \Sigma_{j=1}^{q} \beta_j h_{t-j}}$$

The GARCH-t model has an additional non-integer parameter denoted by $\boldsymbol{m}$, known as the shape parameter.

- APARCH model:

Changing the formulation of volatility can alter the GARCH model. APARCH stands for the asymmetric power of ARCH model because an asymmetry exists in the effect of negative or positive returns. Moreover, the formula for conditional variance also varies for different returns. The APARCH model is denoted by:

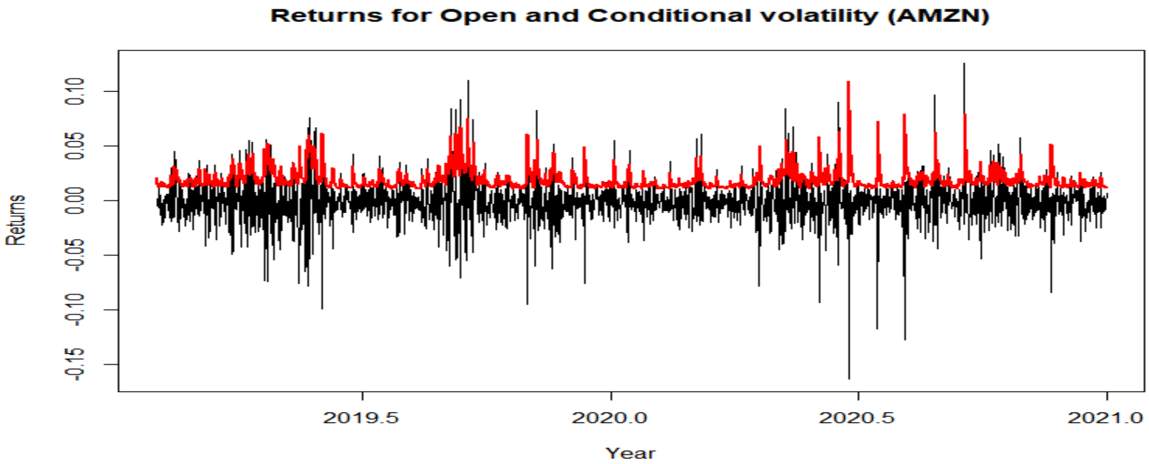
$$\boldsymbol{Y_t \sim APARCH(p,q), Y_t = n_t \sqrt{h_t}}$$

$$\boldsymbol{Y_t = n_t \sigma_t, \sigma_t^{\delta} = \alpha_0 + \Sigma_{i=1}^{p} \alpha_i (\lvert Y_{t-i} \rvert - y_i Y_{t-i})^{\delta} + \Sigma_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta}}$$

$\boldsymbol{\delta}$ in the model refers to Power of absolute returns, but not necessarily for squared returns. Moreover, the coefficient varies for positive or negative returns. The APARCH class includes several other extensions of the GARCH model with different values of δ (1 or 2). Unlike the ARMA time series models, GARCH models takes the extent of noise ϵ_t into consideration to portray a simulated path with its past values[4]. Therefore, periods exhibiting greater volatility will immediately be followed by fluctuations with significantly smaller amplitude. Additionally, large absolute values tend to become

[4] GARCH Models (Page 19)

clustered rather than being uniformly distributed. The permuted coefficients α and β have no effect on the variance in the simulated sequence but has an effect on higher-order moments.



The GARCH model can also be used to estimate the Conditional Standard deviation of a return series. The above plot of Amazon US return series portrays the conditional volatility using the GARCH (1,1) model. The black line represents returns, while the red line represents conditional volatility.

In Chapter-7, we learn about the different approaches to risk measurement based on loss distribution in time-series analysis. With different Risk Measurement approaches, we can predict extreme losses of risk capital. The two most widely used loss functions for risk measurement are VaR (Value at Risk) and ES (Expected Shortfall). The VaR is as a quantile of loss function with probability α, where the value of α is between 0 and 1. It is defined as the maximum expected loss of a portfolio over a specified time horizon assuming a particular confidence interval. Although VaR is a good measure for risks, it cannot accurately quantify tail behaviors of loss distributions and can sometimes give similar values. In those cases, the use of ES is more appropriate. ES is defined as the average loss when VaR is surpassed. It can help obtain vital information regarding frequency and size of losses. Regardless of the value of α, the value of ES is always greater the VaR.

$$\boldsymbol{VaR_\alpha = q_\alpha(F_L) = F_L(\alpha)}$$

$$\boldsymbol{ES_\alpha = E(L \mid L > VaR_\alpha)}$$

Chapter-8 focusses on modelling HF (High Frequency Data). HF data generally refers to datasets that record several observations of data in a financial market in a particular day, and the durations between the observations are no longer equidistant. The number of observations in such datasets are usually very high. A variation of HF data is the is UHF (Ultra High Frequency) data, that records all observations in a particular time frame. Some of the practical applications of HF and UHF data includes estimating the realized volatility and realized correlations, modeling transaction durations and so on.

2.2 a)

$$\boldsymbol{Y_t \mid F_t \sim N(0, h_t)}$$

$$\boldsymbol{h_t = 1 + 0.3Y_{t-1}^2 + 0.67h_{t-1}}$$

- Stationary Condition:

$$\boldsymbol{\Sigma_{i=1}^{p}\alpha_i + \Sigma_{j=1}^{q}\beta_j \ < \ 1}$$

0.3 + 0.67 = 0.97 < 1

As the 0.97 is less than 1, we can conclude that the GARCH model is stationary and has finite variance. Therefore, we can estimate the unconditional variance.

- Estimating Unconditional Variance:

$$\boldsymbol{Var(Y_t) = \frac{\alpha_0}{1 - \Sigma_{i=1}^{p}\alpha_i - \Sigma_{j=1}^{q}\beta_j}}$$

$$\boldsymbol{Var(Y_t) = \frac{1}{1 - 0.3 - 0.67} = 33.33}$$

- Fourth Moment Condition:

$$\boldsymbol{3\alpha_1^2 + 2\alpha_1\beta_1 + \beta_1^2 \ < 1}$$

$$\boldsymbol{3 * 0.3^2 + 2 * 0.3 * 0.67 + 0.67^2 \ = 1.1209 < 1}$$

As the estimated value of 1.1209 is greater than 1, the fourth moment condition has not been met for this GARCH (1,1) model.

- Estimating Fourth Moment:

$$\boldsymbol{E(Y_t)^4 = \frac{3\alpha_0^2\,(1 + \ \alpha_1 + \ \beta_1)}{(1 - \alpha_1 - \beta_1)(1 - \beta_1^2 - 2\,\alpha_1\beta_1 - 3\alpha_1^2\ )}}$$

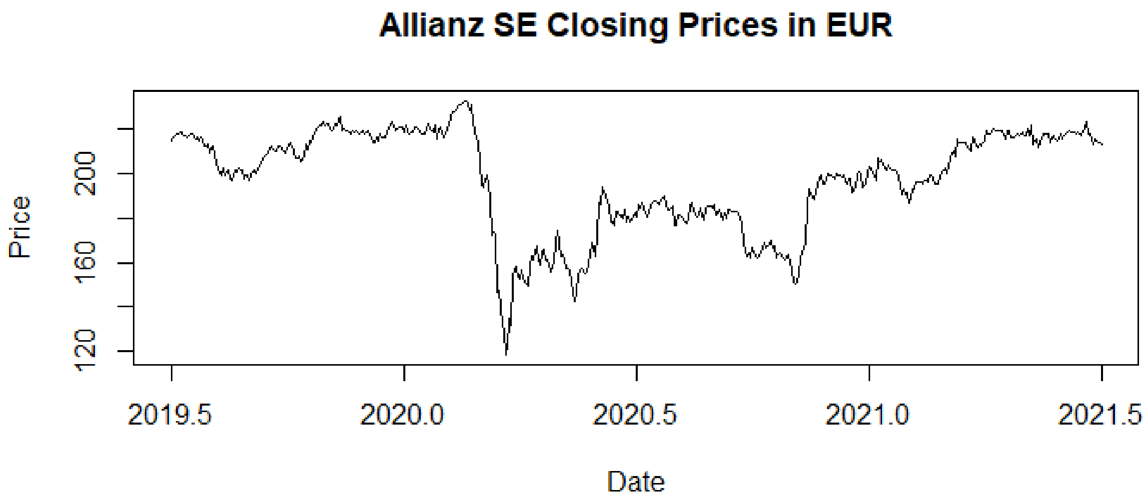
$$\boldsymbol{E(Y_t)^4 = \frac{3 * 1^2\,(1 + \ 0.3 + \ 0.67)}{(1 - 0.3 - 0.67)(1 - 0.67^2 - 2 * 0.3 * 0.67 - 3 * 0.3^2)} = \ -1629.4}$$

As the condition for finite fourth moment is not met, we obtain a negative expected value, making the estimation undefined.

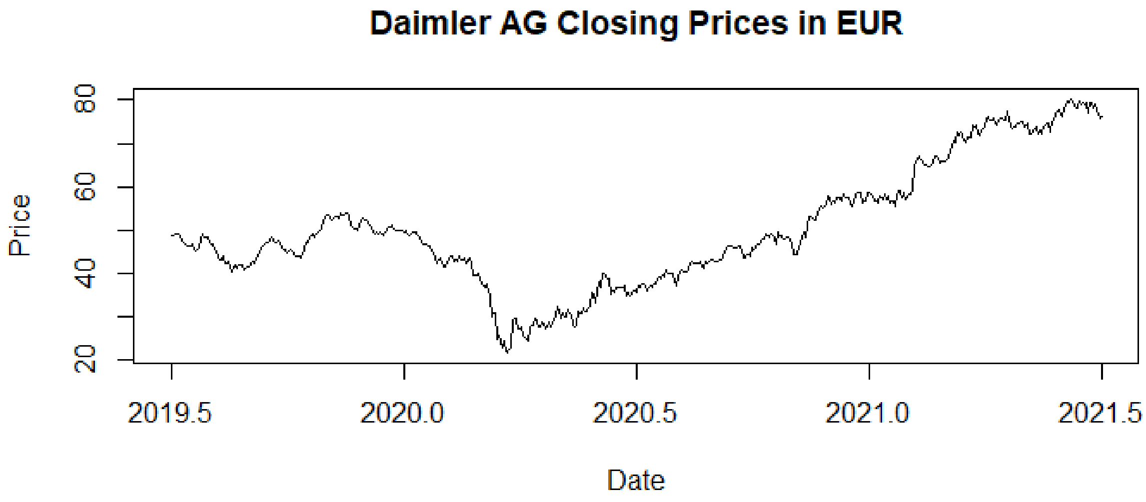
<u>Task 3 – Risk Management</u>

<u>3.1)</u>

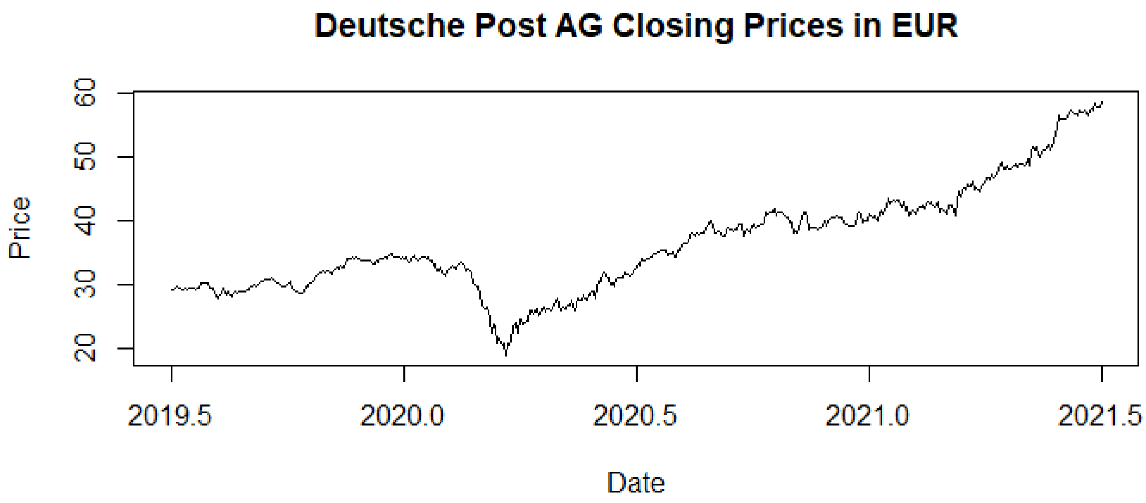
To fulfill the requirements of the task, we have chosen four companies which are represented in the DAX 30. The companies' shares are all traded on the Frankfurt Stock Exchange and are listed there in the same currency (Euro). These same basic conditions for all four companies create a common basis that enables an undistorted comparison of the values. Nevertheless, we have chosen companies from four different sectors to get a diversified view of the market and to highlight differences where necessary. We deliberately set the time horizon of two years from 07/01/2019 to 06/30/2021 so that we could reflect in our data the period before the Covid-19 pandemic, the rapid fall during it and the first recovery period afterwards. We chose to plot the stock prices in four separate charts so that the graphs would not be distorted by the different levels of stocks in a common scale. The order of the companies is chosen alphabetically and has no further meaning for our analysis.



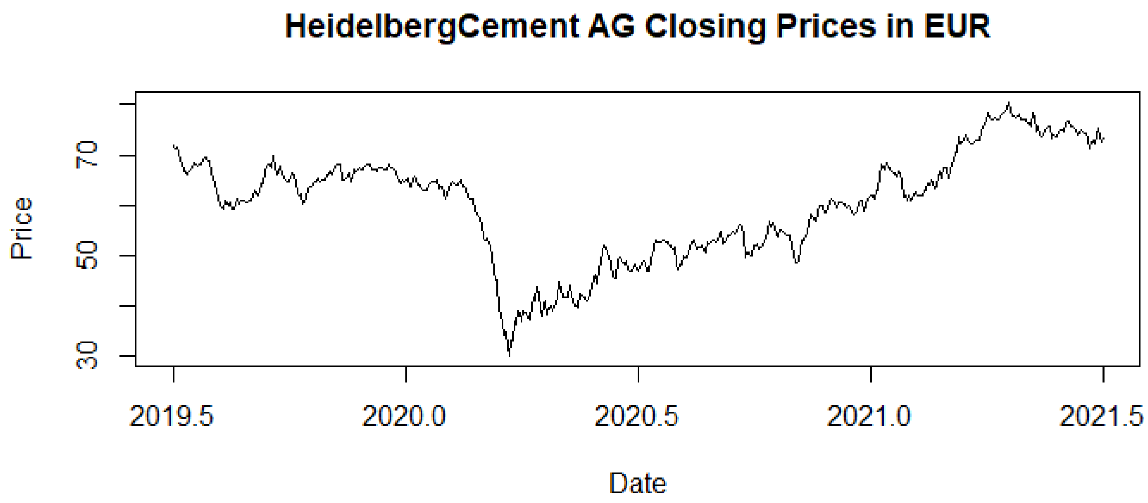
Allianz SE is a german insurance company headquartered in Munich, which operates worldwide and has several subsidiaries.



Dailmer AG is a german automotive group headquartered in Stuttgart, which, among other things, sells cars worldwide under the well-known name Mercedes-Benz.

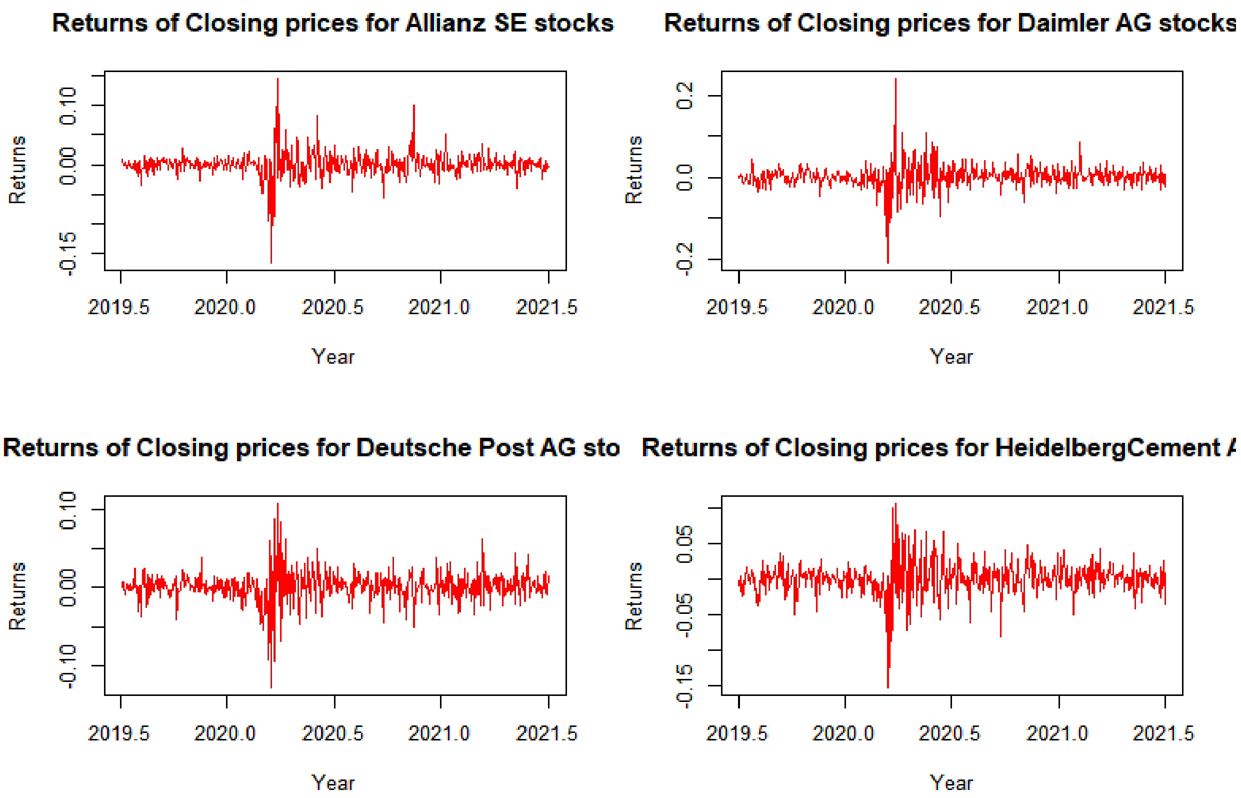


Deutsche Post AG is a logistics group headquartered in Bonn, which operates internationally under the name DHL.



HeidelbergCement AG is a German building materials group that is one of the world's leading companies in its sector.

<u>3.2)</u>



The above plot highlights the log return series for the closing price of all four chosen companies. Here, we can see that there is a distinguishable volatility in returns for all companies in the period between the start of 2020 and May 2020. This could be explained by the onset of the COVID-19 pandemic from March. However, the patterns of volatility in the return series for all companies differ with varying degrees from the onset of the pandemic till May 2021. Looking at the different GARCH (1,2) and APARCH (1,2) models for the four companies, we see that the GARCH (1,2) and APARCH (1,2) models for Allianz SE has the lowest BIC value. Therefore, these models are optimal. The models are as follows:

Companies	GARCH (BIC values)	APARCH (BIC Values)
Allianz SE	-5.4204	-5.426607
Daimler AG	-4.709687	-4.770112
Deutsche Post AG	-5.330374	2.733233
Heidelberg Cement AG	-4.902902	-3.791708

I. GARCH (1,2) model for Allianz SE return series:

$$Y_t = 1.301e^{-4} + E_t\sqrt{h_t}, E_t \sim N(0,1)$$

$$h_t = 1.405e^{-5} + 0.287\, Y_{t-1}^2 + 0.7167\, h_{t-1} + 1.0e^{-8} h_{t-2}$$

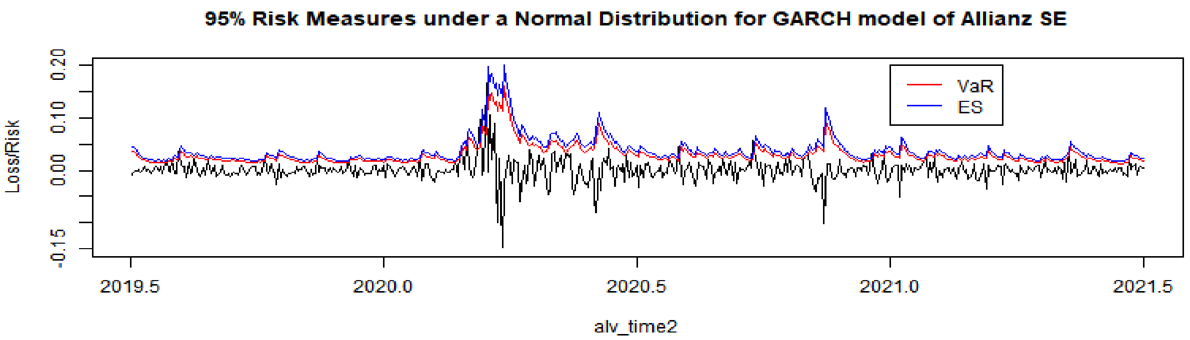
II. APARCH (1,2) model for Allianz SE return series:

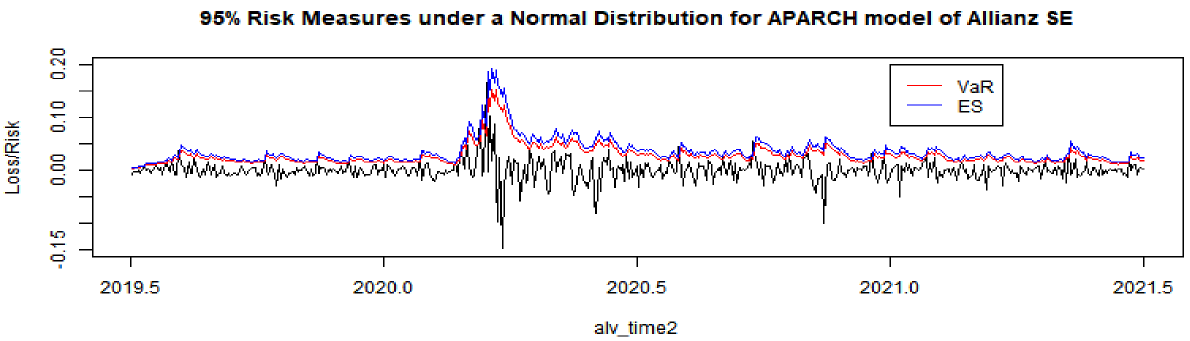
$$Y_t = -1.301e^{-4} + E_t\, \sigma_t, E_t \sim N(0,1)$$

$$\sigma_t^{1.1252} = 4.493e^{-4} + 0.2434(\,|Y_{t-1}| - 0.3665\, Y_{t-1})^{1.1252} + 0.7814\sigma_{t-1}^{1.1252} + 1.0e^{-8}\sigma_{t-2}^{1.1252}$$

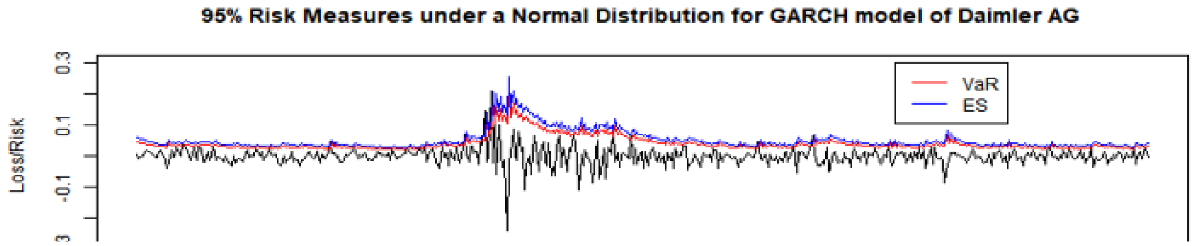
3.3)

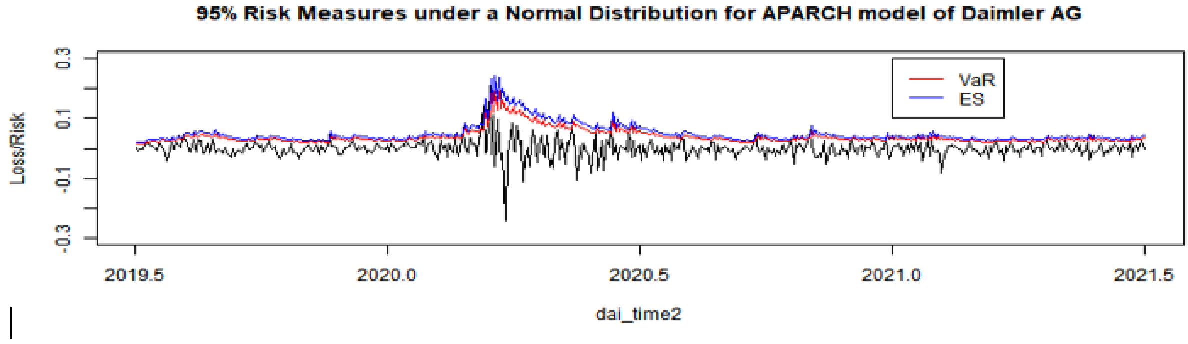
- Plot of return series for Allianz SE with VaR and ES:



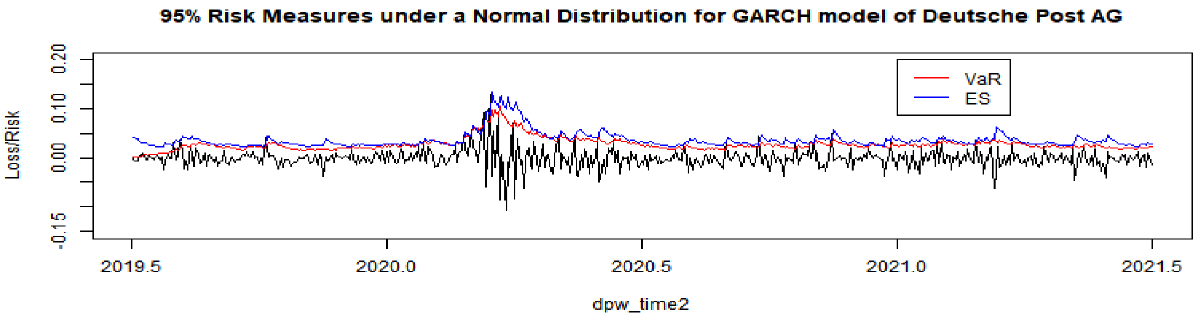


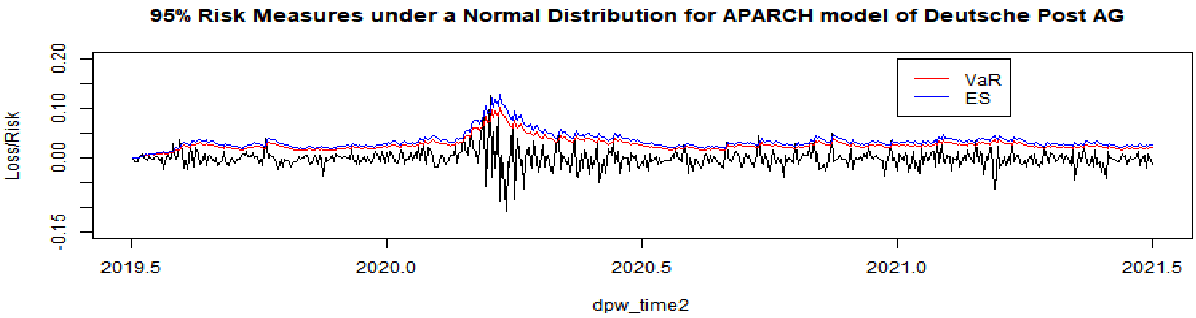
- Plot of return series for Daimler AG with VaR and ES:



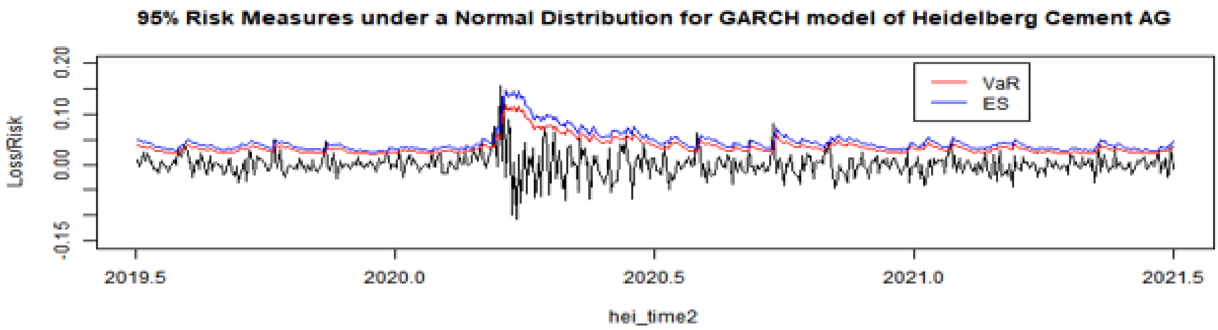


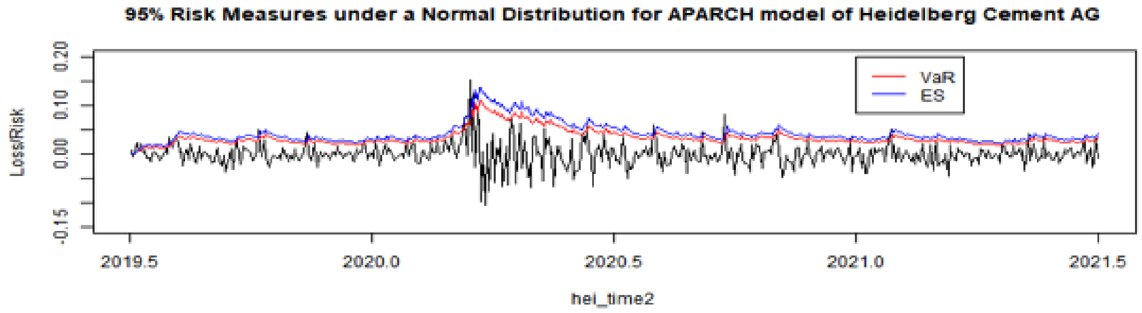
- Plot of return series for Deutsche Post AG with VaR and ES:





- Plot of return series for Heidelberg Cement AG with VaR and ES:





3.4)

Estimate of the total number of exceptions (Point-over-threshold) for GARCH and APARCH model of all companies:

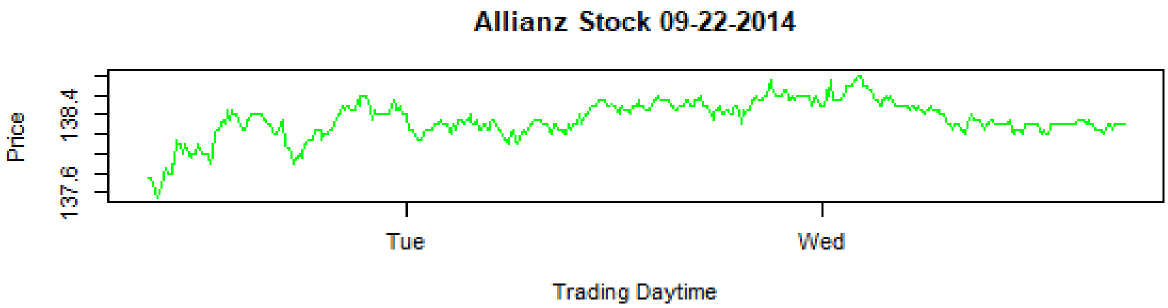
Variables	POT Values	Assigned Traffic Light
POT_VaR_n95_garch_alv	0.1662169	Green
POT_VaR_n95_aparch_alv	0.2945409	Green
POT_ES_n95_garch_alv	0.01768877	Green
POT_ES_n95_aparch_alv	0.009545255	Green
POT_VaR_n95_garch_dai	0.1662169	Green
POT_VaR_n95_aparch_dai	0.07904212	Green
POT_ES_n95_garch_dai	0.000391434	Green
POT_ES_n95_aparch_dai	0.0001419223	Green
POT_VaR_n95_garch_dpw	0.8575038	Green
POT_VaR_n95_aparch_dpw	0.9775878	Yellow
POT_ES_n95_garch_dpw	0.07904212	Green
POT_ES_n95_aparch_dpw	0.07904212	Green
POT_VaR_n95_garch_hei	0.7520121	Green
POT_VaR_n95_aparch_hei	0.9264001	Green
POT_ES_n95_garch_hei	0.004818366	Green
POT_ES_n95_aparch_hei	0.1172943	Green

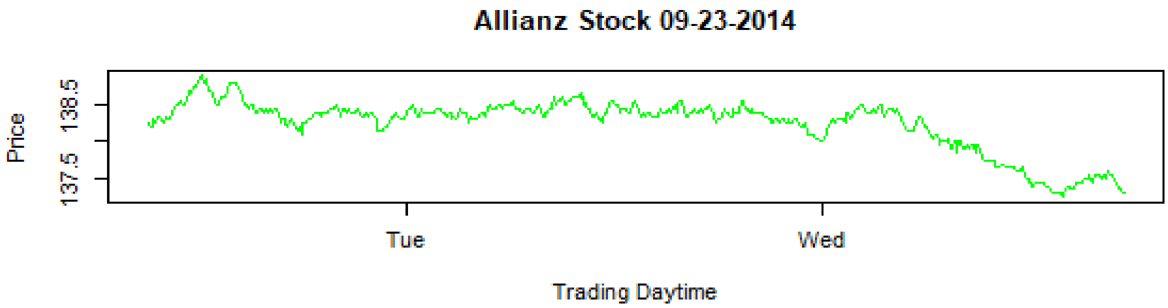
Among all the companies, the APARCH model for Deutsche PostAG was assigned a Yellow traffic light after VaR Risk Measurement was conducted. All other companies and corresponding GARCH and APARCH models have been assigned Green traffic light for all Risk Measurement approaches.

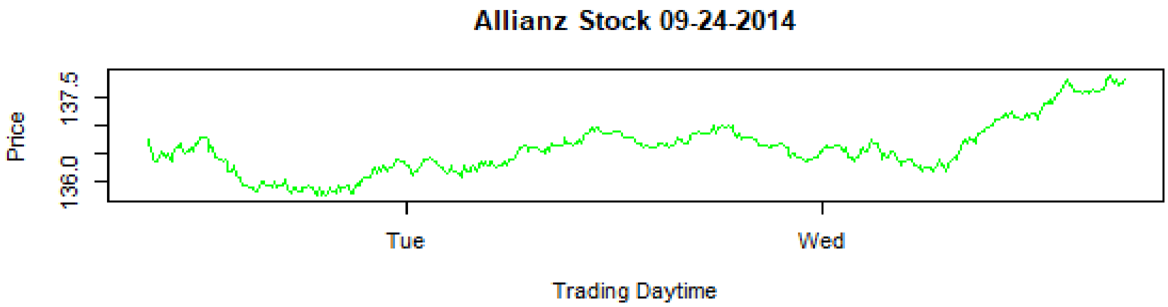
Task 4 - High Frequency Financial Data:

4.1)

We used the dataset of Allianz stock prices for the year of 2014 and this contain 1 minute high frequential data. On our project only 3 days data will be analysed. Allianz market trading time is between 9:30 to 17:30.

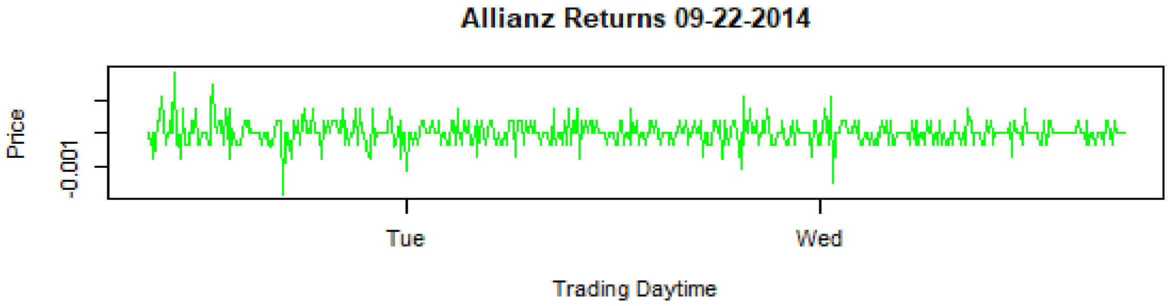


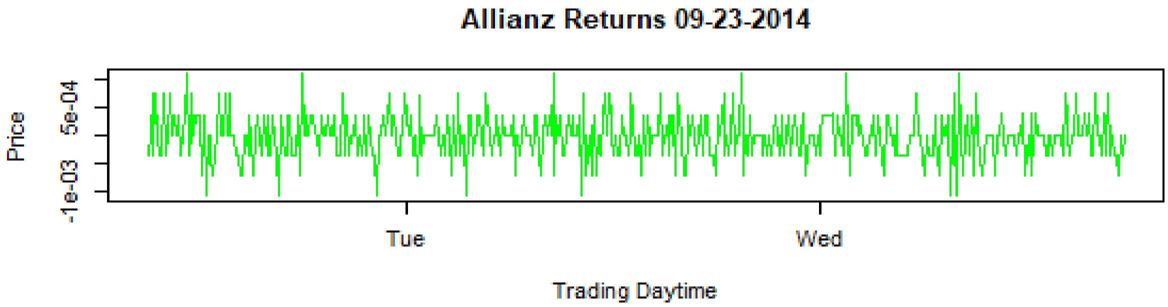


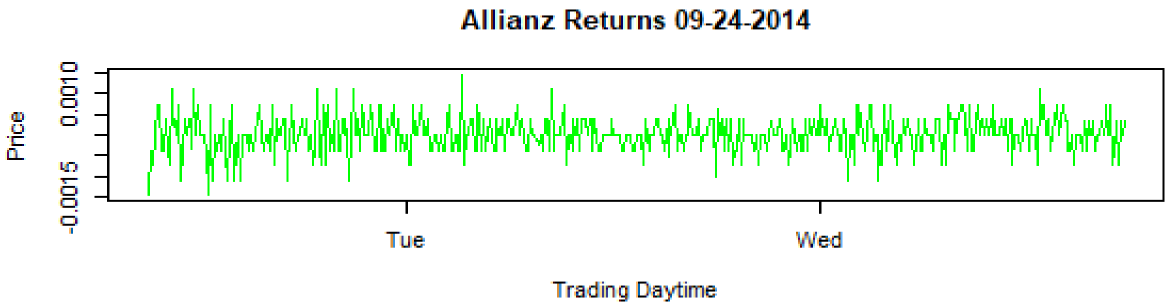


4.2)

The returns for these three days are given below:







Realized Volatility from the returns:

22/09/2014	**1.29e-07**
23/09/2014	1.52e-07
24/09/2014	1.86e-07

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