

# TB Vaccination

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## Data creation

```
library(data.table)
library(pomp)
library(ggplot2)
library(rstan)

rstan_options(auto_write = TRUE)
options(mc.cores = parallel::detectCores())

#everyone enters on jan 1
#results are for dec 31
newPeople <- data.table(id=1:1000,year=2000,age=1,timeNoVax=0,timeWithVax=0,vaccinated=0,ltbi=0,atbi=0,)

pomp::bake("data/peopleNoVax.RDS",{
  set.seed(4)
  peopleNoVax <- vector("list",length=17)
  peopleNoVax[[1]] <- newPeople[1,] [-1,]
  for(i in 1:17){
    # NO VACCINATIONS
    if(i!=1) peopleNoVax[[i]] <- copy(peopleNoVax[[i-1]])
    peopleNoVax[[i]][,age:=age+1]
    peopleNoVax[[i]] <- rbind(peopleNoVax[[i]],newPeople)
    peopleNoVax[[i]][,year:=2000+i]

    N <- nrow(peopleNoVax[[i]])
    peopleNoVax[[i]][sample(1:N,size=round(N*0.050)),ltbi:=1]
    peopleNoVax[[i]][,timeNoVax:=timeNoVax+1]

    Nltbi <- sum(peopleNoVax[[i]]$ltbi)
    peopleNoVax[[i]][atbi==1,ltbi:=0]
    peopleNoVax[[i]][atbi==1,ageAtTB:=age]
    peopleNoVax[[i]][,atbi:=0] # people with ATBI go to the doctor and get cured last year
    peopleNoVax[[i]][sample(which(peopleNoVax[[i]]$ltbi==1),size=round(Nltbi*0.1)),atbi:=1]
  }
  peopleNoVax
}) -> peopleNoVax

pomp::bake("data/peopleWithVax.RDS",{
  set.seed(4)
  peopleWithVax <- vector("list",length=17)
  peopleWithVax[[1]] <- newPeople[1,] [-1,]
  for(i in 1:17){
    # WITH VACCINATIONS, BUT STOPPING THEM AFTERWARDS
    if(i!=1) peopleWithVax[[i]] <- copy(peopleWithVax[[i-1]])
    peopleWithVax[[i]][,age:=age+1]
    peopleWithVax[[i]] <- rbind(peopleWithVax[[i]],newPeople)
```

```

peopleWithVax[[i]][,year:=2000+i]
N <- nrow(peopleWithVax[[i]])
if(i>=10){
  peopleWithVax[[i]][sample(1:N,size=round(N*0.050)),ltbi:=1]
  peopleWithVax[[i]][,timeNoVax:=timeNoVax+1]
} else {
  peopleWithVax[[i]][sample(1:N,size=round(N*0.050)),ltbi:=1]
  peopleWithVax[[i]][,timeWithVax:=timeWithVax+1]
}

Nltbi <- sum(peopleWithVax[[i]]$ltbi)
peopleWithVax[[i]] <- peopleWithVax[[i]][atbi==0]
peopleWithVax[[i]][atbi==1,ltbi:=0]
peopleWithVax[[i]][atbi==1,ageAtTB:=age]
peopleWithVax[[i]][,atbi:=0] # people with ATBI go to the doctor and get cured last year
peopleWithVax[[i]][sample(which(peopleWithVax[[i]]$ltbi==1),size=round(Nltbi*0.1)),atbi:=1]
}
peopleWithVax
}) -> peopleWithVax

```

## Detecting LTBI without vaccination

Lets try an easy model, detecting LTBI in unvaccinated people.

We have a Poisson binomial distribution, where the probability of person  $i$  not having LTBI is:

$$Pr(Y_i = 0|p, T_i) = (1 - p)^{T_i}$$

And the probability of person  $i$  having LTBI is:

$$Pr(Y_i = 1|p, T_i) = 1 - Pr(Y_i = 0|p, T_i)$$

```

stanData <- copy(peopleNoVax[[17]])
stanData[ageAtTB>0,ltbi:=1]
stanData <- stanData[,.(R=N),by=.(timeNoVax,timeWithVax,ltbi)]
data = list(N=nrow(stanData),
           R=stanData$R,
           y=stanData$ltbi*stanData$R,
           timeNoVax=stanData$timeNoVax)

stan_code =
data {
  int<lower=0> N;
  int R[N];
  int y[N];
  vector[N] timeNoVax;
}
parameters {
  real<lower=0,upper=0.5> thetaNoVax;
}
model {
  thetaNoVax ~ beta(0.05, 1);

```

```

for (n in 1:N){
  y[n] ~ binomial(R[n], 1 - ((1-thetaNoVax)^timeNoVax[n]));
}
"
pomp::bake("results/noVaxLTBI.RDS",{
  stan(model_code=stan_code,
        model_name="noVaxLTBI",
        data=data,
        iter=10000, chains=4, init=0, seed=4)
}) -> fit
summary(fit)$summary

##               mean      se_mean       sd      2.5%
## thetaNoVax  4.955827e-02 7.385874e-06 0.0006438153 4.830014e-02
## lp__       -9.870137e+03 7.422981e-03 0.7257379087 -9.872220e+03
##             25%       50%       75%      97.5%
## thetaNoVax  4.912818e-02     0.0495498  4.998994e-02  5.083482e-02
## lp__       -9.870297e+03 -9869.8573114 -9.869680e+03 -9.869630e+03
##             n_eff      Rhat
## thetaNoVax 7598.342 1.001092
## lp__       9558.790 1.000284

```

## Detecting LTBI with vaccination

Lets try a harder model, detecting LTBI when vaccination originally exists, and is then phased out.

We have a Poisson binomial distribution, where the probability of person not  $i$  having LTBI is:

$$Pr(Y_i = 0|p_w, p_n, TW_i, TN_i) = (1 - p_w)^{TW_i} \times (1 - p_n)^{TN_i}$$

And the probability of person  $i$  having LTBI is:

$$Pr(Y_i = 1|p_w, p_n, TW_i, TN_i) = 1 - Pr(Y_i = 0|p_w, p_n, TW_i, TN_i)$$

```

stanData <- copy(peopleWithVax[[17]])
stanData[ageAtTB>0,ltbi:=1]
stanData <- stanData[,.(R=.N),by=.(timeNoVax,timeWithVax,ltbi)]
data = list(N=nrow(stanData),
           R=stanData$R,
           y=stanData$ltbi*stanData$R,
           timeNoVax=stanData$timeNoVax,
           timeWithVax=stanData$timeWithVax
          )

stan_code = "
data {
  int<lower=0> N;
  int R[N];
  int y[N];
  vector[N] timeNoVax;
  vector[N] timeWithVax;

```

```

}

parameters {
  real<lower=0,upper=0.5> thetaNoVax;
  real<lower=0,upper=0.5> thetaWithVax;
}
model {
  thetaWithVax ~ beta(0.05, 1);
  thetaNoVax ~ beta(0.05, 1);

  for (n in 1:N){
    y[n] ~ binomial(R[n], 1-((1-thetaWithVax)^timeWithVax[n])*((1-thetaNoVax)^timeNoVax[n]));
  }
}

pomp::bake("results/withVaxLTBI.RDS",{
  stan(model_code=stan_code,
        model_name="withVaxLTBI",
        data=data,
        iter=10000, chains=4, init=0, seed=4)
}) -> fit
summary(fit)$summary

##               mean      se_mean       sd      2.5%
## thetaNoVax   4.023537e-02 1.193584e-05 0.0009857973 3.830165e-02
## thetaWithVax 1.068818e-02 2.480706e-05 0.0020852201 6.601235e-03
## lp__         -7.533216e+03 1.260362e-02 1.0357132482 -7.535946e+03
##                   25%          50%          75%         97.5%
## thetaNoVax   3.957005e-02 4.023253e-02 4.089666e-02 4.218267e-02
## thetaWithVax 9.275577e-03 1.069942e-02 1.207870e-02 1.482700e-02
## lp__         -7.533628e+03 -7.532893e+03 -7.532477e+03 -7.532201e+03
##               n_eff      Rhat
## thetaNoVax   6821.334 1.000376
## thetaWithVax 7065.667 1.000509
## lp__         6752.873 1.000107

```

## Detecting ATBI without vaccination

Lets try a hard model, detecting ATBI in unvaccinated people. ATBI is incident TB and detected in the year that it occurs.

We have a Poisson binomial distribution, where the probability of person  $i$  not having ATBI is:

$$\begin{aligned}
 Pr(ATBI_i = 0 | p_{atbi}, p_{ltbi}, T_i) &= Pr(LTBI_i = 0 | p_{atbi}, p_{ltbi}, T_i) + Pr(LTBI_i = 1 \text{ without activation} | p_{atbi}, p_{ltbi}, T_i) \\
 &= (1 - p_{ltbi})^{T_i} + \sum_{t=1}^{T_i} Pr(\text{Incident LTBI infection at time } t, \text{ no activation}) \\
 &= (1 - p_{ltbi})^{T_i} + \sum_{t=1}^{T_i} (1 - p_{ltbi})^{t-1} \times p_{ltbi} \times (1 - p_{atbi})^{T_i-t+1}
 \end{aligned}$$

And the probability of person  $i$  having LTBI is:

$$Pr(ATBI_i = 1 | p_{atbi}, p_{ltbi}, T_i) = 1 - Pr(ATBI_i = 0 | p_{atbi}, p_{ltbi}, T_i)$$

```

cases <- list()
for(i in 1:14){
  cases[[i]] <- peopleNoVax[[i]][atbi==1]
}
cases <- rbindlist(cases)
controls <- peopleNoVax[[17]][atbi==0 & ageAtTB==0]

cases[,ageAtTB:=0]
controls[,ageAtTB:=0]

stanData <- rbind(controls[,.(R=.N),by=.(timeNoVax,timeWithVax,atbi)],cases[,.(R=.N),by=.(timeNoVax,ti
data = list(N=nrow(stanData),
            R=stanData$R,
            y=stanData$atbi*stanData$R,
            timeNoVax=stanData$timeNoVax)

stan_code = "
data {
  int<lower=0> N;
  int timeNoVax[N];
  int R[N];
  int y[N];
}
parameters {
  real<lower=0,upper=0.5> ltbiNoVax;
  real<lower=0,upper=0.5> atbiNoVax;
}
model {
real ps;
ltbiNoVax ~ beta(0.05, 1);
atbiNoVax ~ beta(0.05, 1);

{
  int replicates;
  int personTime;
  real prob_ltbi_no_tb;

  for (n in 1:N){
    prob_ltbi_no_tb = (1-ltbiNoVax)^timeNoVax[n];
    personTime = timeNoVax[n];
    replicates = R[n];

    for(t in 1:personTime){
      prob_ltbi_no_tb = prob_ltbi_no_tb + ((1-ltbiNoVax)^(t-1))*ltbiNoVax*((1-atbiNoVax)^(personTime-t));
    }
    y[n] ~ binomial(R[n], 1-prob_ltbi_no_tb);
  }
}
"

```

```

pomp::bake("results/noVaxATBI.RDS", {
  stan(model_code=stan_code,
        model_name="noVaxATBI",
        data=data,
        iter=10000, chains=4, init=0, seed=4)
}) -> fit
summary(fit)$summary

##               mean     se_mean      sd      2.5%      25%
## ltbiNoVax    0.1377507 0.14723672 0.2082359  1.680510e-02 1.735962e-02
## atbiNoVax    0.3782281 0.14725045 0.2082572  1.703154e-02 3.665443e-01
## lp__      -6737.5267238 0.01304701 1.0621821 -6.740365e+03 -6.737951e+03
##             50%      75%      97.5%      n_eff      Rhat
## ltbiNoVax    0.0176942   0.1357622   0.4998091   2.000225 262.854545
## atbiNoVax    0.4982943   0.4993698   0.4999491   2.000262 166.143214
## lp__      -6737.1952609 -6736.7598293 -6736.4833275 6627.899108   1.000793

```