Homework 1 - Applied Machine Learning

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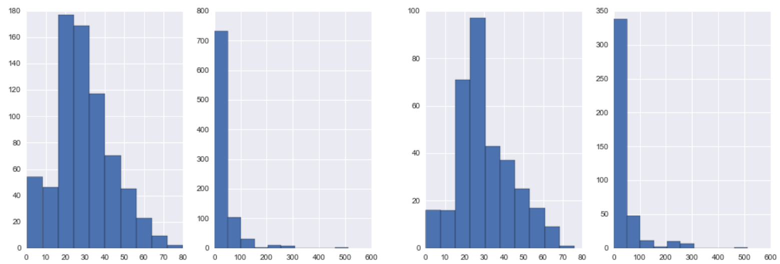

(a) training data (b) test data

Figure 1: Age (left) and fare (right) distributions

2. Titanic Disaster

For this assignment, we competed in the Titanic dataset challenge on Kaggle. Our goal was to train a logistic regression classifier that uses passenger data, such as age and gender, to predict who survived.

It turns out that the `scikit-learn` package contains a `LogisticRegression` model that we can use for this very purpose. Before we can train such a model, however, we need to clean the data provided by Kaggle, fill in missing values, and select a combination of features for the model to examine.

Preparation of the training and test data occurs within `munge_data()` in the attached code. For the logistic regression training and predicting to work, input data must be numeric, so much of our preparation involves mapping categorical data (i.e. `Sex` and `Port of Embarkation`) to numeric values (`Sex_enum` and `Embarked_enum`).

Next, we fill in missing data - in particular, `Age` and `Fare` - with a dummy value derived from the rest of the dataset. To determine the best dummy values, it is helpful to examine the attached histograms in Figure 1. We see that the distributions of ages and fares are both highly skewed, so a median value might be more appropriate than a mean. One sophisticated approach, borrowed from Kaggle's "Getting Started With Python II" guide, is to use the median fare/age for a given gender and passenger's socioeconomic status, which will hopefully better represent typical passengers.

A couple of features in the dataset were ignored, namely `Ticket` and `Cabin`. The difficulty of mapping these categorical values, which seem irrelevant, would likely outweigh any benefits. Additionally, the values for `Cabin` were largely missing.

Following the Kaggle article, we constructed two new features from the

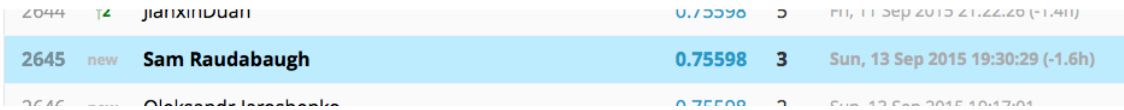

Figure 2: Kaggle result for Titanic challenge

dataset: `FamilySize`, obtained simply by adding together the number of siblings, spouses, parents and children aboard, and `Age*Class`, obtained by multiplying age by the passenger's socioeconomic status (1 for upper class, 2 for middle class, 3 for lower class), since both older and lower class passengers had a lower likelihood of survival.

At each decision step, we test the usefulness of a combination of features with a 10 fold cross-validation technique, which uses the code from Michael Wilber's lecture working with the `cross_validation` module in `scikit-learn` as a starting point. Using `FamilySize` and `Age*Class` in place of `Age` both produced higher cross-validation scores.

We also experimented with pulling out the titles of each passenger's name and categorizing them in different ways. The first strategy was a binary approach with only pedestrian and non-pedestrian titles, while the second strategy used 5 title types: pedestrian, honorary, academic, military, or religious. The latter approach resulted in higher cross-validation scores, though neither approach seemed to affect the Kaggle test results. For reference, the attached code makes use of the second strategy, defined in `get_title_type()`.

After running our solution on the test dataset, we submitted the results to Kaggle and received a score of 0.75598 as shown in Figure 2. For future improvements, experimenting with another model, such as a random forest classifier, is recommended.

Written 1. Variance of a sum

Show var[X+Y] = var[X] + var[Y] + 2cov[X,Y].

$var[X+Y] = cov[X+Y, X+Y]$

By definition of covariance:
$var[X+Y] = E[(X+Y)(X+Y)] - E[X+Y]E[X+Y]$

$var[X+Y] = E[(X^2 + 2XY + Y^2] - (E[X] + E[Y])(E[X] + E[Y])$

$var[X+Y] = E[X^2] - E[X]E[X] + E[Y^2] - E[Y]E[Y] + E[XY] - E[X]E[Y] + E[XY] - E[X]E[Y]$

$var[X+Y] = cov[X,X] + cov[Y,Y] + cov[X,Y] + cov[X,Y]$

$var[X+Y] = var[X] + var[Y] + 2cov[X,Y]$

Written 2. Bayes rule for medical diagnosis

Let D refer to the event that you have the disease, and TP refer to the event that you tested positive. Then:

$P(D|TP) = \frac{P(D)P(TP|D)}{P(TP)}$

Given $P(D) = 0.0001$ and $P(TP|D) = 0.99$, we only need to derive $P(TP)$.

$P(TP) = P(TP|D)P(D) + P(TP|D')P(D')$

Finally, with $P(TP|D') = 0.01$ and $P(D') = 0.9999$:

$P(D|TP) = \frac{P(D)P(TP|D)}{P(TP|D)P(D)+P(TP|D')P(D')} = \frac{0.0001*0.99}{0.99*0.0001+0.01*0.9999} = 0.0098$

Written 3a. Derivative of sigmoid function

Given $\sigma(a) = \frac{1}{1+e^{-a}} = (1 + e^{-a})^{-1}$

$\frac{d\sigma(a)}{da} = -(1 + e^{-a})^{-2} * -e^{-a}$

$\frac{d\sigma(a)}{da} = \frac{e^{-a}}{(1+e^{-a})^2}$

$\frac{d\sigma(a)}{da} = \frac{1}{1+e^{-a}} \left(\frac{e^{-a}}{1+e^{-a}} \right)$

$\frac{d\sigma(a)}{da} = \frac{1}{1+e^{-a}} \left(\frac{1+e^{-a}}{1+e^{-a}} - \frac{1}{1+e^{-a}} \right)$

$\frac{d\sigma(a)}{da} = \sigma(a) \left(1 - \sigma(a)\right)$

Written 3b. Gradient of log likelihood

Given $l(\beta) = \sum_{i=1}^{N}\{y_i logp(x_i;\beta) + (1-y_i)log(1-p(x_i;\beta))\}$

$\frac{\delta l(\beta)}{\delta\beta} = \sum_{i=1}^{N} \frac{\delta}{\delta\beta} y_i logp(x_i;\beta) + \sum_{i=1}^{N} \frac{\delta}{\delta\beta}(1-y_i)log(1-p(x_i;\beta))$

Derive individual $\frac{\delta}{\delta\beta}$ terms:

$\frac{\delta}{\delta\beta} y_i logp(x_i;\beta) = y_i \cdot \frac{1}{p(x_i;\beta)} \cdot \frac{\delta}{\delta\beta} p(x_i;\beta)$

$\frac{\delta}{\delta\beta} y_i logp(x_i;\beta) = x_i y_i \cdot \frac{1}{p(x_i;\beta)} \cdot p(x_i;\beta)(1-p(x_i;\beta)) = x_i y_i(1-p(x_i;\beta))$

and:

$\frac{\delta}{\delta\beta}(1-y_i)log(1-p(x_i;\beta)) = (1-y_i)\frac{1}{1-p(x_i;\beta)} \cdot \frac{\delta}{\delta\beta}(1-p(x_i;\beta))$

$\frac{\delta}{\delta\beta}(1-y_i)log(1-p(x_i;\beta)) = \frac{1-y_i}{1-p(x_i;\beta)} \cdot -x_i p(x_i;\beta)(1-p(x_i;\beta))$

$\frac{\delta}{\delta\beta}(1-y_i)log(1-p(x_i;\beta)) = -x_i(1-y_i)p(x_i;\beta)$

Plugging the terms into the original gradient, we simplify to the desired form:

$\frac{\delta l(\beta)}{\delta\beta} = \sum_{i=1}^{N} x_i y_i(1-p(x_i;\beta)) - x_i(1-y_i)p(x_i;\beta)$

$\frac{\delta l(\beta)}{\delta\beta} = \sum_{i=1}^{N} x_i(y_i - y_i p(x_i;\beta) - (1-y_i)p(x_i;\beta)) = \sum_{i=1}^{N} x_i(y_i - p(x_i;\beta))$

Written 3c. Proof that log likelihood Hessian is positive definite

In order to prove that X^TWX is positive definite, we must show that the scalar $v^TX^TWXv > 0$ for every v, which is any non-zero column vector of real numbers.

Because W is a diagonal matrix of non-negative real values w_i, we can absorb W into X and refactor X^TWX into the following form:

$$X^TWX = Q^TQ$$

where each element of Q is equal to the element at the same i^{th} row and j^{th} column in X, multiplied by $\sqrt{w_i}$.

Plugging this new form into the expression v^TX^TWXv, we find that the expression can be represented as the inner product of Qv:

$$v^TX^TWXv = v^TQ^TQv = (Qv)^TQv = ||Qv||$$

which is a scalar representing the magnitude of a vector in Euclidian space, therefore it cannot be negative. Assuming nondegeneracy in the data (no duplicate inputs x_i), it also cannot be 0. Therefore:

$$||Qv|| = v^TX^TWXv > 0$$