

iterative time complexity tricks

① for nested loop, if the iterators are independent, figure out each loop's time and multiply.

② dependant \rightarrow you can estimate the answer for the largest value of input which is independent, and try to figure out the dependent loop and multiply.

OR, you can use summation of those two loops to get the answer.

\Rightarrow
$$\begin{array}{l} x = n \\ \text{while } x > 0: \\ \quad x = x/2 \end{array} \quad \left. \vphantom{\begin{array}{l} x = n \\ \text{while } x > 0: \\ \quad x = x/2 \end{array}} \right\} O(1)$$

 $\{x = n - x = 0\}$

\Rightarrow
$$\begin{array}{l} y = n \\ \text{while } (y > 0): \\ \quad y = y/2 \end{array} \quad \left. \vphantom{\begin{array}{l} y = n \\ \text{while } (y > 0): \\ \quad y = y/2 \end{array}} \right\} O(\log(n))$$

\Rightarrow
$$\begin{array}{l} i = 1 \\ \text{while } i < n: \\ \quad i = i + i \end{array} \quad \left. \vphantom{\begin{array}{l} i = 1 \\ \text{while } i < n: \\ \quad i = i + i \end{array}} \right\} O(\log(n))$$

 $i = i + i \Rightarrow i = 2i \Rightarrow i \times 2 = 2$

$\Rightarrow x = 0$

$$\text{for } (i = 1; x < n; i++) \{ \quad \left. \vphantom{\text{for } (i = 1; x < n; i++) \{}} \right\} O(n)$$

 $x = x + i$

Explanation:

for 1 iteration $\Rightarrow 1$.

$$2^{\text{nd}} \Rightarrow 1+2$$

$$3^{\text{rd}} \Rightarrow 1+2+3$$

$$k^{\text{th}} \Rightarrow 1+2+3+\dots+k \Rightarrow \frac{k(k+1)}{2}$$

for the loop to break in k^{th} step, $\frac{k(k+1)}{2} \gg n$
 $\Rightarrow k = \sqrt{n}$

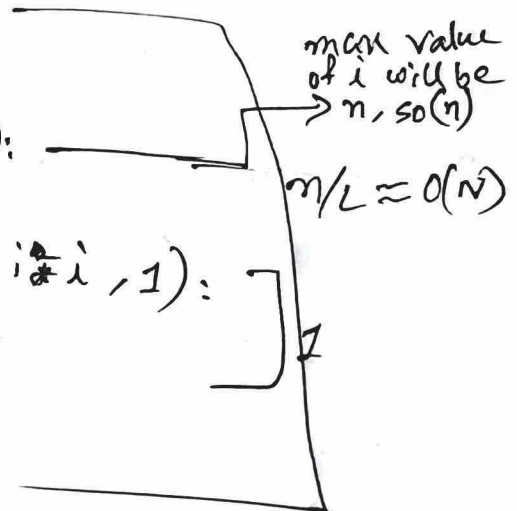
\Rightarrow for i in $\pi(0, n, L)$

for j in $\pi(0, i, 1)$:

for k in $\pi(0, i, 1)$:

break

$\rightarrow O(n^2)$



if we use sum formula for dependant loop:

$$\text{it } 1 \Rightarrow 1$$

$$\text{it } 2 \Rightarrow 1+2$$

$$\text{it } 3 \Rightarrow 1+2+3$$

$$\text{it } n \Rightarrow 1+2+3+\dots+n$$

$$\Rightarrow \frac{n(n+1)}{2}$$

so, outer two loop combines as $\frac{n(n+1)}{2}$ and the innermost loop (1) makes $\Rightarrow O\left(\frac{n(n+1)}{2} + 1\right) \Rightarrow O(n^2)$

divide = 1

for ~~a~~ in $\text{range}(n, 0, -2)$:

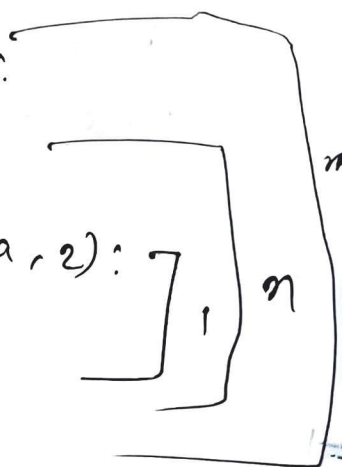
for b in $\text{range}(a, 0, -1)$:

for c in $\text{range}(0, a, 2)$:

d = d // (a/b)

break

same like the previous. $O(N^2)$



Sum - 23 mid

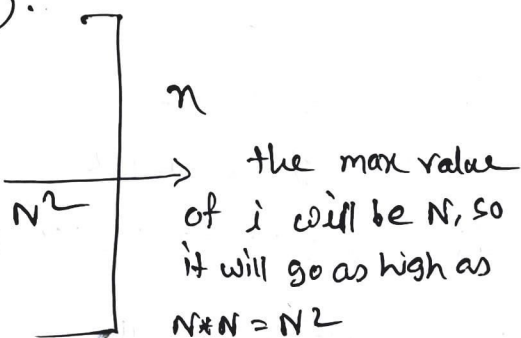
by estimation

for i in $\text{range}(1, n)$:

j = 1

while j < i:

j += 1



$O(N^3)$

using summation:

it 1 $\Rightarrow 1$

it 2 $\Rightarrow 1 + 4$

it 3 $\Rightarrow 1 + 4 + 9$

it n $\Rightarrow 1 + 4 + 9 + \dots + n^2$

$\Rightarrow 1^2 + 2^2 + 3^2 + \dots + n^2$

$$\rightarrow \frac{n(n+1)(2n+1)}{6} \\ \Rightarrow O(n^3)$$

for i in $\mathcal{R}(1, n)$:

$j = 1$

while $j \neq i < i$:

$j++$

if $j^2 = i$, i
will run at
max times.

then j will

run at ~~at~~ max

\sqrt{n} times

(\sqrt{n})

n times

$$O(n\sqrt{n})$$

Summation:

it 1 $\Rightarrow 1$

it 2 $\Rightarrow 1+2$

it 3 $\Rightarrow 1+2+2$

it 4 $\Rightarrow 1+2+2+2$

it 5 $\Rightarrow 1+2+2+2+3$

if you watch carefully,

for each n , its running $N^{10.5}$ times. $[\sqrt{N} = N^{0.5}]$

the series is like:

$$T(N) = 1^{0.5} + 2^{0.5} + 3^{0.5} + 4^{0.5} + 5^{0.5} + \dots N^{0.5}$$

$$\Rightarrow \sqrt{1} + \sqrt{2} + \sqrt{3} + \dots \sqrt{N} \Rightarrow$$

\Rightarrow there is no direct way of doing it, the integration result for this:

$$\int \sqrt{n} \, dn = \frac{2n\sqrt{n}}{3} + C$$

$$\Rightarrow \frac{2}{3} (n\sqrt{n}) + C$$

$$\text{so, } T(n) = \frac{2}{3} \cdot n^{3/2}$$

$$\Rightarrow O(n\sqrt{n})$$

—

tips: you can use multiplying methods since

we are dealing with asymptotic notation, usually don't need the exact value. for example:

for i in $\mathcal{R}(1, n)$: — loop 1

$j = 1$

while $j * j < i$: — loop 2

$j++$

first, independent iterator i can go max n .

so, loop 1 = N , dependant iterator can go as high

as $j * j < i$; since i can go n ; $j = \sqrt{n}$; so loop 2 =

\sqrt{n} . ~~so~~ so, loop 1 * loop 2 $\Rightarrow O(N\sqrt{N})$