Graph Theory

Problem #1:
Lecture material:
https://drive.google.com/file/d/1OkLO91GV4M23e-1EtvFr9yeGWgk6V91q/view?usp=sharing

Some general questions regarding the basic terminology of Graph Theory

1. In a directed graph, if there is an edge from Node A to Node B, and not the other way around, can A and B be called adjacent/neighbor to each other?
2. In an undirected graph of n vertices, how many edges can we construct without allowing loops (edge to self) and multiple edges between a pair of vertices?
3. Same question for directed graphs.
4. How many edges can we construct in an undirected graph with n vertices without introducing any cycle? Loops can also be considered as cycles.
5. Same question for directed graphs.
6. Take any graph, try to construct a path that starts from a vertex, then visits all the edges of the graph exactly once. It will not be possible in all graphs, only in some having a special property. What's that property?
7. Take any graph, try to construct a path that starts from a vertex, then visits all other vertices of the graph exactly once (some edges might be unvisited). Like the previous question, it will not be possible in all graphs. Just find some examples with both cases.
8. Analyze the efficiency of using adjacency list over adjacency matrix for different type of graphs: undirected, directed, weighted, unweighted, sparse, dense…

Problem #2:
Suppose we have the following social network, Friendsbook where:

Person A is friends with Person B, C, D, and E.
Person B is friends with Person A, C, and F.
Person C is friends with Person A, B, D, and G.
Person D is friends with Person A, C, E, G, and H.
Person E is friends with Person A, D, and H.
Person F is friends with Person B and G.
Person G is friends with Person C, D, F, and H.
Person H is friends with Person D, E, and G.

A. Draw a graph to represent the network using each user as a node or vertex and the friendships as edges between the nodes.
B. Create the adjacency list and adjacency matrix for the graph representation
C. For each pair of users, calculate the number of friends they have in mutual.
D. In Friendsbook, a user can share posts. Suppose the CEO of Friendsbook hired you for developing their post display system. You were given the task to implement a system that displays posts from users who are at most two degrees of connection away from the

one who is scrolling his feed. After looking at the requirements you came up with an idea to implement the system using the BFS algorithm. Now, for person F, simulate your algorithm. Can F see the posts of all other users in his feed?

Problem #3:

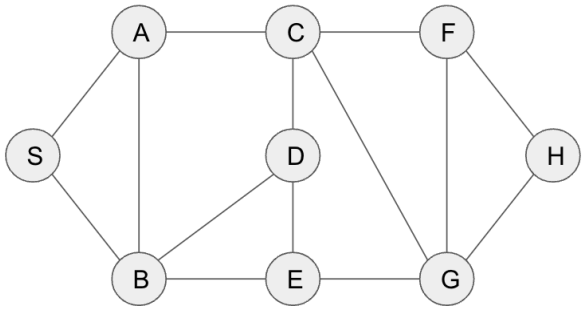
The following information is stored after implementing the DFS algorithm on an undirected graph.

Nodes	1	2	3	4	5	6	7
Parent							
Starting Time	8	11	5	6	4	1	2
Finish Time	9	12	10	7	13	14	3
Distance from Root							

A. Draw the DFS tree.
B. Fill up the empty rows of the table.

Problem #4:

Consider this graph:



A. Justify $\sum_{v \in V} deg(v) = 2m$, where m = number of edges in the graph.
B. Calculate the number of additional edges that can be added between the nodes randomly, keeping the graph 'simple' (without adding multiple edges between any two nodes, loops in a single node).