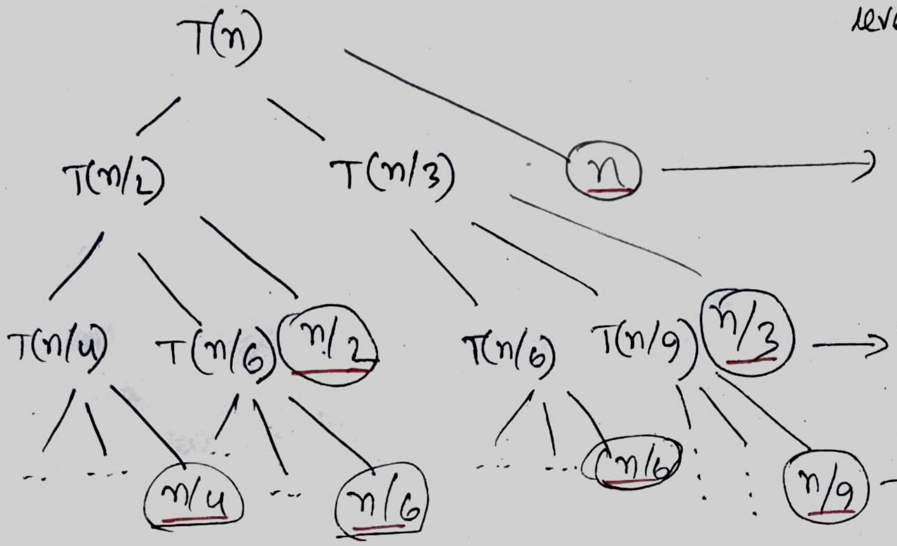
$T(n) = T(n/2) + T(n/3) + O(N)$



Instructions per level

$\rightarrow n$

$\rightarrow n/2 + n/3 = \frac{5n}{6} \Rightarrow n * \left(\frac{5}{6}\right)$

$\rightarrow n/4 + n/6 + n/6 + n/9$

$= \frac{25n}{36} \Rightarrow n * \left(\frac{5}{6}\right)^2$

. . . . next level $\Rightarrow n * \left(\frac{5}{6}\right)^3$

so, at k^{th} level $\Rightarrow n * \left(\frac{5}{6}\right)^K$

total instructions $\Rightarrow n + \frac{5}{6} * n + \left(\frac{5}{6}\right)^2 * n + \left(\frac{5}{6}\right)^3 * n + \cdots \left(\frac{5}{6}\right)^4 * n$

$$\Rightarrow n\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^3 + \cdots \left(\frac{5}{6}\right)^4\right)$$

[G.P. series formula $\Rightarrow a + ax + ax^2 + ax^3 + \cdots ax^n$

$\Rightarrow \frac{a(x^n - 1)}{x - 1}$ / or, $\frac{a(1 - x^n)}{1 - x}$]

depending if $x > 1$ or < 1; we use 2nd one since $x < 1$; $a = 1$, $x = \frac{5}{6}$

So, total instructions,

$$n * \left[\frac{1\left(1-(5/6)^k\right)}{(1-5/6)} \right] \quad \text{—— (1)}$$

now, since, the recursive function was, $T(n) = T(n/2) + T(n/3) + n$

we can safely assume that $T(n/3)$ will branch out and go to the base case long before the $T(n/2)$ term. so the number of level (k) depends on the $T(n/2)$ mostly.,

so, $T(n/2^k) = 1$ [last level, k^{th} level]

$\Rightarrow k = \log_2 N$.

putting it in eq^n 1 $\Rightarrow T(n) = n * \left[\frac{1-(5/6)^{\log_2 N}}{1-5/6} \right]$

we can neglect the constant $(1-5/6)$ & the value $(5/6)^{\log n}$ will be very small. so we decide to ignore this as a constant term as it will have almost no effect compared to n.

so, $T(n) = n * O(1)$

$\Rightarrow O(n)$