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section: 03

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Answer to the ques. no - 1

(a)

$$T(n) = 2T(n/2) + 1/n$$

$$= 2T(n/2) + n^{-1}$$

$\therefore$ $a=2$, $b=2$, $K=-1$

Now,

$$\log_b a = \log_2 2 = 1 > K = -1$$

$\therefore$ $\log_b a > K$.

$\therefore$ The time complexity $= \Theta(n^{\log_b a})$

$$= \Theta(n^1)$$

$$= \Theta(n)$$

(answer)

(b)

$$T(n) = 2T(n/3) + n$$

comparing it with $T(n) = aT(n/b) + \Theta(n^k \log^p n)$

$a=2$, $b=3$, $K=1$, $p=0$

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$\log_b a = \log_3 2 = 0.631 < K = 1$

$\therefore \log_b a < K$ and $P = 0$

$\therefore$ Time complexity $= \Theta(n^K \log^P n)$

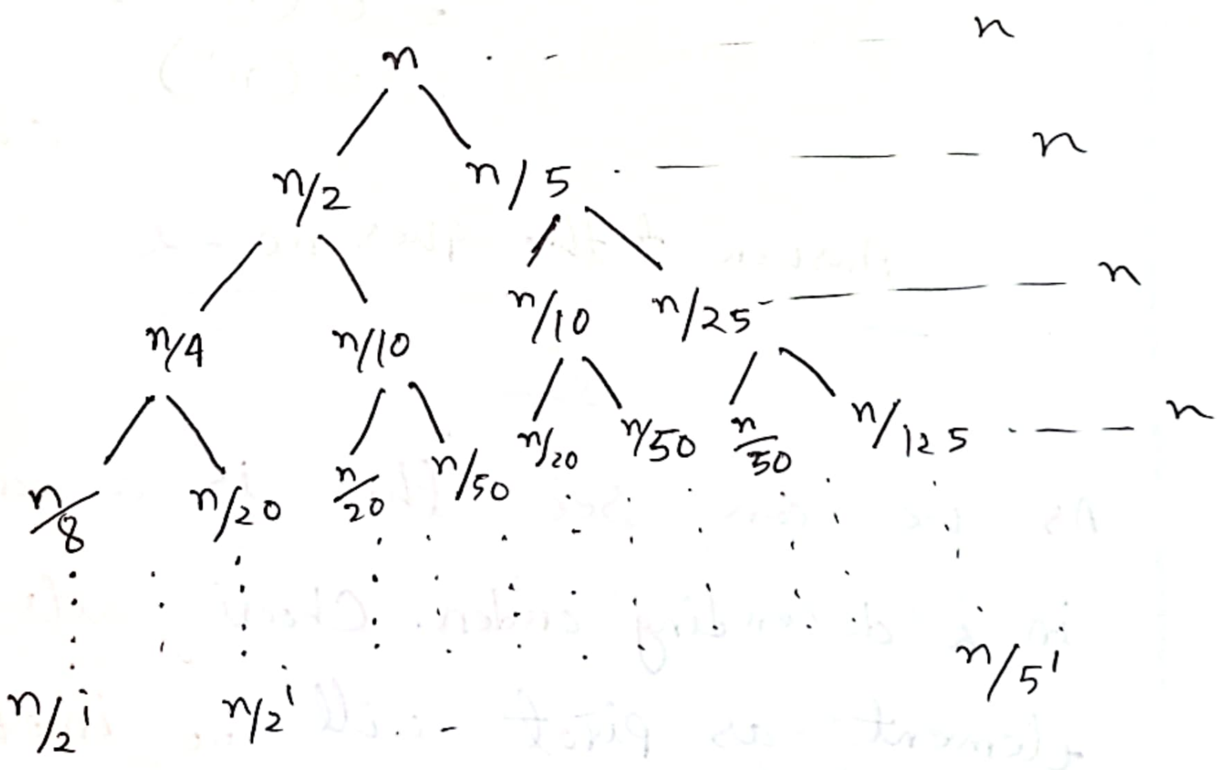
$= \Theta(n^1 \log^0 n)$

$= \Theta(n)$

(answer)

(C)

$T(n) = T(n/2) + T(n/5) + n$



$\therefore \frac{n}{5^i} = 1$

$\therefore i = \log_5(n)$ (answer)

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(d)

$$T(n) = 2T(n/4) + n^2$$

$a = 2, \; b = 4, \; K = 2, \; P = 0$

now,

$\log_b a = \log_4 2 = 0.5 < K = 2$

$\therefore \log_b a < K.$

$\therefore$ Time complexity $= \Theta(n^K \log^P n)$

$= \Theta(n^2 \log^0 n)$

$= \Theta(n^2)$

(answer)

Answer to the ques. no - 2

(a)

As we can see this is a sorted list in a decending order. Choosing its first element as pivot will be inefficient. As we know pivot helps us to divide the

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array into roughly equal halves. But choosing the first/max element of a ~~tis~~ sorted list can't perform the task efficiently causing the time complexity $O(n^2)$ instead of $O(n\log n)$. Meaning it will create a worst case scenario.

(b)

The recurrence relation if we choose the first element,

$$T(n) = T(n-1) + O(n)$$

Now,

$$T(n-1) = T(n-2) + O(n-1)$$
$$T(n-2) = T(n-3) + O(n-2)$$
$$\vdots$$
$$T(2) = T(1) + O(2)$$
$$T(1) = O(1)$$

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Summing the equations,

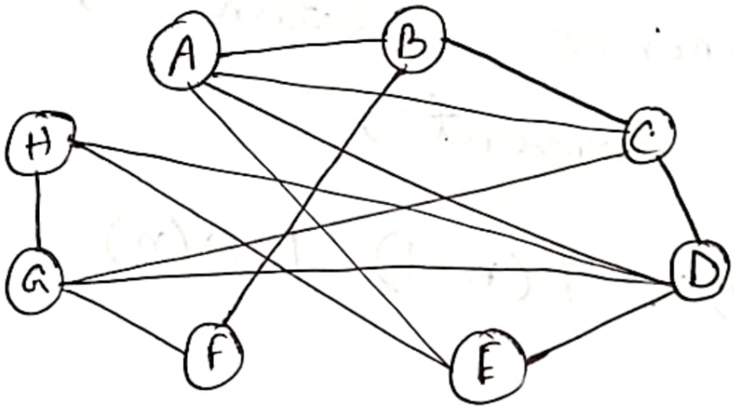
$$T(n) = O(n) + O(n-1) + O(n-2) + \ldots + O(2) + O(1)$$

which eventually ~~ne~~ leads to $O(n^2)$.

which is the worst case complexity of this algorithm.

Answer to the question no – 4

ⓐ



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(b)

Adjancency list —

A → B → C → D → E
B → A → C → F
C → A → B → D → G
D → A → C → E → G → H
E → A → D → H
F → B → G
G → C → D → F → H
H → D → E → G

Adjancy matrix —

	A	B	C	D	E	F	G	H
A	0	1	1	1	1	0	0	0
B	1	0	1	0	0	1	0	0
C	1	1	0	1	0	0	1	0
D	1	0	1	0	1	0	1	1
E	1	0	0	1	0	0	0	1
F	0	1	0	0	0	0	1	0
G	0	0	1	1	0	1	0	1
H	0	0	0	1	1	0	1	0

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(c)

Pairs	Mutual friends
A - B	C
B - F	None
B - C	A
A - C	B, D
A - D	E, C
A - E	D
D - E	A, H
E - H	D
C - D	A, G
D - H	E, G
D - G	H, C
C - G	D
G - H	D
G - F	None

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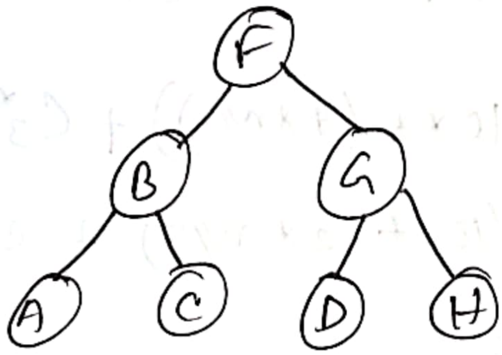
D

If we make a tree which is two degree from F it will be



As we can see E is not two degree away from F, F can't see it's post. So, F can't see everyone's post.

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Answer to the ques. no - 3

(a)

$$A = A_1 \times 10^{2(n/3)} + A_2 \times 10^{n/3} + A_3$$

$$B = B_1 \times 10^{2(n/3)} + B_2 \times 10^{n/3} + B_3$$

(b)

$$A.B = \left(A_1 \times 10^{2(n/3)} + A_2 \times 10^{n/3} + A_3\right) \times \left(B_1 \times 10^{2(n/3)} + B_2 \times 10^{n/3} + B_3\right)$$

$$= A_1 B_1 \times 10^{2.2(n/3)} + A_1.B_2 \times 10^{n} + A_1 B_3 \times 10^{2(n/3)}$$
$$+ A_2 B_1 \times 10^{n} + A_2 B_2 \times 10^{2(n/3)} + A_2 B_3 \times 10^{n/3}$$
$$+ A_3 B_1 \times 10^{2n/3} + A_3 B_2 \times 10^{n/2} + A_3 B_3$$

$$= A_1 B_1 \times 10^{4(n/3)} + (A_1 B_3 + A_3 B_1 + A_2 B_2) \times 10^{2(n/3)}$$
$$+ (A_1 B_2 + A_2 B_1) \times 10^{n} + (A_2 B_3 + A_3 B_2) \times 10^{n/3}$$
$$+ A_3 B_3$$

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(c)

```
def modified_Karatsuba(x, y):
    if x<10 or y<10:
        return x*y
    else:
        n = max(len(str(x)), len(str(y)))
        A1 = x//(10** (2×n//3)
        A2 = (x // (10** n//3)) % (10** n//3)
        A3 = x% ((10** n//3)
        B1 = y//(10** (2* n//3))
        B2 = (y// (10** n//3)) % (10** (n/3)))
        B3 = y% (10** n// 3)
        C4 = modified_Karatsuba (A1, B1)
        C0 = modified_Karatsuba (A3, B3)
        P1 = modified_Karatsuba ((A1+A2+A3),
                                 (B1+B2+B3))
        P2 = modified_Karatsuba( (A1-A2+A3), (B1-B2+B3))
        P3 = modified_Karatsuba((4A1+2A2 +A3), (4B1+2B2+B3))
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$$C_2 = ((P_1 + P_2)/2) - C_0 - C_4$$

$$C_3 = (P_3 - 2P_0 - 14C_4 - 2C_2 + C_0)/6$$

$$C_1 = P_1 - C4 - C3 - C2 - C0$$

return $C_4 * (10 ** (4 * n//3)) + C_3 * (10 ** (3 * n//3)) + C_2 * (10 ** (2 * n//3)) + C_1 * (10 ** n//3)) + C_0$

(ans)

D

Now,

$$T(n) = 5T(n/3) + \theta(n)$$

$$T(n) = aT(n/b) + \theta(n^c \log^d n)$$

$a = 5$, $b = 3$, $c = 1$, $d = 0$

Now,

$\log_b a = \log_3 5 < c$.

$\therefore$ Time complexity $= \theta(n^{\log_b a})$

$= n^{\log_3 5}$

(ans)

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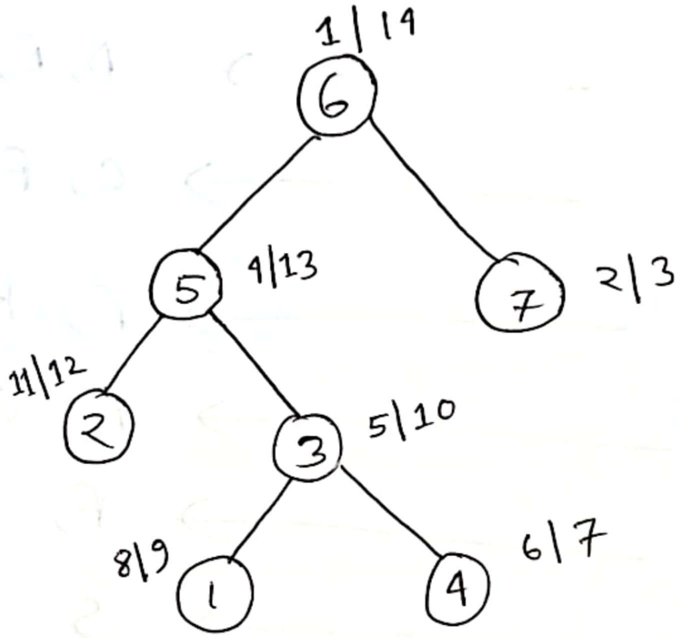
Answer to the ques. no - 5

(a)



(b)

Nodes	1	2	3	4	5	6	7
Parent	3	5	5	3	6		6
starting Time	8	11	5	6	4	1	2
Finish Time	9	12	10	7	13	14	3
Distance from root	3	2	2	3	1	0	1

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Answer to the ques. no – 5

(a)

Here, total ~~eg~~ edge, $m = 14$

$\therefore\ 2m = 2 \times 14 = 28$

Now,

Node		Degree (V)
A	$\longrightarrow$	3
B	$\longrightarrow$	4
C	$\longrightarrow$	4
D	$\longrightarrow$	3
E	$\longrightarrow$	3
F	$\longrightarrow$	3
G	$\longrightarrow$	4
H	$\longrightarrow$	2
S	$\longrightarrow$	2
		Total = 28

$\therefore\ \sum \deg(V) = 2m = 28$

(answer)

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(B)

From		TO
A	$\longrightarrow$	D, E, F, G, H
B	$\longrightarrow$	C, F, G, H
C	$\longrightarrow$	F, H, S
D	$\longrightarrow$	F, G, H, S
E	$\longrightarrow$	F, H, S
F	$\longrightarrow$	S
G	$\longrightarrow$	S
H	$\longrightarrow$	S
S	$\longrightarrow$	None

Total new edge = 22

So we can add 22 new edge.

math matical Proof:

• New edges = $\frac{n(n-1)}{2} - 14 = \frac{9(9-1)}{2} - 14 = 22$

(answer)