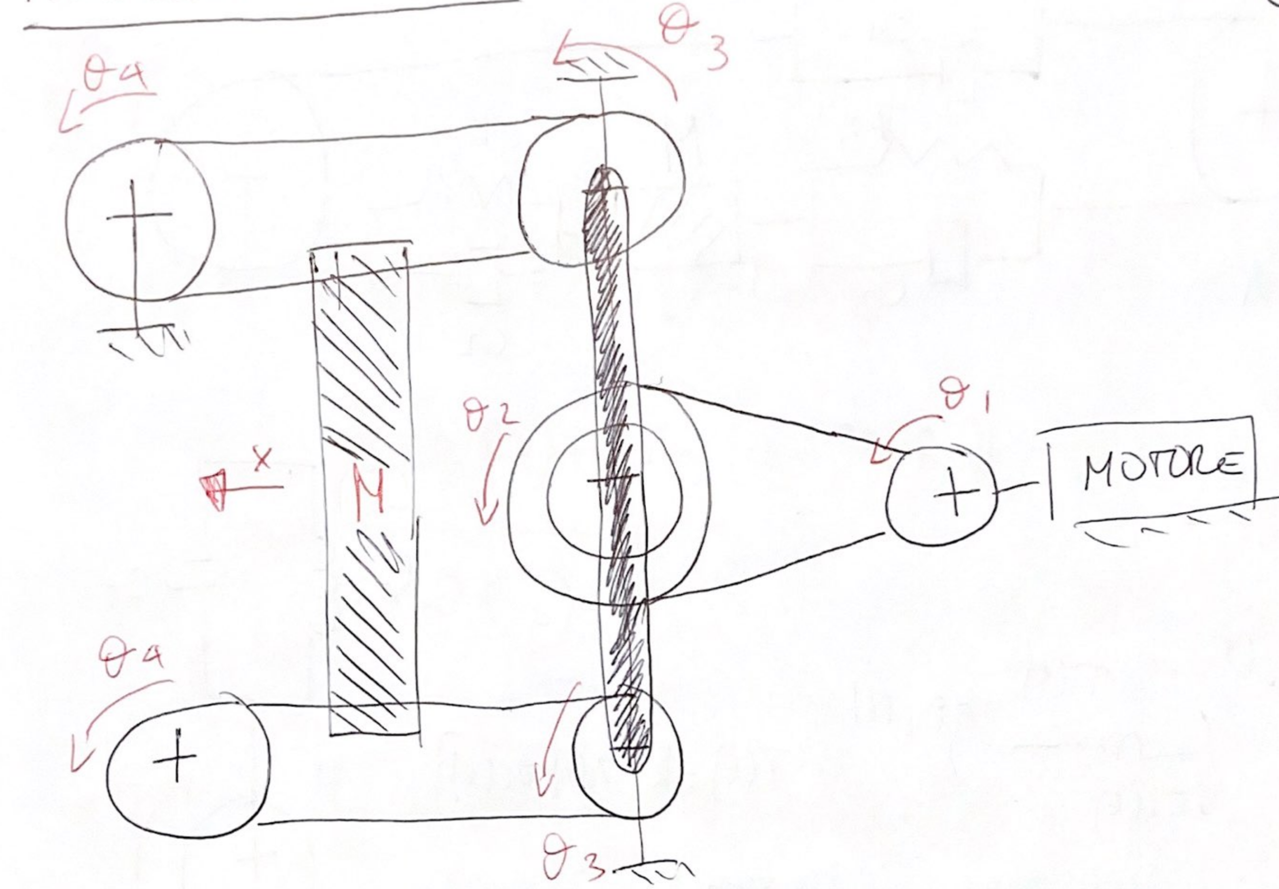
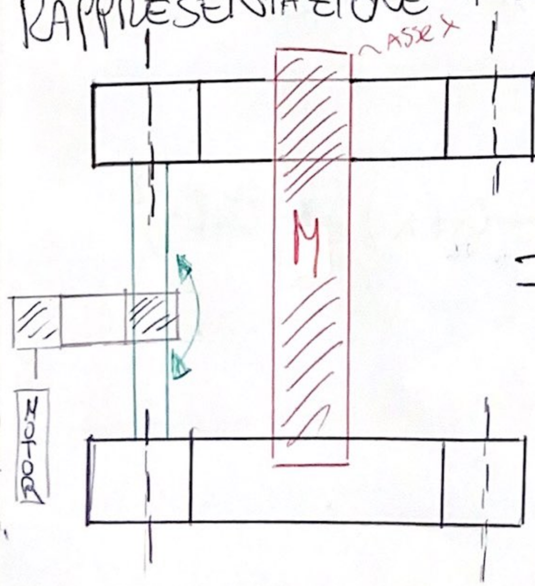
MODELLO ASSE Y

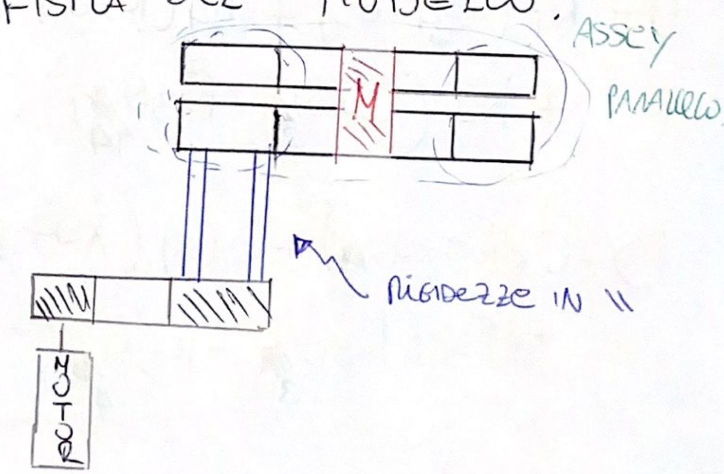


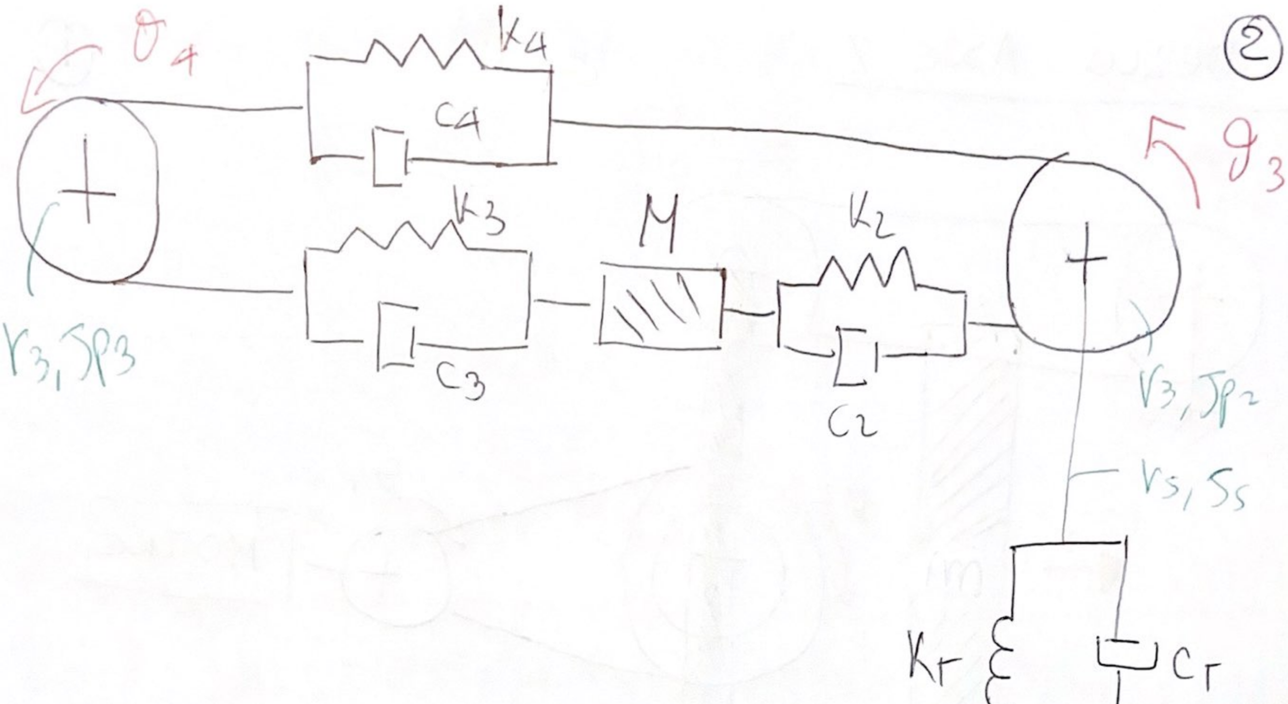
TRATTO ② SIMMETRICO

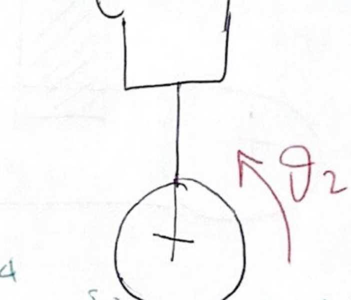
TRATTO ① IDENTICO MODELLO ASSE X, TRATTO ①

RAPPRESENTAZIONE NON FISICA DEL MODELLO:

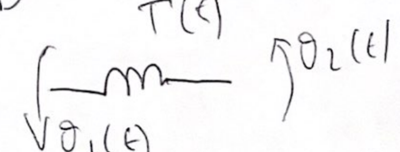








N. B



$$T(t) = k\left[\theta_1(t) - \theta_2(t)\right]$$

{1} PULEGGIA2 {2} PULEGGIA 3 {2} PULEGGIA 4 {1} MASSA {2} ALBERO

$$T = \frac{1}{2} J_{p2}\dot{\theta}_2^2 + J_{p3}\dot{\theta}_3^2 + J_{p3}\dot{\theta}_4^2 + \frac{1}{2} M\dot{x}^2 + J_S\dot{\theta}_2^2$$

{2} RIG. TORS {2} RIG. 2 {2} RIG.3 {2} RIG. 4

$$U = K_T(\theta_3 - \theta_2)^2 + K_2(x)(x - \theta_3 R_3)^2 + K_3(x)(x - \theta_4 R_3)^2 + K_4(R_3\theta_3 - R_3\theta_4)^2$$

$$D = C_T(\dot{\theta}_3 - \dot{\theta}_2)^2 + C_2(x)(\dot{x} - \dot{\theta}_3 R_3)^2 + C_3(x)(\dot{x} - \dot{\theta}_4 R_3)^2 + C_4(R_3\dot{\theta}_3 - R_3\dot{\theta}_4)^2$$

$C_3(x), C_2(x) = C_3, C_2$

$K_3(x), K_2(x) = K_3, K_2$

COORDINATE LIBERE: $x, \theta_2, \theta_3, \theta_4$

$$\frac{\partial T}{\partial x} = 0 \quad , \frac{\partial T}{\partial \dot{x}} = M\dot{x} \quad , \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = M\ddot{x}$$

$$\frac{\partial U}{\partial x} = 2K_2\left(x - \theta_3 R_3\right) + 2K_3\left(x - \theta_4 R_3\right)$$

$$\frac{\partial D}{\partial \dot{x}} = 2C_2\left(\dot{x} - \dot{\theta}_3 R_3\right) + 2C_3\left(\dot{x} - \dot{\theta}_4 R_3\right)$$

$$M\ddot{x} + 2\cdot\left[K_2\left(x - \theta_3 R_3\right) + K_3\left(x - \theta_4 R_3\right) + C_2\left(\dot{x} - \dot{\theta}_3 R_3\right) + C_3\left(\dot{x} - \dot{\theta}_4 R_3\right)\right] = 0$$

$$\frac{\partial T}{\partial \theta_2} = 0, \quad \frac{\partial T}{\partial \dot{\theta}_2} = J_{p2}\dot{\theta}_2 + 2J_5\dot{\theta}_2 = \dot{\theta}_2\left(J_{p2} + 2J_5\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) = \ddot{\theta}_2\left(J_{p2} + 2J_5\right)$$

$$\frac{\partial U}{\partial \theta_2} = -2K_T\left(\theta_3 - \theta_2\right)$$

$$\frac{\partial D}{\partial \dot{\theta}_2} = -2C_T\left(\dot{\theta}_3 - \dot{\theta}_2\right)$$

$$\ddot{\theta}_2\left(J_{p2} + 2J_5\right) - 2\cdot\left[K_T\left(\theta_3 - \theta_2\right) + C_T\left(\dot{\theta}_3 - \dot{\theta}_2\right)\right] = 0$$

$$\frac{\partial T}{\partial \theta_3} = 0 \, , \quad \frac{\partial T}{\partial \dot{\theta}_3} = 2 J_{p_3} \dot{\theta}_3 \quad , \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_3}\right) = \ddot{\theta}_3 \left(2 J_{p_3}\right)$$

$$\frac{\partial U}{\partial \theta_3} = 2 K_T (\theta_3 - \theta_2) - 2 R_3 K_2 (X - \theta_3 R_3) + 2 R_3 K_4 (R_3 \theta_3 - R_3 \theta_4)$$

$$\frac{\partial D}{\partial \dot{\theta}_3} = 2 C_T (\dot{\theta}_3 - \dot{\theta}_2) - 2 R_3 C_2 (\dot{X} - \dot{\theta}_3 R_3) + 2 R_3 C_4 (R_3 \dot{\theta}_3 - R_3 \dot{\theta}_4)$$

$$\ddot{\theta}_3 (2 J_{p_3}) + 2 \cdot \left\{ \left[K_T (\theta_3 - \theta_2) + C_T (\dot{\theta}_3 - \dot{\theta}_2) \right] + R_3 \cdot \left[-K_2 (X - \theta_3 R_3) + K_4 (R_3 \theta_3 - R_3 \theta_4) - C_2 (\dot{X} - \dot{\theta}_3 R_3) + C_4 (R_3 \dot{\theta}_3 - R_3 \dot{\theta}_4) \right] \right\} = 0$$

$$\frac{\partial T}{\partial \theta_4} = 0 \, , \quad \frac{\partial T}{\partial \dot{\theta}_4} = 2 J_{p_3} \dot{\theta}_4 \quad , \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_4}\right) = (2 J_{p_3}) \ddot{\theta}_4$$

$$\frac{\partial U}{\partial \theta_4} = -2 R_3 K_3 (X - \theta_4 R_3) - 2 R_3 K_4 (R_3 \theta_3 - R_3 \theta_4)$$

$$\frac{\partial D}{\partial \dot{\theta}_4} = -2 R_3 C_3 (\dot{X} - \dot{\theta}_4 R_3) - 2 R_3 C_4 (R_3 \dot{\theta}_3 - R_3 \dot{\theta}_4)$$

$$\ddot{\theta}_4 (2 J_{p_3}) - 2 R_3 \cdot \left[K_3 (X - \theta_4 R_3) + K_4 (R_3 \theta_3 - R_3 \theta_4) + C_3 (\dot{X} - \dot{\theta}_4 R_3) + C_4 (R_3 \dot{\theta}_3 - R_3 \dot{\theta}_4) \right] = 0$$

TRATTO 1:

- $\ddot{\theta}_1 (J_{p1}+J_m) + 2R_1\left[-K_1(\theta_2 R_2-\theta_1 R_1) - C_1(\dot{\theta}_2 R_2-\dot{\theta}_1 R_1)\right] = C_m$
- $\ddot{\theta}_2 (J_{p2}) + 2R_2\left[K_1(\theta_2 R_2-\theta_1 R_1) + C_1(\dot{\theta}_2 R_2-\dot{\theta}_1 R_1)\right] = 0$

UNISCO I 2 MODELLI...

$$M\ddot{x} + 2\left[K_2(x-\theta_3 R_3) + K_3(x-\theta_4 R_3) + C_2(\dot{x}-\dot{\theta}_3 R_3) + C_3(\dot{x}-\dot{\theta}_4 R_3)\right] = 0$$

$$\ddot{\theta}_1 (J_{p1}+J_m) + 2R_1\left[-K_1(\theta_2 R_2-\theta_1 R_1) - C_1(\dot{\theta}_2 R_2-\dot{\theta}_1 R_1)\right] = C_m$$

$$\ddot{\theta}_2 (J_{p2}+2J_s) - 2\left[K_T(\theta_3-\theta_2) + C_T(\dot{\theta}_3-\dot{\theta}_2)\right] + 2R_2\left[K_1(\theta_2 R_2-\theta_1 R_1) + C_1(\dot{\theta}_2 R_2-\dot{\theta}_1 R_1)\right] = 0$$

$$\ddot{\theta}_3 (2J_{p3}) + 2\cdot\left\{\left[K_T(\theta_3-\theta_2) + C_T(\dot{\theta}_3-\dot{\theta}_2)\right] + R_3\cdot\left[-K_2(x-\theta_3 R_3) + K_4(R_3\theta_3-R_3\theta_4) - C_2(\dot{x}-\dot{\theta}_3 R_3) + C_4(R_3\dot{\theta}_3-R_3\dot{\theta}_4)\right]\right\} = 0$$

$$\ddot{\theta}_4 (2J_{p3}) - 2R_3\left[K_3(x-\theta_4 R_3) + K_4(R_3\theta_3-R_3\theta_4) + C_3(\dot{x}-\dot{\theta}_4 R_3) + C_4(R_3\dot{\theta}_3-R_3\dot{\theta}_4)\right] = 0$$

NOTAZIONE MATRICIALE

$$M = \begin{bmatrix} M & 0 & 0 & 0 & 0 \\ 0 & (J_{p1}+J_m) & 0 & 0 & 0 \\ 0 & 0 & (J_{p2}+2J_s) & 0 & 0 \\ 0 & 0 & 0 & 2J_{p3} & 0 \\ 0 & 0 & 0 & 0 & 2J_{p3} \end{bmatrix}$$

$$K = \begin{bmatrix} 2(K_2+K_3) & 0 & 0 & -2R_3K_2 & -2R_3K_3 \\ 0 & 2R_1^2K_1 & -2R_1R_2K_1 & 0 & 0 \\ 0 & -2R_2R_1K_1 & 2(K_T+R_2^2K_1) & -2K_T & 0 \\ -2R_3K_2 & 0 & -2K_T & 2[K_T+R_3^2(K_2+K_4)] & -2R_3^2K_4 \\ -2R_3K_3 & 0 & 0 & -2R_3^2K_4 & 2R_3^2(K_3+K_4) \end{bmatrix}$$

K è UNA MATRICE SIMMETRICA

$$C = \begin{bmatrix} 2(C_2+C_3) & 0 & 0 & -2R_3C_2 & -2R_3C_3 \\ 0 & 2R_1^2C_1 & -2R_1R_2C_1 & 0 & 0 \\ 0 & -2R_2R_1C_1 & 2(C_T+R_2^2C_1) & -2C_T & 0 \\ -2R_3C_2 & 0 & -2C_T & 2[C_T+R_3^2(C_2+C_4)] & -2R_3^2K_4 \\ -2R_3K_3 & 0 & 0 & -2R_3^2K_4 & -2R_3^2(C_3+C_4) \end{bmatrix}$$

C è UNA MATRICE DIAGONALE..

$$\boxed{M\ddot{x} + Kx + C\dot{x} = F}$$

$$F = \begin{bmatrix} 0 \\ C_m \\ 0 \\ 0 \\ 0 \end{bmatrix}$$