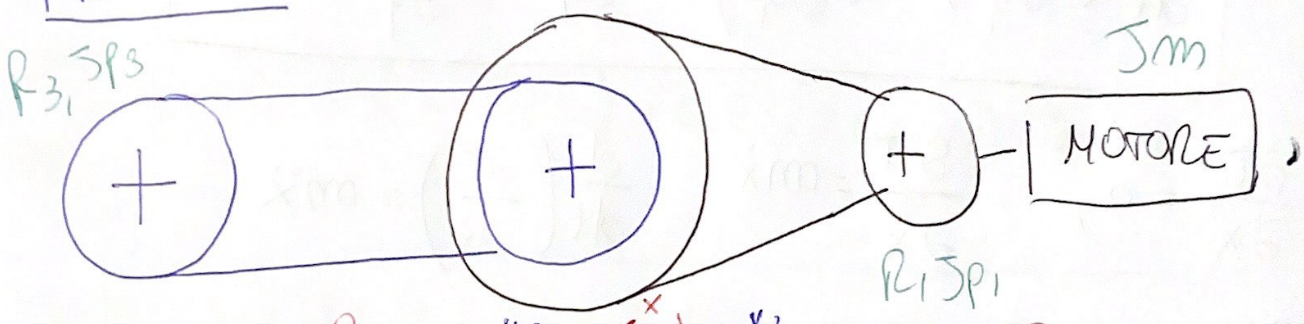
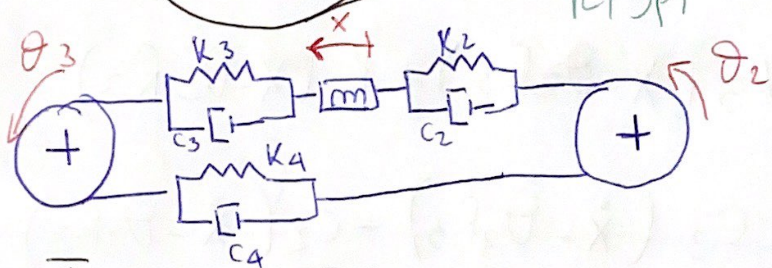
MODELLAZIONE:

ASSE X:

TRATTO 2 — TRATTO 1



TRATTO 2:



$$T = T_{CARRELLO} + T_{PULEGGIA_3} + T_{PULEGGIA_2}$$

$$= \frac{1}{2} J_{p3} \dot{\theta}_3^2 + \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_{p3} \dot{\theta}_2^2$$

$$U = \overset{=0}{U_{GRAV}} + U_{CINGH}$$

$$U_{CINGH} = \frac{1}{2} \underbrace{K_3(x)}_{\text{VARIABILE}} (x - \theta_3 R_3)^2 + \frac{1}{2} \underbrace{K_2(x)}_{\text{VARIABILE}} (x - \theta_2 R_3)^2 + \frac{1}{2} \underbrace{K_4}_{\text{COSTANTE}} (R_3\theta_2 - R_3\theta_3)^2$$

$$D = \frac{1}{2} \underbrace{C_3(x)}_{\text{VARIABILE}} (\dot{x} - \dot{\theta}_3 R_3)^2 + \frac{1}{2} \underbrace{C_2(x)}_{\text{VARIABILE}} (\dot{x} - \dot{\theta}_2 R_3)^2 + \frac{1}{2} \underbrace{C_4}_{\text{COSTANTE}} (R_3\dot{\theta}_2 - R_3\dot{\theta}_3)^2$$

NOTAZIONE: $C_2(x) = C_2$, $C_3(x) = C_3$

$K_2(x) = K_2$, $K_3(x) = K_3$

COORDINATE LIBERE: x, θ_2, θ_3

APPLICO LAGRANGE:

$$\boxed{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)-\left(\frac{\partial T}{\partial q}\right)+\frac{\partial U}{\partial q}+\frac{\partial D}{\partial \dot{q}}=Q_i}$$

$$\frac{\partial T}{\partial x}=0, \quad \frac{\partial T}{\partial \dot{x}}=m\dot{x} \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right)=m\ddot{x}$$

$$\frac{\partial U}{\partial x}=K_3(x-\theta_3R_3)+K_2(x-\theta_2R_3)$$

$$\frac{\partial D}{\partial \dot{x}}=C_3(\dot{x}-\dot{\theta}_3R_3)+C_2(\dot{x}-\dot{\theta}_2R_3)$$

$$\boxed{m\ddot{x}+K_3(x-\theta_3R_3)+K_2(x-\theta_2R_3)+C_3(\dot{x}-\dot{\theta}_3R_3)+C_2(\dot{x}-\dot{\theta}_2R_3)} = Q_x$$

$$\frac{\partial T}{\partial \theta_3}=0, \quad \frac{\partial T}{\partial \dot{\theta}_3}=J_{p3}\dot{\theta}_3 \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_3}\right)=J_{p3}\ddot{\theta}_3$$

$$\frac{dU}{\partial \theta_3}=-K_3R_3(x-\theta_3R_3)-K_4R_3(R_3\theta_2-R_3\theta_3)$$

$$\frac{\partial D}{\partial \dot{\theta}_3}=-C_3R_3(\dot{x}-\dot{\theta}_3R_3)-C_4R_3(R_3\dot{\theta}_2-R_3\dot{\theta}_3)$$

$$= Q_{\theta_3}$$

$$\boxed{J_{p3}\ddot{\theta}_3-K_3R_3(x-\theta_3R_3)-K_4R_3(R_3\theta_2-R_3\theta_3)-C_3R_3(\dot{x}-\dot{\theta}_3R_3)-C_4R_3(R_3\dot{\theta}_2-R_3\dot{\theta}_3)}$$

$$\frac{\partial T}{\partial \theta_2} = 0 \quad , \quad \frac{\partial T}{\partial \dot{\theta}_2} = J_3 \dot{\theta}_2 \quad , \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) = J_{p3} \ddot{\theta}_2$$

$$\frac{\partial U}{\partial \theta_2} = -K_2 R_3 (X - \theta_2 R_3) + K_4 R_3 (R_3 \theta_2 - R_3 \theta_3)$$

$$\frac{\partial D}{\partial \dot{\theta}_2} = -C_2 R_3 (\dot{X} - \dot{\theta}_2 R_3) + C_4 R_3 (R_3 \dot{\theta}_2 - R_3 \dot{\theta}_3)$$

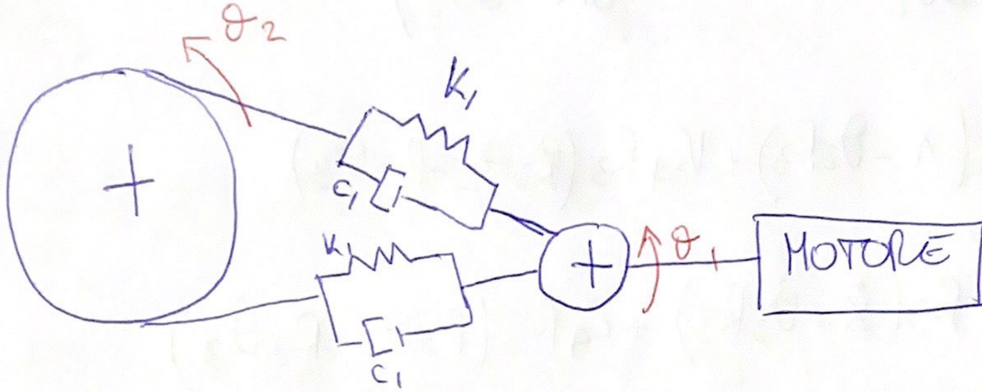
$// \; Q_{\theta_2}$

$$J_{p3} \ddot{\theta}_2 - K_2 R_3 (X - \theta_2 R_3) + K_4 R_3 (R_3 \theta_2 - R_3 \theta_3) - C_2 R_3 (\dot{X} - \dot{\theta}_2 R_3) + C_4 R_3 (R_3 \dot{\theta}_2 - R_3 \dot{\theta}_3)$$

SCRIVO LE 3 EQUAZIONI TROVATE:

$$\begin{cases} m \ddot{X} + K_3 (X - \theta_3 R_3) + K_2 (X - \theta_2 R_3) + C_3 (\dot{X} - \dot{\theta}_3 R_3) + C_2 (\dot{X} - \dot{\theta}_2 R_3) = 0 \\ J_{p3} (\ddot{\theta}_3) - R_3 \cdot \left[K_3 (X - \theta_3 R_3) + K_4 (R_3 \theta_2 - R_3 \theta_3) + C_3 (\dot{X} - \dot{\theta}_3 R_3) + C_4 (R_3 \dot{\theta}_2 - R_3 \dot{\theta}_3) \right] = 0 \\ J_{p3} (\ddot{\theta}_2) + R_3 \cdot \left[-K_2 (X - \theta_2 R_3) + K_4 (R_3 \theta_2 - R_3 \theta_3) - C_2 (\dot{X} - \dot{\theta}_2 R_3) + C_4 (R_3 \dot{\theta}_2 - R_3 \dot{\theta}_3) \right] = 0 \end{cases}$$

TRATTO (1) :



$$\boxed{\omega_m = \dot{\theta}_1}$$

$$T = T_{PULEGGIA\,1} + T_{PULEGGIA\,2} + T_{MOTORE}$$

$$= \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2 + \frac{1}{2} J_m \dot{\theta}_1^2$$

$$= \frac{1}{2} \dot{\theta}_1^2 \left(J_{p1} + J_m \right) + \frac{1}{2} J_{p2} \dot{\theta}_2^2$$

$$U = \cancel{2} \cdot \left[\frac{1}{\cancel{2}} K_1 \left(\theta_2 R_2 - \theta_1 R_1 \right)^2 \right]$$

$$D = \cancel{2} \cdot \left[\frac{1}{\cancel{2}} c_1 \left(\dot{\theta}_2 R_2 - \dot{\theta}_1 R_1 \right)^2 \right]$$

$$\frac{\partial T}{\partial \theta_1} = 0 \quad , \quad \frac{\partial T}{\partial \dot{\theta}_1} = \dot{\theta}_1 \left(J_{p1} + J_m \right) , \quad \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}_1} \right) = \ddot{\theta}_1 \left(J_{p1} + J_m \right)$$

$$\frac{\partial U}{\partial \theta_1} = -2 K_1 R_1 \left(\theta_2 R_2 - \theta_1 R_1 \right)$$

$$\frac{\partial D}{\partial \theta_1} = -2 c_1 R_1 \left(\dot{\theta}_2 R_2 - \dot{\theta}_1 R_1 \right)$$

$$\boxed{\ddot{\theta}_1 \left(J_{p1} + J_m \right) - R_1 \left[2 K_1 \left(\theta_2 R_2 - \theta_1 R_1 \right) + 2 c_1 \left(\dot{\theta}_2 R_2 - \dot{\theta}_1 R_1 \right) \right] = 0}$$

$$\frac{\partial T}{\partial \theta_2} = 0 \,, \quad \frac{\partial T}{\partial \dot{\theta}_2} = J_{p_2}\dot{\theta}_2 \,, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) = J_{p_2}\ddot{\theta}_2$$

$$\frac{\partial U}{\partial \theta_2} = 2K_1R_2(\theta_2R_2-\theta_1R_1)$$

$$\frac{\partial D}{\partial \dot{\theta}_2} = 2C_1R_2(\dot{\theta}_2R_2-\dot{\theta}_1R_1)$$

$$\boxed{J_{p_2}\ddot{\theta}_2 + R_2\cdot\left[2K_1(\theta_2R_2-\theta_1R_1) + 2C_1(\dot{\theta}_2R_2-\dot{\theta}_1R_1)\right] = 0}$$

UNISCO I 2 SOTTO-MODELLI:

$$\begin{cases} m\ddot{x} + K_3(x-\theta_3R_3) + K_2(x-\theta_2R_3) + C_3(\dot{x}-\dot{\theta}_3R_3) + C_2(\dot{x}-\dot{\theta}_2R_3) = 0 \\ \ddot{\theta}_1(J_{p_1}+J_m) - R_1\left[2K_1(\theta_2R_2-\theta_1R_1) + 2C_1(\dot{\theta}_2R_2-\dot{\theta}_1R_1)\right] = C_m \\ \ddot{\theta}_2(J_{p_2}+J_{p_3}) + R_2\cdot\left[2K_1(\theta_2R_2-\theta_1R_1) + 2C_1(\dot{\theta}_2R_2-\dot{\theta}_1R_1)\right] + \\ \quad + R_3\cdot\left[-K_2(x-\theta_2R_3) + K_4(R_3\theta_2-R_3\theta_3) - C_2(\dot{x}-\dot{\theta}_2R_3) + C_4(R_3\dot{\theta}_2-R_3\dot{\theta}_3)\right] = 0 \\ \ddot{\theta}_3(J_{p_3}) - R_3\left[K_3(x-\theta_3R_3) + K_4(R_3\theta_2-R_3\theta_3) + C_3(\dot{x}-\dot{\theta}_3R_3) + C_4(R_3\dot{\theta}_2-R_3\dot{\theta}_3)\right] = 0 \end{cases}$$

ESPRIMO IN NOTAZIONE MATRICIALE...

(6)

$$M = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & J_{P1}+J_m & 0 & 0 \\ 0 & 0 & J_{P2}+J_{P3} & 0 \\ 0 & 0 & 0 & J_{P3} \end{bmatrix} \begin{pmatrix} \ddot{X} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{pmatrix}$$

$$K = \begin{bmatrix} (K_3+K_2) & 0 & (-K_2R_3) & (-K_3R_3) \\ 0 & (2K_1R_1^2) & (-2K_1R_2R_1) & 0 \\ (-R_3K_2) & (-2K_1R_1R_2) & (2K_1R_2^2)+(K_2+K_4)R_3^2 & (-K_4R_3^2) \\ (-K_3R_3) & 0 & (-K_4R_3^2) & (K_3+K_4)R_3^2 \end{bmatrix} \begin{pmatrix} X \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$$

$$C = \begin{bmatrix} (C_3+C_2) & 0 & (-C_2R_3) & (-C_3R_3) \\ 0 & (2C_1R_1^2) & (-2C_1R_2R_1) & 0 \\ (-R_3C_2) & (-2C_1R_1R_2) & (2C_1R_2^2)+(C_2+C_4)R_3^2 & (-C_4R_3^2) \\ (-C_3R_3) & 0 & (-C_4R_3^2) & (C_3+C_4)R_3^2 \end{bmatrix} \begin{pmatrix} \dot{X} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix}$$

$$F = \begin{bmatrix} 0 \\ C_m \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{M\ddot{X} + KX + C\dot{X} = F}$$