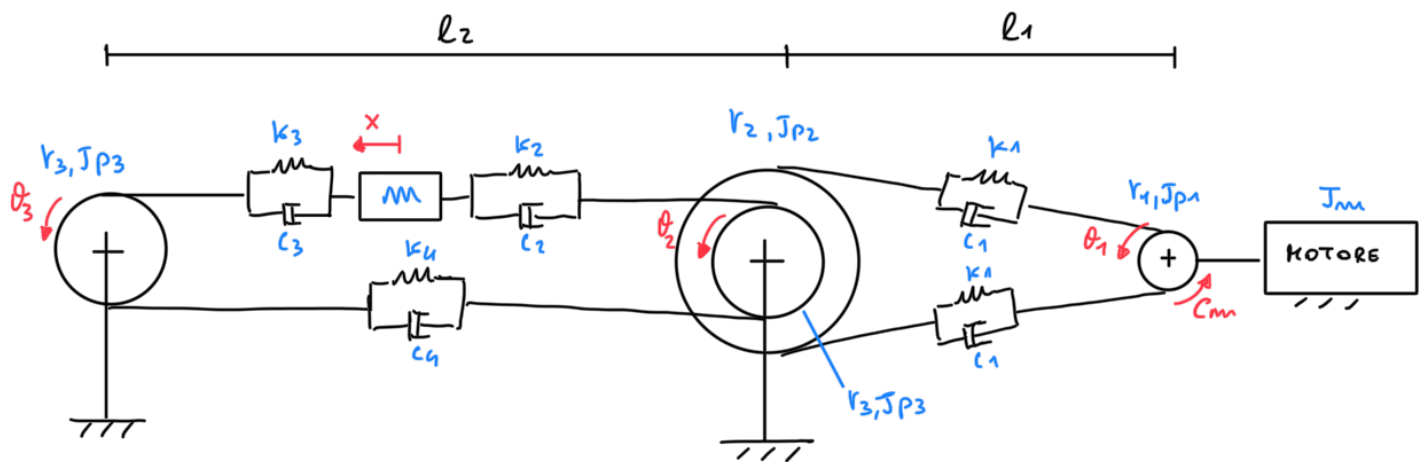


MODELLAZIONE ASSI

• Asse x



$$R\theta = x$$

$$U = 2 \left[\frac{1}{2} k_1 (\theta_1 r_1 - \theta_2 r_2)^2 \right] + \frac{1}{2} k_2 (\theta_2 r_3 - x)^2 + \frac{1}{2} k_3 (x - \theta_3 r_3)^2 + \frac{1}{2} k_4 (\theta_3 r_3 - \theta_2 r_2)^2$$

$$T = \frac{1}{2} J_m \dot{\theta}_1^2 + \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2 + \frac{1}{2} J_{p3} \dot{\theta}_2^2 + \frac{1}{2} J_{p3} \dot{\theta}_3^2 + \frac{1}{2} m \dot{x}^2$$

$$\text{LAGRANGE: } \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = Q_q$$

$$\begin{cases} \theta_1: J_m \ddot{\theta}_1 + J_{p1} \ddot{\theta}_1 + 2k_1 (\theta_1 r_1 - \theta_2 r_2) r_1 + 2c_1 (\dot{\theta}_1 r_1 - \dot{\theta}_2 r_2) r_1 = C_m \\ \theta_2: J_{p2} \ddot{\theta}_2 + J_{p3} \ddot{\theta}_2 + k_2 (\theta_2 r_3 - x) r_3 + c_2 (\dot{\theta}_2 r_3 - \dot{x}) r_3 + k_4 (\theta_3 r_3 - \theta_2 r_2) (-r_2) + c_4 (\dot{\theta}_3 r_3 - \dot{\theta}_2 r_2) (-r_2) + 2k_1 (\theta_1 r_1 - \theta_2 r_2) (-r_2) + 2c_1 (\dot{\theta}_1 r_1 - \dot{\theta}_2 r_2) (-r_2) = 0 \\ \theta_3: J_{p3} \ddot{\theta}_3 - k_3 (x - \theta_3 r_3) r_3 - c_3 (\dot{x} - \dot{\theta}_3 r_3) r_3 + k_4 (\theta_3 r_3 - \theta_2 r_2) r_3 + c_4 (\dot{\theta}_3 r_3 - \dot{\theta}_2 r_2) r_3 = 0 \\ x: m \ddot{x} + k_2 (\theta_2 r_3 - x) (-1) + c_2 (\dot{\theta}_2 r_3 - \dot{x}) (-1) + k_3 (x - \theta_3 r_3) + c_3 (\dot{x} - \dot{\theta}_3 r_3) = 0 \end{cases}$$

$$\bar{X} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ x \end{bmatrix}$$

$$\dot{\bar{X}} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{x} \end{bmatrix}$$

$$\ddot{\bar{X}} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{x} \end{bmatrix}$$

$$\bar{T}_m = \begin{bmatrix} C_m \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$M \ddot{\bar{X}} + K \bar{X} + C \dot{\bar{X}} = \bar{T}_m$$

$$\begin{bmatrix} J_m + J_{p1} & 0 & 0 & 0 \\ 0 & J_{p2} + J_{p3} & 0 & 0 \\ 0 & 0 & J_{p3} & 0 \\ 0 & 0 & 0 & m \end{bmatrix}$$

[M]

$$\ddot{\bar{X}} + \dots$$

$$\bar{X} +$$

$$\frac{\dot{\lambda}}{\lambda} = \overline{T}_m$$

The diagram illustrates a mechanical system with a central vertical shaft labeled 'ASSE X'. Four rotors are mounted on this shaft at different positions, separated by distances l_4 and l_3 . The rotors are represented by circles with a cross, indicating their angular displacement. The angular displacements are labeled θ_3 (downward) and θ_4 (upward) for the outer rotors, and θ_2 (downward) for the central rotor. The moments of inertia are J_{p3} for the outer rotors and J_{p2} for the central rotor. The shaft is supported by bearings, represented by vertical lines with cross-hatching at the ends. The bearings are modeled with mass-spring-damper elements, with stiffness k_1 , k_2 , k_3 , and k_4 , and damping c_1 , c_2 , c_3 , and c_4 . A motor, labeled 'MOTORE', is connected to the central rotor through a gear train. The motor has a moment of inertia J_m and a torque C_m . The gear train consists of two gears, each with a moment of inertia J_{p1} and a torque C_{p1} . The motor torque is applied to the central rotor through the gear train. A red arrow points to the shaft with the text 'COME SE FOSSE UN UNICO ASSE', indicating that the shaft is treated as a single rigid body. A coordinate system X_x is shown on the shaft, with a moment M_x applied.

RIGIDEZZA ALBERO

2 GUIDE CON
STESSI PARAMETRI

TRANSCRIBIBLE ?

GOL IN PIV

$$+ \frac{1}{2} k_3 (x - \theta_3 r_3)^2 + \frac{1}{2} k_4 (\theta_3 r_3 - \theta_4 r_3)^2]$$

$$T = \frac{1}{2} J_m \dot{\theta}_1^2 + \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2 + J_{p3} \dot{\theta}_4^2 + J_{p3} \dot{\theta}_3^2 + \frac{1}{2} M_x \dot{x}^2 + \frac{1}{2} J_s \dot{\theta}_2^2$$

MASSA ASSE x
 VELOCITA' ASSE x
 J ALBERO

$$\begin{aligned} \theta_1: & J_m \ddot{\theta}_1 + J_{p1} \ddot{\theta}_1 + 2k_1(\theta_1 r_1 - \theta_2 r_2) r_1 + 2c_1(\dot{\theta}_1 r_1 - \dot{\theta}_2 r_2) r_1 = C_m \\ \theta_2: & J_{p2} \ddot{\theta}_2 + J_s \ddot{\theta}_2 + 2k_1(\theta_1 r_1 - \theta_2 r_2)(-r_2) + k_5(\theta_2 r_3 - \theta_4 r_3) r_3 + \\ & + 2c_1(\dot{\theta}_1 r_1 - \dot{\theta}_2 r_2)(-r_2) + c_5(\dot{\theta}_2 r_3 - \dot{\theta}_4 r_3) r_3 = 0 \\ \theta_3: & 2J_{p3} \ddot{\theta}_3 + 2k_3(x - \theta_3 r_3)(-r_3) + 2c_3(\dot{x} - \dot{\theta}_3 r_3)(-r_3) + 2k_4(\theta_3 r_3 - \theta_4 r_3) r_3 + \\ & + 2c_4(\dot{\theta}_3 r_3 - \dot{\theta}_4 r_3) r_3 = 0 \\ \theta_4: & 2J_{p3} \ddot{\theta}_4 + k_5(\theta_2 r_3 - \theta_4 r_3)(-r_3) + c_5(\dot{\theta}_2 r_3 - \dot{\theta}_4 r_3)(-r_3) + \\ & + 2k_2(\theta_4 r_3 - x) r_3 + 2c_2(\dot{\theta}_4 r_3 - \dot{x}) r_3 + 2k_4(\theta_3 r_3 - \theta_4 r_3)(-r_3) + \\ & + 2c_4(\dot{\theta}_3 r_3 - \dot{\theta}_4 r_3)(-r_3) = 0 \\ x: & M_x \ddot{x} + 2k_2(\theta_4 r_3 - x)(-1) + 2c_2(\dot{\theta}_4 r_3 - \dot{x})(-1) + 2k_3(x - \theta_3 r_3) + \\ & + 2c_3(\dot{x} - \dot{\theta}_3 r_3) = 0 \end{aligned}$$

$$\bar{X} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ x \end{bmatrix}$$

$$\dot{\bar{X}} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{x} \end{bmatrix}$$

$$\ddot{\bar{X}} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{\theta}_4 \\ \ddot{x} \end{bmatrix}$$

$$\bar{T}_m = \begin{bmatrix} C_m \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} J_m + J_{p1} & 0 & 0 & 0 & 0 \\ 0 & J_{p2} + J_s & 0 & 0 & 0 \\ 0 & 0 & 2J_{p3} & 0 & 0 \\ 0 & 0 & 0 & 2J_{p3} & 0 \\ 0 & 0 & 0 & 0 & M_x \end{bmatrix}$$

[M]

$\ddot{\bar{X}} +$

$$\begin{bmatrix} 2r_1^2 k_1 & -2k_1 r_1 r_2 & 0 & 0 & 0 \\ -2r_1 r_2 k_1 & 2r_2^2 k_1 + r_3^2 k_5 & 0 & -r_3^2 k_5 & 0 \\ 0 & 0 & 2r_3^2 (k_3 + k_4) & -2r_3^2 k_4 & -2r_3 k_3 \\ 0 & -r_3^2 k_5 & -2r_3^2 k_4 & r_3^2 k_5 + 2r_3 k_2 + 2r_3^2 k_4 & -2r_3 k_2 \end{bmatrix}$$

$\bar{X} +$

$$\begin{bmatrix} 0 & 0 & -2r_3 k_3 & -2r_3 k_2 & 2k_2 + 2k_3 \end{bmatrix}$$

$[k]$

$$\begin{bmatrix} 2r_1^2 c_1 & -2c_1 r_1 r_2 & 0 & 0 & 0 \\ -2r_1 r_2 c_1 & 2r_2^2 c_1 + r_3^2 c_5 & 0 & -r_3^2 c_5 & 0 \\ 0 & 0 & 2r_3^2 (c_3 + c_4) & -2r_3^2 c_4 & -2r_3 c_3 \\ 0 & -r_3^2 c_5 & -2r_3^2 c_4 & r_3^2 c_5 + 2r_3 c_2 + 2r_3^2 c_4 & -2r_3 c_2 \\ 0 & 0 & -2r_3 c_3 & -2r_3 c_2 & 2c_2 + 2c_3 \end{bmatrix}$$

$[c]$

$$\dot{\bar{X}} = \bar{Y}_m$$