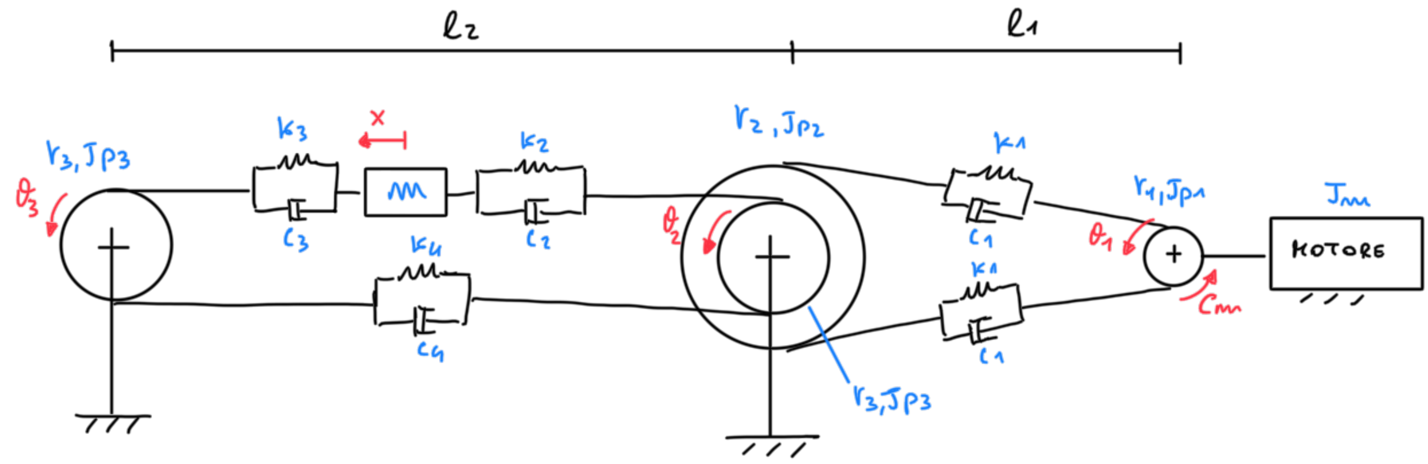
Modellazione assi

- <u>Asse x</u>



$$R\theta = x$$

$$U = 2\left[\frac{1}{2} k_1 \left(\theta_1 r_1 - \theta_2 r_2\right)^2\right] + \frac{1}{2} k_2 \left(\theta_2 r_3 - x\right)^2 + \frac{1}{2} k_3 \left(x - \theta_3 r_3\right)^2 + \frac{1}{2} k_4 \left(\theta_3 r_3 - \theta_2 r_3\right)^2$$

$$T = \frac{1}{2} J_m \dot{\theta}_1^2 + \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2 + \frac{1}{2} J_{p3} \dot{\theta}_2^2 + \frac{1}{2} J_{p3} \dot{\theta}_3^2 + \frac{1}{2} m \dot{x}^2$$

Lagrange: $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = Q_q$

$$\begin{cases}
\theta_1: & J_m \ddot{\theta}_1 + J_{p1} \ddot{\theta}_1 + 2k_1 \left(\theta_1 r_1 - \frac{\theta_2}{\tau_1} r_2\right) r_1 + 2c_1 \left(\dot{\theta}_1 r_1 - \frac{\dot{\theta}_2}{\tau_1} r_2\right) r_1 = \frac{C_m}{R} \\
\theta_2: & J_{p2} \ddot{\theta}_2 + J_{p3} \ddot{\theta}_2 + k_2 (\theta_2 r_3 - x) r_3 + c_2 (\dot{\theta}_2 r_3 - \dot{x}) r_3 + k_4 (\theta_3 r_3 - \theta_2 r_3)(-r_3) + \\
 & + c_4 (\dot{\theta}_3 r_3 - \dot{\theta}_2 r_3)(-r_3) + 2k_1 \left(\theta_1 r_1 - \frac{\theta_2}{\tau_1} r_2\right)\left(-\frac{r_2}{\tau_1}\right) + 2c_1 \left(\dot{\theta}_1 r_1 - \frac{\dot{\theta}_2}{\tau_1} r_2\right)\left(-\frac{r_2}{\tau_1}\right) = 0 \\
\theta_3: & J_{p3} \ddot{\theta}_3 - k_3 (x - \theta_3 r_3) r_3 - c_3 (\dot{x} - \dot{\theta}_3 r_3) r_3 + k_4 (\theta_3 r_3 - \theta_2 r_3) r_3 + \\
 & + c_4 (\dot{\theta}_3 r_3 - \dot{\theta}_2 r_3) r_3 = 0 \\
x: & m \ddot{x} + k_2 (\theta_2 r_3 - x)(-1) + c_2 (\dot{\theta}_2 r_3 - x)(-1) + k_3 (x - \theta_3 r_3) + c_3 (\dot{x} - \dot{\theta}_3 r_3)
\end{cases}$$

$$\bar{X} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ x \end{bmatrix} \qquad \dot{\bar{X}} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{x} \end{bmatrix} \qquad \ddot{\bar{X}} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{x} \end{bmatrix} \qquad \bar{T}_m = \begin{bmatrix} C_m \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

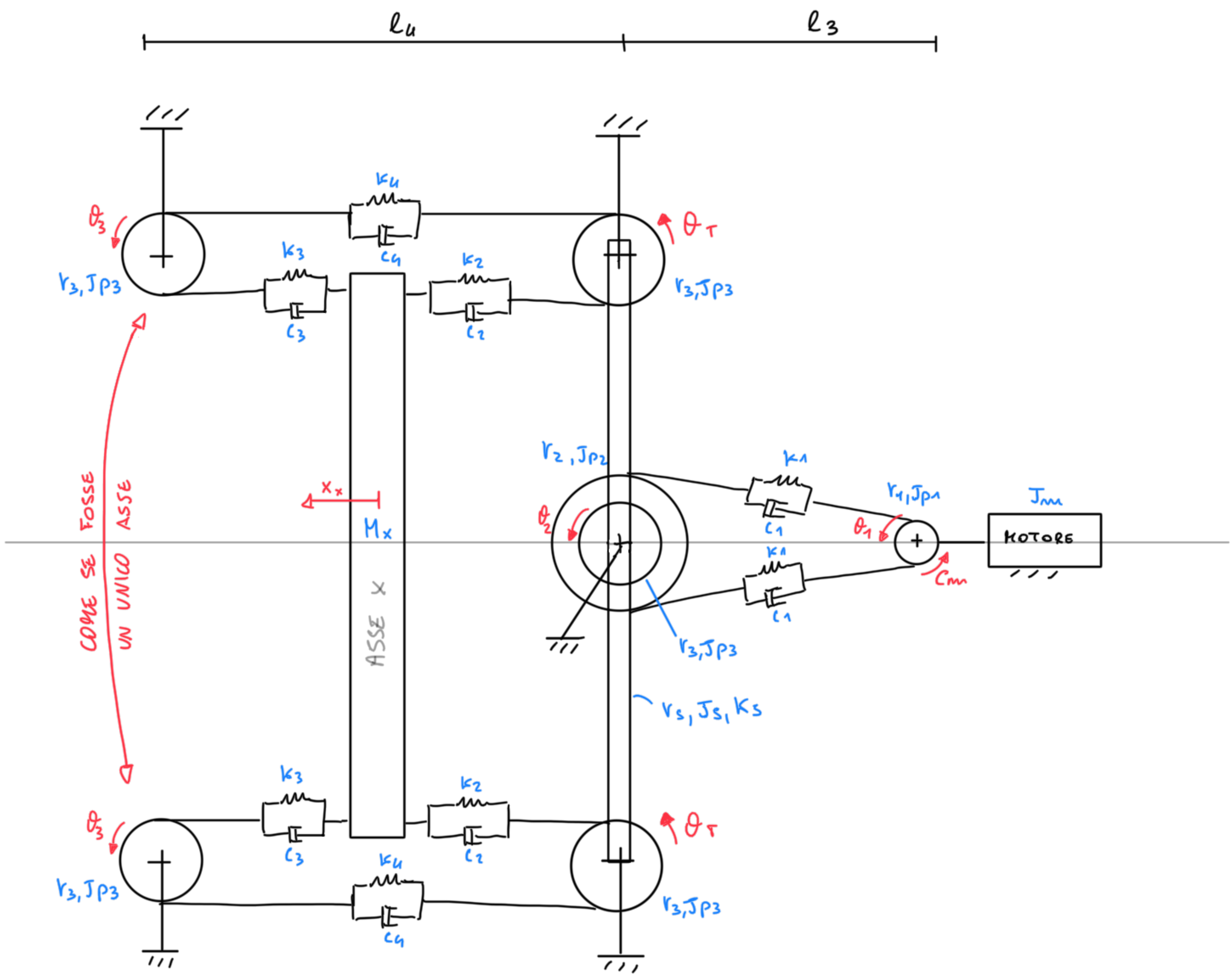
$$M \ddot{\bar{X}} + K \bar{X} + C \dot{\bar{X}} = \bar{T}_m$$

$$\underbrace{\begin{bmatrix} J_m + J_{p1} & 0 & 0 & 0 \\ 0 & J_{p2} + J_{p3} & 0 & 0 \\ 0 & 0 & J_{p3} & 0 \\ 0 & 0 & 0 & m \end{bmatrix}}_{[M]} \ddot{\bar{X}} +$$

$$\underbrace{\begin{bmatrix} 2r_1^2 k_1 & -2r_1r_2\,k_1 & 0 & 0 \\ -2r_1r_2\,k_1 & 2r_2^2k_1 + r_3^2(k_2+k_4) & -r_3^2k_4 & -r_3k_2 \\ 0 & -r_3^2k_4 & r_3^2(k_3+k_4) & -r_3k_3 \\ 0 & -r_3k_2 & -r_3k_3 & k_2+k_3 \end{bmatrix}}_{[K]} \bar{x} \; +$$

$$\underbrace{\begin{bmatrix} 2r_1^2 c_1 & -2r_1r_2\,c_1 & 0 & 0 \\ -2r_1r_2\,c_1 & 2r_2^2c_1 + r_3^2(c_2+c_4) & -r_3^2c_4 & -r_3c_2 \\ 0 & -r_3^2c_4 & r_3^2(c_3+c_4) & -r_3c_3 \\ 0 & -r_3c_2 & -r_3c_3 & c_2+c_3 \end{bmatrix}}_{[C]} \dot{\bar{x}} = \bar{T}_m$$

- <u>ASSE y</u>



RIGIDEZZA ALBERO

2 GUIDE CON STESSI PARAMETRI

$$U = 2\left[\frac{1}{2}k_1(\theta_1 r_1 - \theta_2 r_2)^2\right] + \underbrace{\frac{1}{2}k_s(\theta_2 r_3 - \theta_T r_3)^2}_{\text{TRASCURABILE ?}} + 2\left[\frac{1}{2}k_2(\theta_2 r_3 - x)^2 + \right.$$

$$+ \frac{1}{2} k_3 \left(x - \theta_3 r_3 \right)^2 + \frac{1}{2} k_4 \left(\theta_3 r_3 - \theta_2 r_3 \right)^2 \Big]$$

$$T = \frac{1}{2} J_m \dot{\theta}_1^{\,2} + \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2 + J_{p3} \dot{\theta}_T^{\,2} + J_{p3} \dot{\theta}_3^{\,2} + \frac{1}{2} \overset{\text{MASSA ASSE X}}{M_x} \underset{\text{VELOCITA' ASSE X}}{\dot{x}_x^{\,2}} + \underbrace{\frac{1}{2} J_s \dot{\theta}_3^{\,2}}_{\text{J ALBERO}}$$