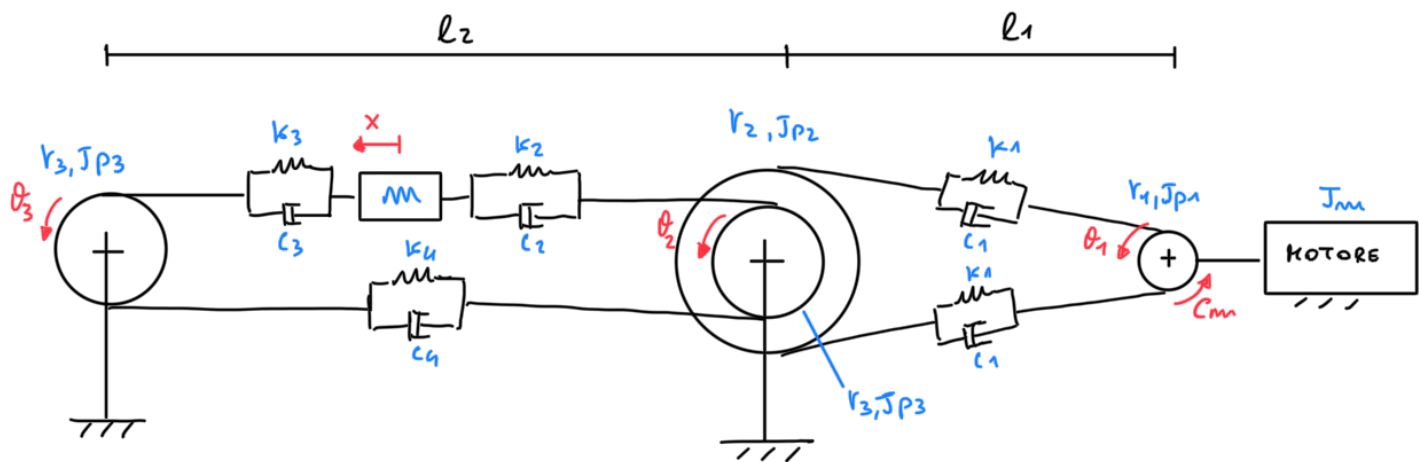


MODELLAZIONE ASSI

• Asse x



$$R\theta = x$$

$$U = 2 \left[\frac{1}{2} k_1 (\theta_1 r_1 - \theta_2 r_2)^2 \right] + \frac{1}{2} k_2 (\theta_2 r_3 - x)^2 + \frac{1}{2} k_3 (x - \theta_3 r_3)^2 + \frac{1}{2} k_4 (\theta_3 r_3 - \theta_2 r_3)^2$$

$$T = \frac{1}{2} J_m \dot{\theta}_1^2 + \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2 + \frac{1}{2} J_{p3} \dot{\theta}_2^2 + \frac{1}{2} J_{p3} \dot{\theta}_3^2 + \frac{1}{2} m \dot{x}^2$$

$$\text{LAGRANGE: } \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = Q_q$$

$$\begin{cases} \theta_1: J_m \ddot{\theta}_1 + J_{p1} \ddot{\theta}_1 + 2k_1 \left(\theta_1 r_1 - \frac{\theta_2 r_2}{r_1} \right) r_1 + 2c_1 \left(\dot{\theta}_1 r_1 - \frac{\dot{\theta}_2 r_2}{r_1} \right) r_1 = \frac{C_m}{R} \\ \theta_2: J_{p2} \ddot{\theta}_2 + J_{p3} \ddot{\theta}_2 + k_2 (\theta_2 r_3 - x) r_3 + c_2 (\dot{\theta}_2 r_3 - \dot{x}) r_3 + k_4 (\theta_3 r_3 - \theta_2 r_3) (-r_3) + \\ + c_4 (\dot{\theta}_3 r_3 - \dot{\theta}_2 r_3) (-r_3) + 2k_1 \left(\theta_1 r_1 - \frac{\theta_2 r_2}{r_1} \right) \left(-\frac{r_2}{r_1} \right) + 2c_1 \left(\dot{\theta}_1 r_1 - \frac{\dot{\theta}_2 r_2}{r_1} \right) \left(-\frac{r_2}{r_1} \right) = 0 \\ \theta_3: J_{p3} \ddot{\theta}_3 - k_3 (x - \theta_3 r_3) r_3 - c_3 (\dot{x} - \dot{\theta}_3 r_3) r_3 + k_4 (\theta_3 r_3 - \theta_2 r_3) r_3 + \\ + c_4 (\dot{\theta}_3 r_3 - \dot{\theta}_2 r_3) r_3 = 0 \\ x: m \ddot{x} + k_2 (\theta_2 r_3 - x) (-1) + c_2 (\dot{\theta}_2 r_3 - \dot{x}) (-1) + k_3 (x - \theta_3 r_3) + c_3 (\dot{x} - \dot{\theta}_3 r_3) \end{cases}$$

$$\bar{X} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ x \end{bmatrix}$$

$$\dot{\bar{X}} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{x} \end{bmatrix}$$

$$\ddot{\bar{X}} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{x} \end{bmatrix}$$

$$\bar{T}_m = \begin{bmatrix} C_m \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$M \ddot{\bar{X}} + K \bar{X} + C \dot{\bar{X}} = \bar{T}_m$$

$$\begin{bmatrix} J_m + J_{p1} & 0 & 0 & 0 \\ 0 & J_{p2} + J_{p3} & 0 & 0 \\ 0 & 0 & J_{p3} & 0 \\ 0 & 0 & 0 & m \end{bmatrix} \ddot{\bar{X}} + \begin{bmatrix} 2k_1 r_1 & -2k_1 r_2 & 0 & 0 \\ 2c_1 r_1 & -2c_1 r_2 & 0 & 0 \\ 0 & 0 & k_4 r_3 & 0 \\ 0 & 0 & 0 & k_2 r_3 + k_3 r_3 \end{bmatrix} \dot{\bar{X}} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \bar{X} = \bar{T}_m$$

[n]

$$\begin{bmatrix} 2r_1^2 c_1 & -2r_1 r_2 c_1 & 0 & 0 \\ -2r_1 r_2 c_1 & 2r_2^2 c_1 + r_3^2 (c_2 + c_4) & -r_3^2 c_4 & -r_3 c_2 \\ 0 & -r_3^2 c_4 & r_3^2 (c_3 + c_4) & -r_3 c_3 \\ 0 & -r_3 c_2 & -r_3 c_3 & c_2 + c_3 \end{bmatrix} \quad \frac{\cdot}{X} = \bar{T}_m$$

Hand-drawn mechanical diagram of a motorized assembly. The diagram is divided into two sections, l_4 and l_3 , by a vertical line. A central vertical shaft is labeled "ASSE X" and has a coordinate x^* and moment M_x . Four rotors are connected to this central shaft via springs and dampers. The rotors are labeled with moments of inertia J_{p1} , J_{p2} , J_{p3} , and J_{p4} . The motor is labeled "MOTORE" and has a moment of inertia J_m . The diagram also shows various spring constants (k_1, k_2, k_3, k_4) and damping coefficients (c_1, c_2, c_3, c_4). A red arrow points to the central shaft with the text "COME SE FOSSE UN UNICO ASSE" (As if it were a single shaft).

$$U = 2 \left[\frac{1}{2} K_1 (\theta_1 r_1 - \theta_2 r_2)^2 \right] + \underbrace{\frac{1}{2} k_s (\theta_2 r_3 - \theta_r r_3)^2}_{\text{TRASCURSIBILE?}} + 2 \left[\frac{1}{2} K_2 (\theta_2 r_3 - x)^2 \right] +$$

$$+ \frac{1}{2} k_3 (x - \theta_3 r_3)^2 + \frac{1}{2} k_u (\theta_3 r_3 - \theta_2 r_3)^2]$$

$$T = \frac{1}{2} J_m \dot{\theta}_1^2 + \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2 + J_{p3} \dot{\theta}_T^2 + J_{p3} \dot{\theta}_3^2 + \frac{1}{2} M_x \dot{x}_x^2 + \frac{1}{2} J_s \dot{\theta}_3^2$$

MASSA ASSE x
 VELOCITA' ASSE x
 J ALBERO