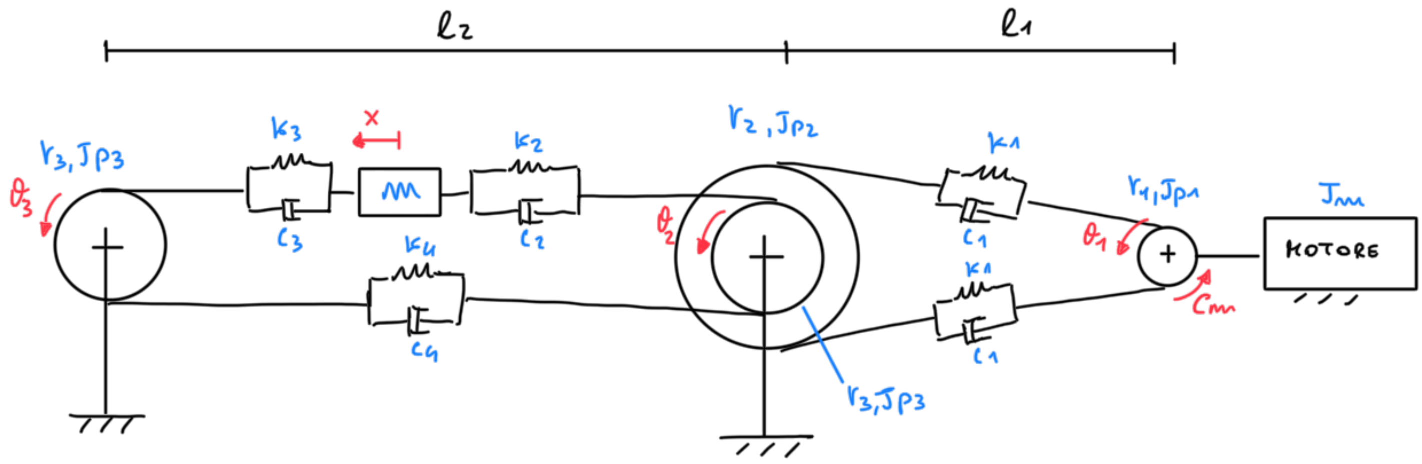
Modellazione assi

• **Asse x**



$$R\theta = x$$

$$U = 2\left[\frac{1}{2} k_1 \left(\theta_1 r_1 - \theta_2 r_2\right)^2\right] + \frac{1}{2} k_2 \left(\theta_2 r_3 - x\right)^2 + \frac{1}{2} k_3 \left(x - \theta_3 r_3\right)^2 + \frac{1}{2} k_4 \left(\theta_3 r_3 - \theta_2 r_3\right)^2$$

$$T = \frac{1}{2} J_m \dot{\theta}_1^2 + \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2 + \frac{1}{2} J_{p3} \dot{\theta}_2^2 + \frac{1}{2} J_{p3} \dot{\theta}_3^2 + \frac{1}{2} m \dot{x}^2$$

Lagrange: $$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = Q_q$$

$$\begin{cases}
\theta_1: \; J_m \ddot{\theta}_1 + J_{p1} \ddot{\theta}_1 + 2k_1 \left(\theta_1 r_1 - \theta_2 r_2\right) r_1 + 2c_1 \left(\dot{\theta}_1 r_1 - \dot{\theta}_2 r_2\right) r_1 = C_m \\
\theta_2: \; J_{p2} \ddot{\theta}_2 + J_{p3} \ddot{\theta}_2 + k_2 (\theta_2 r_3 - x) r_3 + c_2 (\dot{\theta}_2 r_3 - \dot{x}) r_3 + k_4 (\theta_3 r_3 - \theta_2 r_3)(-r_3) + \\
\quad + c_4 (\dot{\theta}_3 r_3 - \dot{\theta}_2 r_3)(-r_3) + 2k_1 (\theta_1 r_1 - \theta_2 r_2)(-r_2) + 2c_1 (\dot{\theta}_1 r_1 - \dot{\theta}_2 r_2)(-r_2) = 0 \\
\theta_3: \; J_{p3} \ddot{\theta}_3 - k_3 (x - \theta_3 r_3) r_3 - c_3 (\dot{x} - \dot{\theta}_3 r_3) r_3 + k_4 (\theta_3 r_3 - \theta_2 r_3) r_3 + \\
\quad + c_4 (\dot{\theta}_3 r_3 - \dot{\theta}_2 r_3) r_3 = 0 \\
x: \; m \ddot{x} + k_2 (\theta_2 r_3 - x)(-1) + c_2 (\dot{\theta}_2 r_3 - x)(-1) + k_3 (x - \theta_3 r_3) + c_3 (\dot{x} - \dot{\theta}_3 r_3)
\end{cases}$$

$$\bar{X} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ x \end{bmatrix} \qquad \dot{\bar{X}} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{x} \end{bmatrix} \qquad \ddot{\bar{X}} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{x} \end{bmatrix} \qquad \bar{T}_m = \begin{bmatrix} C_m \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

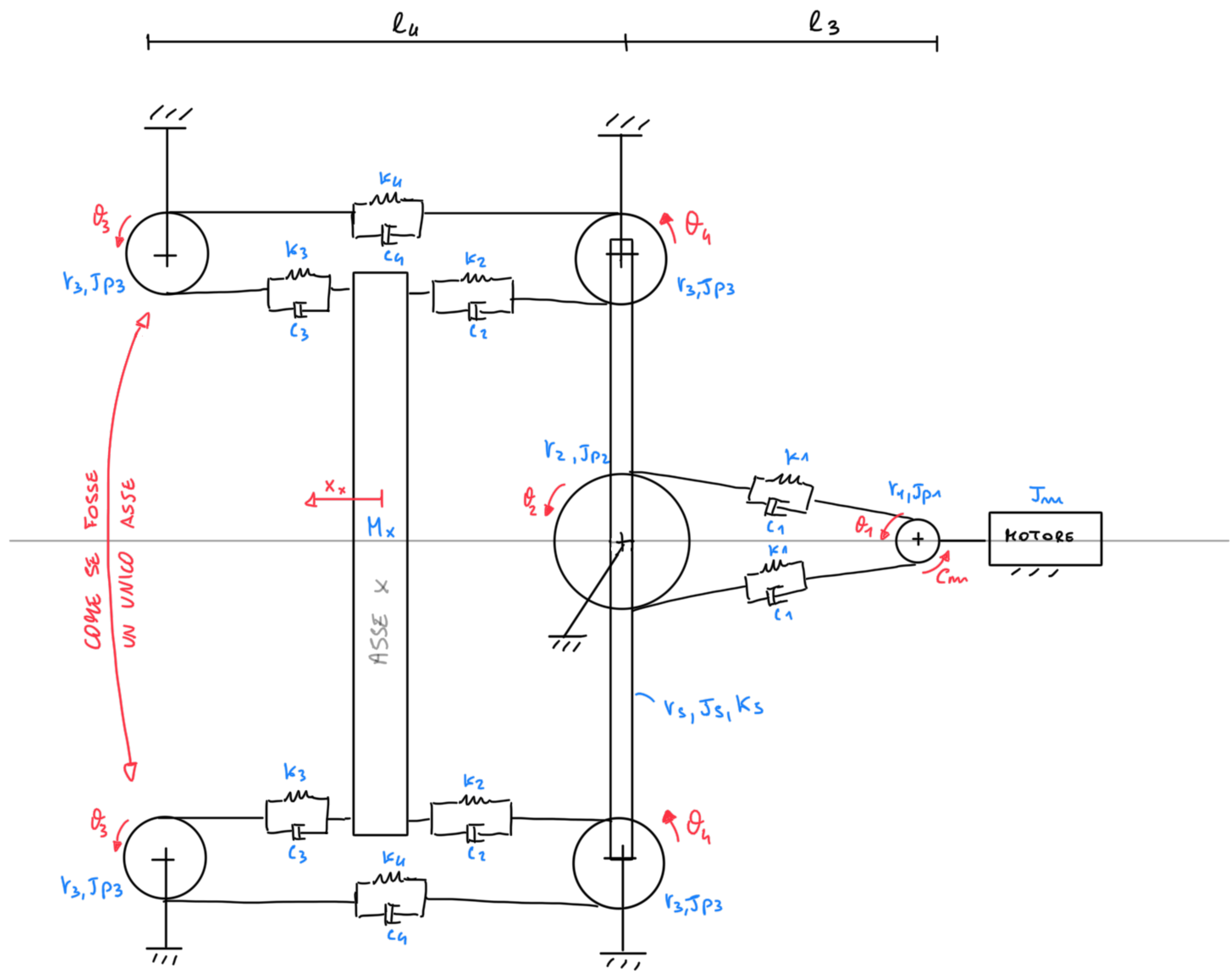
$$M \ddot{\bar{X}} + K \bar{X} + C \dot{\bar{X}} = \bar{T}_m$$

$$\underbrace{\begin{bmatrix} J_m + J_{p1} & 0 & 0 & 0 \\ 0 & J_{p2} + J_{p3} & 0 & 0 \\ 0 & 0 & J_{p3} & 0 \\ 0 & 0 & 0 & m \end{bmatrix}}_{[M]} \ddot{\bar{X}} +$$

$$
\underbrace{\begin{bmatrix}
2r_1^2k_1 & -2r_1r_2k_1 & 0 & 0\\
-2r_1r_2k_1 & 2r_2^2k_1+r_3^2(k_2+k_4) & -r_3^2k_4 & -r_3k_2\\
0 & -r_3^2k_4 & r_3^2(k_3+k_4) & -r_3k_3\\
0 & -r_3k_2 & -r_3k_3 & k_2+k_3
\end{bmatrix}}_{[K]}\bar{X}\;+
$$

$$
\underbrace{\begin{bmatrix}
2r_1^2c_1 & -2r_1r_2c_1 & 0 & 0\\
-2r_1r_2c_1 & 2r_2^2c_1+r_3^2(c_2+c_4) & -r_3^2c_4 & -r_3c_2\\
0 & -r_3^2c_4 & r_3^2(c_3+c_4) & -r_3c_3\\
0 & -r_3c_2 & -r_3c_3 & c_2+c_3
\end{bmatrix}}_{[C]}\dot{\bar{X}} = \bar{T}_m
$$

• **Asse y**



RIGIDEZZA ALBERO

2 GUIDE CON STESSI PARAMETRI

$$
U = 2\left[\frac{1}{2}k_1(\theta_1 r_1 - \theta_2 r_2)^2\right] + \underbrace{\frac{1}{2}k_s(\theta_2 r_3 - \theta_4 r_3)^2}_{\text{TRASCURABILE ?}} + 2\left[\frac{1}{2}k_2(\theta_4 r_3 - x)^2 + \right.
$$

GDL IN PIU'

$$+\frac{1}{2}k_3\left(x-\theta_3 r_3\right)^2+\frac{1}{2}k_4\left(\theta_3 r_3-\theta_4 r_3\right)^2\Big]$$

MASSA ASSE x

$$T=\frac{1}{2}J_m\dot\theta_1^{\,2}+\frac{1}{2}J_{p1}\dot\theta_1^2+\frac{1}{2}J_{p2}\dot\theta_2^2+J_{p3}\dot\theta_4^{\,2}+J_{p3}\dot\theta_3^{\,2}+\frac{1}{2}M_x\dot x_x^{\,2}+\underbrace{\frac{1}{2}J_s\dot\theta_2^{\,2}}_{J\ \text{ALBERO}}$$

VELOCITA' ASSE x

$$\begin{cases}
\theta_1: & J_m\ddot\theta_1+J_{p1}\ddot\theta_1+2k_1(\theta_1 r_1-\theta_2 r_2)r_1+2c_1(\dot\theta_1 r_1-\dot\theta_2 r_2)r_1=C_m\\
\theta_2: & J_{p2}\ddot\theta_2+J_s\ddot\theta_s+2k_1(\theta_1 r_1-\theta_2 r_2)(-r_2)+k_5(\theta_2 r_3-\theta_4 r_3)r_3+\\
& +2c_1(\dot\theta_1 r_1-\dot\theta_2 r_2)(-r_2)+c_5(\dot\theta_2 r_3-\dot\theta_4 r_3)r_3=0\\
\theta_3: & 2J_{p3}\ddot\theta_3+2k_3(x-\theta_3 r_3)(-r_3)+2c_3(\dot x-\dot\theta_3 r_3)(-r_3)+2k_4(\theta_3 r_3-\theta_4 r_3)(r_3)+\\
& 2c_4(\dot\theta_3 r_3-\dot\theta_4 r_3)(r_3)=0\\
\theta_4: & 2J_{p3}\ddot\theta_4+k_5(\theta_2 r_3-\theta_4 r_3)(-r_3)+c_5(\dot\theta_2 r_3-\dot\theta_4 r_3)(-r_3)+\\
& +2k_2(\theta_4 r_3-x)r_3+2c_2(\dot\theta_4 r_3-\dot x)r_3+2k_4(\theta_3 r_3-\theta_4 r_3)(-r_3)+\\
& +2c_4(\dot\theta_3 r_3-\dot\theta_4 r_3)(-r_3)=0\\
x: & M_x\ddot x_x+2k_2(\theta_4 r_3-x)(-1)+2c_2(\dot\theta_4 r_3-\dot x)(-1)+2k_3(x-\theta_3 r_3)+\\
& +2c_3(\dot x-\dot\theta_3 r_3)=0
\end{cases}$$

$$\bar X=\begin{bmatrix}\theta_1\\\theta_2\\\theta_3\\\theta_4\\x\end{bmatrix}\qquad \dot{\bar X}=\begin{bmatrix}\dot\theta_1\\\dot\theta_2\\\dot\theta_3\\\dot\theta_4\\\dot x\end{bmatrix}\qquad \ddot{\bar X}=\begin{bmatrix}\ddot\theta_1\\\ddot\theta_2\\\ddot\theta_3\\\ddot\theta_4\\\ddot x\end{bmatrix}\qquad \bar T_M=\begin{bmatrix}C_m\\0\\0\\0\\0\end{bmatrix}$$

$$\underbrace{\begin{bmatrix}J_m+J_{p1}&0&0&0&0\\0&J_{p2}+J_s&0&0&0\\0&0&2J_{p3}&0&0\\0&0&0&2J_{p3}&0\\0&0&0&0&M_x\end{bmatrix}}_{[M]}\ddot{\bar X}+$$

$$\begin{bmatrix}2r_1^2k_1&-2k_1r_1r_2&0&0&0\\-2r_1r_2k_1&2r_2^2k_1+r_3^2k_5&0&-r_3^2k_5&0\\0&0&2r_3^2(k_3+k_4)&-2r_3^2k_4&-2r_3k_3\\0&-r_3^2k_5&-2r_3^2k_4&r_3^2k_5+2r_3k_2+2r_3^2k_4&-2r_3k_2\end{bmatrix}\bar X+$$

$$\underbrace{\begin{bmatrix} 0 & 0 & -2r_3k_3 & -2r_3k_2 & 2k_2+2k_3 \end{bmatrix}}_{[K]}$$

$$\underbrace{\begin{bmatrix} 2r_1^2c_1 & -2c_1r_1r_2 & 0 & 0 & 0 \\ -2r_1r_2c_1 & 2r_2^2c_1+r_3^2c_5 & 0 & -r_3^2c_5 & 0 \\ 0 & 0 & 2r_3^2(c_3+c_4) & -2r_3^2c_4 & -2r_3c_3 \\ 0 & -r_3^2c_5 & -2r_3^2c_4 & r_3^2c_5+2r_3c_2+2r_3^2c_4 & -2r_3c_2 \\ 0 & 0 & -2r_3c_3 & -2r_3c_2 & 2c_2+2c_3 \end{bmatrix}}_{[C]} \dot{\bar{X}} = \bar{T}_m$$