PROBLEM 2 – OPERATIONS RESEARCH

Maximizing profits from the production of washing machines

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A manufacturer of household appliances produces four models of washing machines W4, W5, W40 and W50. These appliances consist mainly of a metal drum, covered with a sheet metal casing, which is rotated by an electric motor, and an electric programmer is used to control the operation of the washing machines.

The W4 and W40 models have a lower load capacity (4 kg), requiring 5 m^2 of sheet metal, while the W5 and W50 models are slightly larger (5 kg), requiring 8.5 m^2 of sheet metal. The available quantity of sheet metal is 10,000 m^2.

The W4 and W5 models have a motor called M1 and a programmer called P1. The W40 and W50 models have an M2 motor and a P2 type programmer. The M1 motor is less powerful than the M2 and the P1 programmer has fewer options than the P2 programmer. The material required to manufacture the motors can be obtained with virtually no limitations.

The motors are assembled in an assembly unit with a working capacity of 3,000 hours, 1 hour being necessary to assemble an M1 motor and 1.5 hours to assemble an M2 motor.

As for the programmers, they can either be manufactured in-house in a section of the assembly plant, or they can be ordered from an electronics manufacturer. In the first case, they compete with motor assembly (the motor manufacturing process takes hours of assembly in the assembly unit), requiring 0.3 hours to manufacture P1 at a cost of 6 euros and 0.75 hours to manufacture P2 at a cost of 11 euros. In the second case, the supplier can provide any quantity of P1 and P2 at a price of 11 and 21 euros, respectively.

Finally, the washing machines are assembled in another finishing unit with a capacity of 5,000 working hours, requiring 1.5 hours for the W4 model, 2.3 hours for the W5 model, 3 hours for the W40 model and 4.2 hours for the W50 model.

In order to satisfy all sectors, the manufacturer has decided that the minimum production of each model should be 300 units. As an additional fact, it is known, according to a report from the marketing department, that the demand for the higher capacity models is always higher than the demand for the lower capacity models. Therefore, the combined production of W5 and W50 must be higher than the combined production of W4 and W40.

The profit provided by each model excluding labor and materials, except the programmer, is 96 euros for the W4 model, 102 euros for the W5 model, 108 euros for the W40 model and 147 euros for the W50 model.

Pose and solve a linear programming model for the production planning of washing machines aiming at profit maximization.

Variables of the washing machines problem in *Gusek*:

- $x['W4']$, $x['W5']$, $x['W40']$, $x['W50']$ are the number of washing machines that will be built of models W4, W5, W40 and W50, respectively.
- $m['M1']$, $m['M2']$, $m['P1']$, $m['P2']$ refer to the number of **MANUFACTURED** parts (M1 motors, M2 motors, P1 programmers and P2 programmers, respectively).
- $b['P1']$, $b['P2']$ refer to the number of **BOUGHT** parts (P1 programmers and P2 programmers, respectively).

Code preview (available in a separate file):

```
set washing_machines;
set parts;
set programmers;
param m2_sheet_metal{washing_machines};
param manufacturing_hours{parts};
param finishing_hours{washing_machines};

var x{washing_machines}>=0;
var m{parts}>=0;
var b{programmers}>=0;

maximize Profit: 96*x['W4'] + 102*x['W5'] + 108*x['W40'] + 147*x['W50']
- 6*m['P1'] - 11*b['P1'] - 11*m['P2'] - 21*b['P2'];

s.t. SheetMetal: m2_sheet_metal['W4']*x['W4'] + m2_sheet_metal['W5']*x['W5'] +
m2_sheet_metal['W40']*x['W40'] + m2_sheet_metal['W50']*x['W50'] <= 10000;
s.t. ManufacturingHours: manufacturing_hours['M1']*m['M1'] + manufacturing_hours['M2']*m['M2'] +
manufacturing_hours['P1']*m['P1'] + manufacturing_hours['P2']*m['P2'] <= 3000;
s.t. FinishingHours: finishing_hours['W4']*x['W4'] + finishing_hours['W5']*x['W5'] +
finishing_hours['W40']*x['W40'] + finishing_hours['W50']*x['W50'] <= 5000;
s.t. MinimumW4: x['W4']>=300;
s.t. MinimumW5: x['W5']>=300;
s.t. MinimumW40: x['W40']>=300;
s.t. MinimumW50: x['W50']>=300;
s.t. CapacityDemand: x['W5'] + x['W50'] >= x['W4'] + x['W40'];
s.t. Motor1: m['M1'] >= x['W4'] + x['W5'];
s.t. Motor2: m['M2'] >= x['W40'] + x['W50'];
s.t. Programmer1: m['P1'] + b['P1'] >= x['W4'] + x['W5'];
s.t. Programmer2: m['P2'] + b['P2'] >= x['W40'] + x['W50'];

end;
```

The data are located in a separate *.dat* file containing the following information:

```
data;
set washing_machines:=W4 W5 W40 W50;
set parts:=M1 M2 P1 P2;
set programmers:=P1 P2;
param m2_sheet_metal:=W4 5 W5 8.5 W40 5 W50 8.5;
param manufacturing_hours:=M1 1 M2 1.5 P1 0.3 P2 0.75;
param finishing_hours:=W4 1.5 W5 2.3 W40 3 W50 4.2;
end;
```

<u>Solution</u>:

```
Problem:    problem2
Rows:       13
Columns:    10
Non-zeros:  42
Status:     OPTIMAL
Objective:  Profit = 158492.5926 (MAXimum)

   No.   Row name   St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 Profit       B         158493
     2 SheetMetal   NU         10000                       10000       17.2593
     3 ManufacturingHours
                    B        2763.33                        3000
     4 FinishingHours
                    B        4313.33                        5000
     5 MinimumW4    NL           300           300                          -7
     6 MinimumW5    NL           300           300                         -40
     7 MinimumW40   B        440.741           300
     8 MinimumW50   B        440.741           300
     9 CapacityDemand
                    NL             0            -0                    -10.7037
    10 Motor1       NL             0            -0                       < eps
    11 Motor2       NL             0            -0                       < eps
    12 Programmer1  NL             0            -0                          -6
    13 Programmer2  NL             0            -0                         -11

   No. Column name  St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 x[W50]       B        440.741             0
     2 x[W40]       B        440.741             0
     3 x[W5]        B            300             0
     4 x[W4]        B            300             0
     5 m[P2]        B        881.481             0
     6 m[P1]        B            600             0
     7 m[M2]        B        881.481             0
     8 m[M1]        B            600             0
     9 b[P2]        NL             0             0                         -10
    10 b[P1]        NL             0             0                          -5
```

According to this solution, the maximum profit is **158,492.5926 euros** and:

- 300 washing machines of type W4 are to be built.
- 300 washing machines of type W5 are to be built.
- 440.741 washing machines of type W40 are to be built.
- 440.741 washing machines of type W50 are to be built.

- 600 M1 motors are to be manufactured.
- 881.481 M2 motors are to be manufactured.
- 600 P1 programmers are to be manufactured.
- 881.481 P2 programmers are to be manufactured.

- No P1 programmers need to be bought.
- No P2 programmers need to be bought.

In addition,

- The whole 10,000 m^2 of available sheet metal are consumed.
- Only 2,763.33 working hours are used in the assembly unit out of the 3,000 hours available.
- Only 4,313.33 working hours are spent in the finishing unit out of the 5,000 hours available.
- Having been told to build an equal or greater number of 5 kg capacity washing machines than 4 kg capacity washing machines, in reality, the same number of both are built.
- To ensure that there are always enough motors and programmers to build the number of washing machines we want, there were several conditions. We had imposed that there should always be an equal or greater number of M1 motors than the sum of W4 and W5 washing machines, an equal or greater number of M2 motors than the sum of W40 and W50 washing machines, an equal or greater number of P1 programmers than the sum of W4 and W5 washing machines, and an equal or greater number of P2 programmers than the sum of W40 and W50 washing machines. In fact, in this optimal solution, no motors and no programmers are wasted. All the motors and programmers obtained end up installed in washing machines.

Here, a linear programming model has been proposed. All variables take real values. Nevertheless, in reality, the number of washing machines, motors and programmers should be integer numbers. Integer programming problems are more costly than linear programming problems, but would provide a more *realistic* solution.