PROBLEM 1 – OPERATIONS RESEARCH

Maximizing the benefits of agricultural production

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An agricultural association has two plots. Plot P1 has 400 hectares of usable land and 500 m^3 of water, while plot P2 has 900 hectares of usable land and 1200 m^3 of water.

The recommended crops are beet and cotton. Beet consumes 3 m^3 of water per hectare, with a profit of 700 monetary units per hectare. Cotton consumes 2 m^3 of water per hectare, with a profit of 500 monetary units per hectare.

A maximum production portion has been established for each crop equal to 800 hectares for beet and 600 hectares for cotton, and the total percentage of cultivated land must be the same in each of the two plots (if 40% is cultivated in P1 then also 40% in P2).

Pose and solve the problem that allows to determine the production plan that maximizes profit.

Solving the problem using *Lingo*:

Code preview (available in a separate file):

```
!Variables:

x11 = "Number of hectares cultivated with beet in plot P1"
x12 = "Number of hectares cultivated with beet in plot P2"
x21 = "Number of hectares cultivated with cotton in plot P1"
x22 = "Number of hectares cultivated with cotton in plot P2";

!Objective function;

[Profit] max = 700*(x11 + x12) + 500*(x21 + x22);

!Constraints;

[HectaresP1] x11 + x21 <= 400;
[HectaresP2] x12 + x22 <= 900;
[WaterP1] 3*x11 + 2*x21 <= 500;
[WaterP2] 3*x12 + 2*x22 <= 1200;
[BeetPortion] x11 + x12 <= 800;
[CottonPortion] x21 + x22 <= 600;
[SamePercentage] ((x11 + x21)/400)*100 = ((x12 + x22)/900)*100;
```

Solution:

```
Global optimal solution found.
Objective value:                              416666.7
Infeasibilities:                              0.000000
Total solver iterations:                             4
Elapsed runtime seconds:                          0.51

Model Class:                                        LP

Total variables:                       4
Nonlinear variables:                   0
Integer variables:                     0

Total constraints:                     8
Nonlinear constraints:                 0

Total nonzeros:                       20
Nonlinear nonzeros:                    0
```

Variable	Value	Reduced Cost
X11	28.20513	0.000000
X12	138.4615	0.000000
X21	207.6923	0.000000
X22	392.3077	0.000000

Row	Slack or Surplus	Dual Price
PROFIT	416666.7	1.000000
HECTARESP1	164.1026	0.000000
HECTARESP2	369.2308	0.000000
WATERP1	0.000000	233.3333
WATERP2	0.000000	233.3333
BEETPORTION	633.3333	0.000000
COTTONPORTION	0.000000	33.33333
SAMEPERCENTAGE	0.000000	0.000000

According to this solution, the maximum profit is **416666.7** monetary units and we have to cultivate:

- 28.20513 hectares of beet in P1.
- 138.4615 hectares of beet in P2.
- 207.6923 hectares of cotton in P1.
- 392.3077 hectares of cotton in P2.

We would not need:

- 164.1026 hectares of land in P1.
- 369.2308 hectares of land in P2.
- 633.3333 hectares with respect to the maximum land available for beet.

Solving the problem using *Gusek*:

Code preview (available in a separate file):

```
/*Variables:
x11 = "Number of hectares cultivated with beet in plot P1"
x12 = "Number of hectares cultivated with beet in plot P2"
x21 = "Number of hectares cultivated with cotton in plot P1"
x22 = "Number of hectares cultivated with cotton in plot P2"*/

var x11>=0;
var x12>=0;
var x21>=0;
var x22>=0;

/*Objective function*/

maximize Profit: 700*(x11 + x12) + 500*(x21 + x22);

/*Constraints*/

s.t. HectaresP1: x11 + x21 <= 400;
s.t. HectaresP2: x12 + x22 <= 900;
s.t. WaterP1: 3*x11 + 2*x21 <= 500;
s.t. WaterP2: 3*x12 + 2*x22 <= 1200;
s.t. BeetPortion: x11 + x12 <= 800;
s.t. CottonPortion: x21 + x22 <= 600;
s.t. SamePercentage: ((x11 + x21)/400)*100 = ((x12 + x22)/900)*100;

end;
```

Solution:

```
Problem:    problem1
Rows:       8
Columns:    4
Non-zeros:  20
Status:     OPTIMAL
Objective:  Profit = 416666.6667 (MAXimum)
```

No.	Row name	St	Activity	Lower bound	Upper bound	Marginal
1	Profit	B	416667			
2	HectaresP1	B	235.897		400	
3	HectaresP2	B	530.769		900	
4	WaterP1	NU	500		500	233.333
5	WaterP2	NU	1200		1200	233.333
6	BeetPortion	B	166.667		800	
7	CottonPortion	NU	600		600	33.3333
8	SamePercentage	NS	0	-0	=	< eps

No.	Column name	St	Activity	Lower bound	Upper bound	Marginal
1	x11	B	28.2051	0		
2	x12	B	138.462	0		
3	x21	B	207.692	0		
4	x22	B	392.308	0		

According to this solution, the maximum profit is **416666.6667** monetary units and we have to cultivate:

- 28.2051 hectares of beet in P1.
- 138.462 hectares of beet in P2.
- 207.692 hectares of cotton in P1.
- 392.308 hectares of cotton in P2.

And we must use:

- 235.897 hectares of land in P1.
- 530.769 hectares of land in P2.
- 500 m^3 of water in P1.
- 1200 m^3 of water in P2.
- 166.667 hectares of beet.
- 600 hectares of cotton.