Logică EXAMEN – 20.01.2021

Rândul 3

Subiectul 1. a) Definiţiile reuniunii, intersecţiei şi compunerii a două relaţii.

b) Fie $A = \{1, 2, 3, 4, 5\}$ şi R, S, $S' \subseteq A \times A$, unde
$R = \{(1, 2), (1, 3), (1, 5), (2, 3), (2, 4), (2, 5), (4, 1), (4, 3), (5, 2), (5, 4), (3, 2)\}$,
$S = \{(1, 1), (1, 5), (4, 3), (2, 4), (3, 4)\}$, $S' = \{(1, 4), (5, 4), (1, 2), (2, 5), (3, 3), (4, 4)\}$.
Să se determine relaţia $(R \circ S) \cap (R \circ S')$.

c) Fie relaţiile $\tau = (C, D, S)$, $\sigma = (C, D, S)$ şi $\rho = (A, B, S)$. Să se arate că: $(\sigma \cup \tau) \circ \rho = (\sigma \circ \rho) \cup (\tau \circ \rho)$. Să se precizeze toate tautologiile care au fost folosite în demonstraţie.

Subiectul 2. a) Teorema I de factorizare (enunţ).

b) Să se aplice în cazul funcţiei $f : A \to B$, unde $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{a, b, c, d\}$ şi

x	1	2	3	4	5	6	7	8
$f(x)$	b	d	b	d	a	b	b	d

c) Fie $f : A \to B$ o funcţie. Să se arate că $\ker f$ este relaţie de echivalenţă pe A.

Subiectul 3. a) Mulţimi ordonate, bine ordonate (2 definiţii).

b) Principiul inducţiei complete pentru mulţimi bine ordonate (enunţul corolarului).

c) Să se arate că $(\mathbb{N}, \leq)$ este bine ordonată.

Subiectul 4. Mulţimea numerelor raţionale:
a) Construcţie, definiţiile operaţiilor şi a relaţiei de ordine.
b) Să se verifice proprietatea de distributivitate.

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Row 3

Question 1. a) State the definitions of the union, intersection and composition of two relations.

b) Let $A = \{1, 2, 3, 4, 5\}$, and let R, S, $S' \subseteq A \times A$, where
$R = \{(1, 2), (1, 3), (1, 5), (2, 3), (2, 4), (2, 5), (4, 1), (4, 3), (5, 2), (5, 4), (3, 2)\}$,
$S = \{(1, 1), (1, 5), (4, 3), (2, 4), (3, 4)\}$, $S' = \{(1, 4), (5, 4), (1, 2), (2, 5), (3, 3), (4, 4)\}$. Find the relation $(R \circ S) \cap (R \circ S')$.

c) Consider the relations $\tau = (C, D, S)$, $\sigma = (C, D, S)$ and $\rho = (A, B, S)$. Prove that $(\sigma \cup \tau) \circ \rho = (\sigma \circ \rho) \cup (\tau \circ \rho)$. State separately all the tautologies which have been used in the proof.

Question 2. a) State the 1^{st} Factorization Theorem.

b) Apply the theorem in the case of the function $f : A \to B$, where $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{a, b, c, d\}$ and

x	1	2	3	4	5	6	7	8
$f(x)$	b	d	b	d	a	b	b	d

c) Let $f : A \to B$ o function. Prove that $\ker f$ is an equivalence relation on A.

Question 3. a) Ordered sets, totally ordered sets, well-ordered sets (3 definitions).

b) The principle of complete induction for well-ordered sets (statement of the corollary).

c) Prove that $(\mathbb{N}, \leq)$ is well-ordered.

Question 4. The set of rational numbers:
a) Construction, definitions of the operations and of the order relation.
b) Verify the distributive property.