Hidden Markov Models

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A *hidden Markov model* (HMM) is a temporal probabilistic model in which the state of the process is described by a single discrete random variable. Loosely speaking, it is a Markov chain observed in noise. The theory of hidden Markov models was developed in the late 1960s and early 1970s by Baum, Eagon, Petrie, Soules and Weiss (Baum and Petrie 1966; Baum and Eagon 1967; Baum, *et al.* 1970; Baum 1972), whilst the name 'hidden Markov model' was coined by L. P. Neuwirth. For more information on HMMs, see the tutorial papers Rabiner and Juang (1986); Poritz (1988); Rabiner (1989); Eddy (2004) and the books Mac-Donald and Zucchini (1997); Durbin, *et al.* (1999); Elliot, Aggoun and Moore (2004); Cappé, Moulines and Rydén (2005). HMMs have earned their popularity largely from successful application to speech recognition (Rabiner 1989), but have also been applied to handwriting recognition, gesture recognition, musical score following and bioinformatics.

Formally, a hidden Markov model is a bivariate discrete time process $\{X_k, Y_k\}_{k \geq 0}$, where X_k is a Markov chain and, conditional on X_k, Y_k is a sequence of independent random variables such that the conditional distribution of Y_k only depends on X_k.

The successful application of HMMs to markets is referenced as far back as Kemeny, Snell and Knapp (1976) and Juang (1985). The books Bhar and Hamori (2004) and Mamon and Elliott (2007) cover HMMs in finance.

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