Seminar

① Pe parabola de ecuație $y^2=2px$ se iau trei puncte distincte A, B, C. Tangentele în punctele A, B, C la parabolă determină un triunghi $A'B'C'$. a) Să se demonstreze că dreapta care unește centrele de greutate ale triunghiurilor ABC și $A'B'C'$ este paralelă cu axa Ox.

b). Să se demonstreze că $\mathcal{A}_{[ABC]}=2\mathcal{A}_{[A'B'C']}$.

Soluție a) Fie $A\left(\frac{\alpha^2}{2p},\alpha\right)$, $B\left(\frac{\beta^2}{2p},\beta\right)$, $C\left(\frac{\gamma^2}{2p},\gamma\right)$.

Ecuația tangentei la parabolă în punctul $M_0(x_0,y_0)$ este $yy_0=p(x+x_0)$.

Ec. tangentei în A : $t_A: \alpha y=p\left(x+\frac{\alpha^2}{2p}\right)$

—— || —— B : $t_B: \beta y=p\left(x+\frac{\beta^2}{2p}\right)$

—— || —— C : $t_C: \gamma y=p\left(x+\frac{\gamma^2}{2p}\right)$

$\{A'\}=t_B\cap t_C : (\beta-\gamma)y=\frac{\beta^2-\gamma^2}{2} \Rightarrow$

$\Rightarrow y_{A'}=\frac{\beta+\gamma}{2}$. Analog $y_{B'}=\frac{\gamma+\alpha}{2}$, $y_{C'}=\frac{\alpha+\beta}{2}$.

$$y_G = \frac{\alpha+\beta+\gamma}{3} \; ; \; y_{G'} = \frac{\frac{\alpha+\beta}{2} + \frac{\beta+\gamma}{2} + \frac{\gamma+\alpha}{2}}{3} = \frac{\alpha+\beta+\gamma}{3}$$

$\Rightarrow GG' \parallel Ox.$

b). $x_{A'}$: $\beta \frac{\beta+\gamma}{2} = px + \frac{\beta^2}{2} \Rightarrow$

$\Rightarrow x_{A'} = \frac{\beta\gamma}{2p}$; Analog $x_{B'} = \frac{\gamma\alpha}{2p}$, $x_{C'} = \frac{\alpha\beta}{2p}$.

$$A_{\Delta A'B'C'} = \frac{1}{2} \left| \begin{vmatrix} \frac{\beta\gamma}{2p} & \frac{\beta+\gamma}{2} & 1 \\ \frac{\gamma\alpha}{2p} & \frac{\gamma+\alpha}{2} & 1 \\ \frac{\alpha\beta}{2p} & \frac{\alpha+\beta}{2} & 1 \end{vmatrix} \right| =$$

$$= \frac{1}{8p} \left| \begin{vmatrix} \beta\gamma & \beta+\gamma & 1 \\ \gamma\alpha & \gamma+\alpha & 1 \\ \alpha\beta & \alpha+\beta & 1 \end{vmatrix} \right| = \frac{1}{8p} \left| \begin{vmatrix} \beta\gamma & \beta+\gamma & 1 \\ \gamma(\alpha-\beta) & \alpha-\beta & 0 \\ \beta(\alpha-\gamma) & \alpha-\gamma & 0 \end{vmatrix} \right| =$$

$$= \frac{1}{8p} |(\alpha-\beta)(\alpha-\gamma)| \cdot \left| \begin{vmatrix} \gamma & 1 \\ \beta & 1 \end{vmatrix} \right| = \frac{1}{8p} |(\alpha-\beta)(\alpha-\gamma)(\gamma-\beta)|.$$

$$A_{\Delta ABC} = \frac{1}{2} \left| \begin{vmatrix} \frac{\alpha^2}{2p} & \alpha & 1 \\ \frac{\beta^2}{2p} & \beta & 1 \\ \frac{\gamma^2}{2p} & \gamma & 1 \end{vmatrix} \right| = \frac{1}{4p} \left| \begin{vmatrix} \alpha^2 & \alpha & 1 \\ \beta^2 & \beta & 1 \\ \gamma^2 & \gamma & 1 \end{vmatrix} \right| =$$

$$= \frac{1}{4p} \left| \begin{vmatrix} \alpha^2 & \alpha & 1 \\ \beta^2-\alpha^2 & \beta-\alpha & 0 \\ \gamma^2-\alpha^2 & \gamma-\alpha & 0 \end{vmatrix} \right| = \frac{1}{4p} |(\beta-\alpha)(\gamma-\alpha)| \left| \begin{vmatrix} \beta+\alpha & 1 \\ \gamma+\alpha & 1 \end{vmatrix} \right| =$$

$$= \frac{1}{4p} |(\beta-\alpha)(\gamma-\alpha)(\beta-\gamma)| \Rightarrow A_{\Delta ABC} = 2 A_{\Delta A'B'C'}.$$