# Generatoare rectilinii

① Formați ecuațiile generatoarelor rectilinii ale hiperboloidului cu o pânză

$$\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$$

paralele cu planul

$$\Pi: 6x+4y+3z-17=0.$$

Soluție $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1 \Leftrightarrow \frac{x^2}{4}-\frac{z^2}{16}=1-\frac{y^2}{9} \Leftrightarrow$

$$\left(\frac{x}{2}+\frac{z}{4}\right)\left(\frac{x}{2}-\frac{z}{4}\right)=\left(1+\frac{y}{3}\right)\left(1-\frac{y}{3}\right).$$

$$(D_\lambda): \begin{cases} \frac{x}{2}+\frac{z}{4}=\lambda\left(1+\frac{y}{3}\right) & |\cdot 12 \\ \frac{x}{2}-\frac{z}{4}=\frac{1}{\lambda}\left(1-\frac{y}{3}\right) & |\cdot 12\lambda \end{cases}$$

$$(D_\mu): \begin{cases} \frac{x}{2}+\frac{z}{4}=\mu\left(1-\frac{y}{3}\right) & |\cdot 12 \\ \frac{x}{2}-\frac{z}{4}=\frac{1}{\mu}\left(1+\frac{y}{3}\right) & |\cdot 12\mu \end{cases}$$

Vectorul normal al planului este $\vec{n}(A,B,C)=$

$=\vec{n}(6,4,3)$.

$$(D_\lambda): \begin{cases} 6x-4\lambda y+3z-12\lambda=0 \\ 6\lambda x+4y-3\lambda z-12=0 \end{cases} \quad (*)$$

$\vec{D}_\lambda(p_\lambda, q_\lambda, r_\lambda)$

$$\begin{matrix} P_\lambda & Q_\lambda & R_\lambda \\ 6 & -4\lambda & 3 \\ 6\lambda & 4 & -3\lambda \end{matrix}$$

$$P_\lambda = \begin{vmatrix} -4\lambda & 3 \\ 4 & -3\lambda \end{vmatrix} = 12(\lambda^2-1)$$

$$Q_\lambda = -\begin{vmatrix} 6 & 3 \\ 6\lambda & -3\lambda \end{vmatrix} = 36\lambda$$

$$R_\lambda = \begin{vmatrix} 6 & -4\lambda \\ 6\lambda & 4 \end{vmatrix} = 24(\lambda^2+1)$$

$\vec{D}_\lambda \left(12(\lambda^2-1), 36\lambda, 24(\lambda^2+1)\right)$

Alt vector director $\vec{D'}_\lambda \left((\lambda^2-1), 3\lambda, 2(\lambda^2+1)\right)$

$D_\lambda \parallel \pi \iff \vec{D'}_\lambda \perp \vec{N} \iff \vec{D'}_\lambda \cdot \vec{N} = 0 \iff$

$\iff 6(\lambda^2-1) + 12\lambda + 6(\lambda^2+1) = 0 \iff$

$\lambda^2 - 1 + 2\lambda + \lambda^2 + 1 = 0 \iff 2\lambda^2 + 2\lambda = 0 \iff$

$\iff \boxed{\lambda = 0}$ $\boxed{\lambda = -1}$

$$\Rightarrow D: \begin{cases} \frac{x'}{2} + \frac{z}{4} = -1 - \frac{y}{3} & |\cdot 12 \\ \frac{x}{2} - \frac{z}{4} = -1 + \frac{y}{3} & |\cdot 12 \end{cases}$$

$$\Rightarrow D: \begin{cases} 6x + 4y + 3z + 12 = 0 \\ 6x - 4y - 3z + 12 = 0 \end{cases}$$

Cazul $\lambda = 0$ în sistemul (*)

$$\begin{cases} 6x + 3z = 0 \\ 4y - 12 = 0 \end{cases} \Leftrightarrow \boxed{\begin{cases} 2x + z = 0 \\ y - 3 = 0 \end{cases}}$$

Analog $\mu$: ...