# Seminar 6

## Lista 5+6

② Folosind Teorema lui Rouché, să se discute după parametrul $\alpha \in \mathbb{R}$ compatibilitatea sistemelor, apoi să se rezolve:

a) $\begin{cases} 5x_1 - 3x_2 + 2x_3 + 4x_4 = 3 \\ 4x_1 - 2x_2 + 3x_3 + 7x_4 = 1 \\ 8x_1 - 6x_2 - x_3 - 5x_4 = 9 \\ 7x_1 - 3x_2 + 7x_3 + 17x_4 = \alpha \end{cases}$ (în $\mathbb{R}^4$)

$$\bar{A} = \left(\begin{array}{cccc|c} 5 & -3 & 2 & 4 & 3 \\ 4 & -2 & 3 & 7 & 1 \\ 8 & -6 & -1 & -5 & 9 \\ 7 & -3 & 7 & 17 & \alpha \end{array}\right)$$

- căutăm minorul principal.

$$\begin{vmatrix} 5 & -3 \\ 4 & -2 \end{vmatrix} = 2 \neq 0$$

liniile sunt proporționale ($\frac{4}{3}$).

$$\begin{vmatrix} 5 & -3 & 2 \\ 4 & -2 & 3 \\ 8 & -6 & -1 \end{vmatrix} = \begin{vmatrix} 21 & -15 & 0 \\ 28 & -20 & 0 \\ 8 & -6 & -1 \end{vmatrix} = (-1)^{3+3} \cdot (-1) \begin{vmatrix} 21 & -15 \\ 28 & -20 \end{vmatrix} = 0$$

$L_1 + 2L_3;\ L_2 + 3L_3$

- bordăm cu altă linie și coloană

$$\begin{vmatrix} 5 & -3 & 2 \\ 4 & -2 & 3 \\ 7 & -3 & 7 \end{vmatrix} = 0 \quad ; \quad \begin{vmatrix} 5 & -3 & 4 \\ 4 & -2 & 7 \\ 7 & -3 & 17 \end{vmatrix} = 0 \ ;$$

$$\begin{vmatrix} 5 & -3 & 4 \\ 4 & -2 & 7 \\ 8 & -6 & -5 \end{vmatrix} = 0 .$$

Prin urmare $\begin{vmatrix} 5 & -3 \\ 4 & -2 \end{vmatrix}$ este minorul principal ($\Rightarrow \operatorname{rang} A = 2$).

Există 2 posibilități de a borda minorul principal pt. a obține minori caracteristici :

$$\begin{vmatrix} 5 & -3 & 3 \\ 4 & -2 & 1 \\ 8 & -6 & 9 \end{vmatrix} = \begin{vmatrix} -7 & 3 & 3 \\ 0 & 0 & 1 \\ -28 & 12 & 9 \end{vmatrix} = 0$$

$C_1 - 4C_3 ; C_2 + 2C_3$ $\quad \left(-\frac{7}{3} = -\frac{28}{9}\right)$

$$\begin{vmatrix} 5 & -3 & 3 \\ 4 & -2 & 1 \\ 7 & -3 & \alpha \end{vmatrix} = \begin{vmatrix} -7 & 3 & 3 \\ 0 & 0 & 1 \\ 7-4\alpha & -3+2\alpha & \alpha \end{vmatrix} = (-1)^{2+3} \cdot (\cancel{21} - 14\alpha - \cancel{21} + 12\alpha) = 2\alpha$$

$C_1 - 4C_3 ; C_2 + 2C_3$

I Dacă $\alpha \neq 0 \Rightarrow$ sistem incompatibil

II Dacă $\alpha = 0 \Rightarrow$ sistem compatibil nedeterminat echivalent cu :

$$\begin{cases} 5x_1 - 3x_2 = 3 - 2x_3 - 4x_4 \ /\cdot 2 \\ 4x_1 - 2x_2 = 1 - 3x_3 - 7x_4 \ /\cdot(-3) \end{cases} \Leftrightarrow \begin{cases} 10x_1 - 6x_2 = 6 - 4x_3 - 8x_4 \\ -12x_1 + 6x_2 = -3 + 9x_3 + 21x_4 \end{cases}$$

$$-2x_1 \quad / \quad = 3 + 5x_3 + 13x_4$$

$$\Rightarrow x_1 = -\frac{3}{2} - \frac{5}{2}x_3 - \frac{13}{2}x_4$$

$$-2x_2 = 1 - 3x_3 - 7x_4 - (-6 - 10x_3 - 26x_4)$$

$$-2x_2 = 1 - 3x_3 - 7x_4 + 6 + 10x_3 + 26x_4$$

$$-2x_2 = 7 + 7x_3 + 19x_4$$

$$x_2 = -\frac{7}{2} - \frac{7}{2}x_3 - \frac{19}{2}x_4$$

$$\Rightarrow S = \left\{\left(-\frac{3}{2} - \frac{5}{2}x_3 - \frac{13}{2}x_4 ; -\frac{7}{2} - \frac{7}{2}x_3 - \frac{19}{2}x_4 ; x_3 ; x_4\right) \middle| x_3, x_4 \in \mathbb{R}\right\}$$

## II Metoda lui Gauss

$$\left(\begin{array}{cccc|c} 5 & -3 & 2 & 4 & 3 \\ 4 & -2 & 3 & 7 & 1 \\ 8 & 6 & -1 & -5 & 9 \\ 7 & 3 & 7 & 17 & \alpha \end{array}\right) \underset{l_1 - l_2}{\sim} \left(\begin{array}{cccc|c} 1 & -1 & -1 & -3 & 2 \\ 4 & -2 & 3 & 7 & 1 \\ 8 & 6 & -1 & -5 & 9 \\ 7 & 3 & 7 & 17 & \alpha \end{array}\right) \underset{\substack{l_2 - 4l_1 \\ l_3 - 8l_1 \\ l_4 - 7l_1}}{\sim}$$

$$\sim\left(\begin{array}{cccc|c} 1 & -1 & -1 & -3 & 2 \\ 0 & 2 & 7 & 19 & -7 \\ 0 & 2 & 7 & 19 & -7 \\ 0 & 4 & 14 & 38 & \alpha-14 \end{array}\right) \underset{\substack{l_3-l_2 \\ l_4-2l_2}}{\sim} \left(\begin{array}{cccc|c} 1 & -1 & -1 & -3 & 2 \\ 0 & 2 & 7 & 19 & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha \end{array}\right) \underset{l_3 \leftrightarrow l_4}{\sim}$$

$$\sim\left(\begin{array}{cccc|c} 1 & -1 & -1 & -3 & 2 \\ 0 & 2 & 7 & 19 & -7 \\ 0 & 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1, x_2$ = nec. principale

Dacă $\alpha \in \mathbb{R} \setminus \{0\} \Rightarrow$ sistemul este incompatibil

Dacă $\alpha = 0 \Rightarrow$ sistemul este comp. nedeterminat, echivalent cu:

$$\begin{cases} x_1 - x_2 - x_3 - 3x_4 = 2 \\ 2x_2 + 7x_3 + 19x_4 = -7 \end{cases} \quad \ldots \text{ (temă)}.$$

b) $$\begin{cases} 2x_1 - x_2 + 3x_3 + 4x_4 = 5 \\ 4x_1 - 2x_2 + 5x_3 + 6x_4 = 7 \\ 6x_1 - 3x_2 + 7x_3 + 8x_4 = 9 \\ \alpha x_1 - 4x_2 + 9x_3 + 10x_4 = 11 \end{cases} \quad (\text{în } \mathbb{R}^4)$$

<u>Metoda I</u> (T. Rouché)

$$\bar{A} = \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ 6 & -3 & 7 & 8 & 9 \\ \alpha & -4 & 9 & 10 & 11 \end{array}\right)$$

$$\begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix} = -5 + 6 = 1 \neq 0; \quad \begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ 6 & -3 & 7 \end{vmatrix} = 0; \quad \begin{vmatrix} -1 & 3 & 4 \\ -2 & 5 & 6 \\ -3 & 7 & 8 \end{vmatrix} = 0;$$

$2L_2 = L_1 + L_3$

$$\begin{vmatrix} -1 & 3 & 4 \\ -2 & 5 & 6 \\ -4 & 9 & 10 \end{vmatrix} = \begin{vmatrix} -1 & 3 & 4 \\ 0 & -1 & -2 \\ 0 & -3 & -6 \end{vmatrix} = (-1)^2 \cdot (-1) \cdot 0 = 0;$$

$L_2 - L_1;\ L_3 - 4L_1$

$$\begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ \alpha & -4 & 9 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 3 \\ 0 & -2 & 5 \\ \alpha-8 & -4 & 9 \end{vmatrix} = (-1)^{3+1} \cdot (\alpha - 8) \cdot \begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix} = (\alpha-8)\cdot 1 = \underline{\alpha - 8.}$$

$C_1 + 2C_2$

<u>Cazul I</u> : $\alpha = 8 \Rightarrow \begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix}$ minor principal

Există 2 minori caracteristici corespunzători :

$$\begin{vmatrix} -1 & 3 & 5 \\ -2 & 5 & 7 \\ -3 & 7 & 9 \end{vmatrix} = 0 \;;\; \begin{vmatrix} -1 & 3 & 5 \\ -2 & 5 & 7 \\ -4 & 9 & 11 \end{vmatrix} = \begin{vmatrix} -1 & 3 & 5 \\ 0 & -1 & -3 \\ 0 & -3 & -9 \end{vmatrix} = 0.$$

$L_1 + L_3 = 2L_2$ $\qquad L_2 - 2L_1;\ L_3 - 4L_1$ $\qquad$ proporționale

$\Rightarrow$ sistemul este compatibil nedeterminat echivalent cu:

$$\begin{cases} -x_2 + 3x_3 = 5 - 2x_1 - 4x_4 \ /\cdot(-2) \\ -2x_2 + 5x_3 = 7 - 4x_1 - 6x_4 \end{cases} \quad +$$

$$/ \quad -x_3 = -3 + 2x_4$$

$x_3 = 3 - 2x_4$ ; $\quad -x_2 + 9 - 6x_4 = 5 - 2x_1 - 4x_4 \Rightarrow -x_2 = -4 - 2x_1 + 2x_4$

$S = \{(x_1; 4 + 2x_1 - 2x_4; 3 - 2x_4; x_4) \mid x_1, x_4 \in \mathbb{R}\}$ $\qquad x_2 = 4 + 2x_1 - 2x_4$

<u>Cazul II</u> : $\alpha \neq 8$ atunci $\overbrace{\text{minorul}}^{\text{verificăm dacă}}$ principal este $\begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ \alpha & -4 & 9 \end{vmatrix}$.

$$\begin{vmatrix} 2 & -1 & 3 & 4 \\ 4 & -2 & 5 & 6 \\ 6 & -3 & 7 & 8 \\ \alpha & -4 & 9 & 10 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ \alpha & -4 & 9 \end{vmatrix} \text{ este minorul principal.}$$

$\Rightarrow \text{Rang } A = 3.$

Există un singur minor caracteristic corespunzător:

$$\begin{vmatrix} 2 & -1 & 3 & 5 \\ 4 & -2 & 5 & 7 \\ 6 & -3 & 7 & 9 \\ \alpha & -4 & 9 & 11 \end{vmatrix} = 0 \Rightarrow \text{sistemul este compatibil nedeterminal echivalen}$$

cu următorul sistem:

$$\begin{cases} 2x_1 - x_2 + 3x_3 = 5 - 4x_4 \\ 4x_1 - 2x_2 + 5x_3 = 7 - 6x_4 \\ \alpha x_1 - 4x_2 + 9x_3 = 11 - 10x_4 . \end{cases} \quad \Rightarrow \text{ sistem de tip Cramer (temă).}$$

## II Metoda lui Gauss

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \phantom{5} \end{array} \\ \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ 6 & -3 & 7 & 8 & 9 \\ \alpha & -4 & 9 & 10 & 11 \end{array}\right) \end{array} \underset{c_1 \leftrightarrow c_2}{\sim} \begin{array}{c} \begin{array}{ccccc} x_2 & x_1 & x_3 & x_4 & \phantom{5} \end{array} \\ \left(\begin{array}{cccc|c} -1 & 2 & 3 & 4 & 5 \\ -2 & 4 & 5 & 6 & 7 \\ -3 & 6 & 7 & 8 & 9 \\ -4 & \alpha & 9 & 10 & 11 \end{array}\right) \end{array} \underset{c_2 \leftrightarrow c_4}{\sim}$$

$$\sim \begin{array}{c} \begin{array}{ccccc} x_2 & x_4 & x_3 & x_1 & \phantom{5} \end{array} \\ \left(\begin{array}{cccc|c} -1 & 4 & 3 & 2 & 5 \\ -2 & 6 & 5 & 4 & 7 \\ -3 & 8 & 7 & 6 & 9 \\ -4 & 10 & 9 & \alpha & 11 \end{array}\right) \end{array} \underset{\substack{l_2 - 2l_1 \\ l_3 - 3l_1 \\ l_4 - 4l_1}}{\sim} \begin{array}{c} \begin{array}{ccccc} x_2 & x_4 & x_3 & x_1 & \phantom{5} \end{array} \\ \left(\begin{array}{cccc|c} -1 & 4 & 3 & 2 & 5 \\ 0 & -2 & -1 & 0 & -3 \\ 0 & -4 & -2 & 0 & -6 \\ 0 & -6 & -3 & \alpha-8 & -9 \end{array}\right) \end{array} \underset{\substack{l_3 - 2l_2 \\ l_4 - 3l_2}}{\sim}$$

$$\sim \left(\begin{array}{cccc|c} -1 & 4 & 3 & 2 & 5 \\ 0 & -2 & -1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha-8 & 0 \end{array}\right) \underset{l_4 \leftrightarrow l_3}{\sim} \left(\begin{array}{cccc|c} -1 & 4 & 3 & 2 & 5 \\ 0 & -2 & -1 & 0 & -3 \\ 0 & 0 & 0 & \alpha-8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \underset{c_3 \leftrightarrow c_4}{\sim}$$

(pt. ca matricea să aibă formă trapezoidală).

$$\sim \begin{array}{c} \begin{array}{ccccc} x_2 & x_4 & x_1 & x_3 & \phantom{5} \end{array} \\ \left(\begin{array}{cccc|c} -1 & 4 & 2 & 3 & 5 \\ 0 & -2 & 0 & -1 & -3 \\ 0 & 0 & \alpha-8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

Dacă $\alpha \neq 8 \Rightarrow$ sistemul este comp. nedeterminat, echivalent cu:

$$\begin{cases} -x_2 + 4x_4 + 2x_1 + 3x_3 = 5 \\ \quad -2x_4 - x_3 = -3 \\ \quad (\alpha-8)x_1 = 0 \end{cases} \iff \begin{cases} x_1 = 0 \\ x_2 = -5 + 3x_3 - 2(-3 + x_3) \\ x_3 \in \mathbb{R} \\ x_4 = -\frac{1}{2}(-3 + x_3) \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 1 + x_3 \\ x_3 \in \mathbb{R} \\ x_4 = \frac{1}{2}(3 - x_3) \end{cases}$$

Dacă $\alpha = 8 \Rightarrow$ sist. este comp. nedeterminat, echivalent cu:

$$\begin{cases} -x_2 + 4x_4 + 2x_1 + 3x_3 = 5 \\ -2x_4 \quad\quad - x_3 = -3 \end{cases} \quad \ldots \quad (\text{temă})$$

c) $$\begin{cases} \alpha x + y + z = 1 \\ x + \alpha y + z = 1 \\ x + y + \alpha z = 1 \end{cases} \quad (\text{în } \mathbb{R}^3)$$

Metoda I

$$A = \begin{pmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{pmatrix}; \quad \bar{A} = \begin{pmatrix} \alpha & 1 & 1 & 1 \\ 1 & \alpha & 1 & 1 \\ 1 & 1 & \alpha & 1 \end{pmatrix}$$

$$\det A = \underset{L_1+L_2+L_3}{\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}} = (\alpha+2) \cdot \underset{L_2-L_1}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}} = (\alpha+2) \cdot \begin{vmatrix} 1 & 1 & 1 \\ 0 & \alpha-1 & 0 \\ 1 & 1 & \alpha \end{vmatrix} = (\alpha+2)(\alpha-1)^2$$

$\det A = 0 \Leftrightarrow \alpha = -2$ sau $\alpha = 1$.

1) Dacă $\alpha \in \mathbb{R} \setminus \{-2, 1\}$ sistemul compatibil determinat.

$$d_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = (\alpha-1)^2; \quad d_3 = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & 1 \end{vmatrix} = (\alpha-1)^2.$$

$$d_2 = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & \alpha \end{vmatrix} = (\alpha-1)^2;$$

$$x = \frac{1}{\alpha+2}; \quad y = \frac{1}{\alpha+2}; \quad z = \frac{1}{\alpha+2}$$

Mulțimea soluțiilor sistemului este:

$$S = \left\{ \left( \frac{1}{\alpha+2}; \frac{1}{\alpha+2}; \frac{1}{\alpha+2} \right) \right\}$$

2) Dacă $\alpha = 1$ sist. este echivalent cu:

$$x + y + z = 1$$

Mulțimea sol. este: $S = \{(1-y-z;\ y;\ z) \mid y, z \in \mathbb{R}\}$.

3) Dacă $\alpha = -2$ atunci $\det A = 0$; $\bar{A} = \left(\begin{array}{cc|cc} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 \\ \hline 1 & 1 & -2 & 1 \end{array}\right)$

$\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 \neq 0 \Rightarrow$ minor principal (evident)

Minorul caracteristic corespunzător:

$$\underbrace{\begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix}}_{=(\alpha-1)^2} = 9 \neq 0 \Rightarrow \text{sistemul este incompatibil.}$$

## II Metoda lui Gauss

$$\left(\begin{array}{ccc|c} \alpha & 1 & 1 & 1 \\ 1 & \alpha & 1 & 1 \\ 1 & 1 & \alpha & 1 \end{array}\right) \underset{l_1 <-> l_3}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & \alpha & 1 \\ 1 & \alpha & 1 & 1 \\ \alpha & 1 & 1 & 1 \end{array}\right) \underset{\substack{l_2 - l_1 \\ l_3 - \alpha l_1}}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & \alpha & 1 \\ 0 & \alpha-1 & 1-\alpha & 0 \\ 0 & 1-\alpha & 1-\alpha^2 & 1-\alpha \end{array}\right)$$

$$\underset{l_3 + l_2}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & \alpha & 1 \\ 0 & \alpha-1 & 1-\alpha & 0 \\ 0 & 0 & (1-\alpha)(2+\alpha) & 1-\alpha \end{array}\right)$$

$1-\alpha^2 = (1-\alpha)(1+\alpha)$
$(1-\alpha)(1+\alpha) + (1-\alpha) =$
$= (1-\alpha)(1+\alpha+1)$
$= (1-\alpha)(2+\alpha)$.

Dacă $\alpha = -2 \Rightarrow$ Sistemul este incompatibil.

Dacă $\alpha = 1 \Rightarrow$ Sistemul este comp. nedet. echivalent cu:

$$x_1 + x_2 + x_3 = 0$$

Mulțimea sol. este : $S = \{ (1 - x_2 - x_3, x_2, x_3) \mid x_2, x_3 \in \mathbb{R} \}$

Dacă $\alpha \in \mathbb{R} \setminus \{-2, 1\}$ sistemul este comp. det. echivalent cu:

$$\begin{cases} x_1 + x_2 + \alpha x_3 = 1 \\ (\alpha-1)x_2 + (1-\alpha)x_3 = 0 \\ (1-\alpha)(2+\alpha)x_3 = 1-\alpha \end{cases} \iff \begin{cases} x_1 = \dfrac{1}{2+\alpha} \\ x_2 = \dfrac{1}{2+\alpha} \\ x_3 = \dfrac{1}{2+\alpha} \end{cases}$$

Mulțimea sol. este : $S = \left\{ \left( \dfrac{1}{2+\alpha}, \dfrac{1}{2+\alpha}, \dfrac{1}{2+\alpha} \right) \right\}$

## Completare curs 6:

Fie $A, B \in \mathcal{M}_n(k)$ $(n \in \mathbb{N}^*)$ a.î. $AB = I_n$.

S.s.a.c. $A$ inversabilă în $\mathcal{M}_n(k)$.

Soluție: Considerăm sistemul omogen de $n$ ecuații cu $n$ necunoscute:

$$A \cdot \Big| \; B \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \Rightarrow$$

$$A\left(B \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}\right) = A \cdot \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \Rightarrow \underbrace{(AB)}_{I_n} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \Leftrightarrow \underline{\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}}$$

Prin urmare sistemul este compatibil determinat $\Rightarrow$ rang $B = n \Leftrightarrow \det B \neq 0 \Rightarrow B$ inversabilă în $\mathcal{M}_n(k) \Rightarrow$ $\exists\, B^{-1} \in \mathcal{M}_n(k)$ a.î. $B \cdot B^{-1} = B^{-1} \cdot B = I_n$.

$$AB = I_n \;\Big|\cdot B^{-1} \Rightarrow (AB)B^{-1} = I_n \cdot B^{-1} \Rightarrow A(BB^{-1}) = B^{-1} \Rightarrow$$

$$\Rightarrow A \cdot I_n = B^{-1} \Rightarrow A = \underbrace{B^{-1}}_{\text{inversabilă}} \Rightarrow A \text{ inversabilă.}$$

$\square$