17. 11. 2020

Seminar 7 gr. 311

1. Se dau dreptele $d_1: M_1M_2$ unde $M_1(-1,0,1)$ și $M_2(-2,1,0)$ și

$$d_2: \begin{cases} x+y+z=1 \\ 2x-y-5z=0 \end{cases}.$$

Să se afle distanța dintre cele două drepte și ecuațiile perpendicularei comune.

<u>Soluție</u>. $\overrightarrow{M_1M_2}(-2+1, 1-0, 0-1) \Leftrightarrow \overrightarrow{M_1M_2}(-1,1,-1) = \vec{d_1}(-1,1,-1)$

p	q	r
1	1	1
2	-1	-5

$$p = \begin{vmatrix} 1 & 1 \\ -1 & -5 \end{vmatrix} = -4.$$

$$q = -\begin{vmatrix} 1 & 1 \\ 2 & -5 \end{vmatrix} = 7 \qquad \vec{d_2}(-4, 7, -3)$$

$$r = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3$$

Un punct P pe dreapta d_2 se găsește rezolvând sistemul

$$\begin{cases} x+y+z=1 \\ 2x-y-5z=0 \end{cases}$$

$z = \alpha$ necunoscută secundară

$$\begin{cases} x + y = 1 - \alpha \\ 2x - y = 5\alpha \end{cases}$$

$$3x = 1 + 4\alpha \Rightarrow x = \frac{4\alpha + 1}{3}$$

$$-3y = -2 + 7\alpha \Rightarrow y = \frac{-7\alpha + 2}{3}$$

$$S = \left\{ \left(\frac{4\alpha + 1}{3}, \frac{-7\alpha + 2}{3}, \alpha \right) \middle| \alpha \in \mathbb{R} \right\}$$

pt $\alpha = -1$ obținem $P(-1, 3, -1)$

$$d(d_1, d_2) = \frac{|(\overrightarrow{PM_1}, \vec{d_1}, \vec{d_2})|}{\|\vec{d_1} \times \vec{d_2}\|} \quad \left(= \frac{\text{Volum}}{A_{\text{bazei}}} \right)$$

$$\overrightarrow{PM_1}(-1+1, 0-3, 1+1) \Leftrightarrow \overrightarrow{PM_1}(0, -3, 2).$$

$$\left| \begin{vmatrix} 0 & -3 & 2 \\ -1 & 1 & -1 \\ -4 & 7 & -3 \end{vmatrix} \right| = \left| \begin{vmatrix} -3 & -3 & -1 \\ 0 & 1 & 0 \\ 3 & 7 & 4 \end{vmatrix} \right| = \left| \begin{vmatrix} -3 & -1 \\ 3 & 4 \end{vmatrix} \right| =$$

$$= |-12 + 3| = |-9| = 9. \quad \left(|(\overrightarrow{PM_1}, \vec{d_1}, \vec{d_2})| \right).$$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & -1 \\ -4 & 7 & -3 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 7 & -3 \end{vmatrix} \vec{i} - \begin{vmatrix} -1 & -1 \\ -4 & -3 \end{vmatrix} \vec{j} +$$

$$+ \begin{vmatrix} -1 & 1 \\ -4 & 7 \end{vmatrix} \vec{k} = 4\vec{i} + 1\vec{j} - 3\vec{k}$$

$$\|\vec{d_1} \times \vec{d_2}\| = \sqrt{16 + 1 + 9} = \sqrt{26}$$

$$\Rightarrow d(d_1, d_2) = \frac{9}{\sqrt{26}} = \frac{9\sqrt{26}}{26}.$$

Ecuațiile perpendicularei comune:

$$d: \begin{cases} \Pi_1: \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ p_1 & q_1 & r_1 \\ \begin{vmatrix} q_1 & r_1 \\ q_2 & r_2 \end{vmatrix} & \begin{vmatrix} r_1 & p_1 \\ r_2 & p_2 \end{vmatrix} & \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \end{vmatrix} = 0 \\ \\ \Pi_2: \begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ p_2 & q_2 & r_2 \\ \begin{vmatrix} q_1 & r_1 \\ q_2 & r_2 \end{vmatrix} & \begin{vmatrix} r_1 & p_1 \\ r_2 & p_2 \end{vmatrix} & \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \end{vmatrix} = 0 \end{cases}$$

adică d este intersecția dintre planul Π_1 determinat de dreapta d_1 prin M_1 și $\vec{d_1}$ și vectorul

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{vmatrix} = \begin{vmatrix} q_1 & r_1 \\ q_2 & r_2 \end{vmatrix} \vec{i} - \begin{vmatrix} p_1 & r_1 \\ p_2 & r_2 \end{vmatrix} \vec{j} + \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \vec{k}.$$

$$d: \begin{cases} \Pi_1: \begin{vmatrix} x+1 & y & z-1 \\ -1 & 1 & -1 \\ 4 & 1 & -3 \end{vmatrix} = 0 \\ \\ \Pi_2: \begin{vmatrix} x+1 & y-3 & z+1 \\ -4 & 7 & -3 \\ 4 & 1 & -3 \end{vmatrix} = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \begin{vmatrix} 1 & -1 \\ 1 & -3 \end{vmatrix}(x+1) - \begin{vmatrix} -1 & -1 \\ 4 & -3 \end{vmatrix} y + \begin{vmatrix} -1 & 1 \\ 4 & 1 \end{vmatrix}(z-1) = 0 \\ \begin{vmatrix} 7 & -3 \\ 1 & -3 \end{vmatrix}(x+1) - \begin{vmatrix} -4 & -3 \\ 4 & -3 \end{vmatrix}(y-3) + \begin{vmatrix} -4 & 7 \\ 4 & 1 \end{vmatrix}(z+1) = 0 \end{cases}$$

$$d: \begin{cases} -2(x+1) - 7y - 5(z-1) = 0 & \mid (-1) \\ -18(x+1) - 24(y-3) - 32(z+1) = 0 & \mid : (-2) \end{cases}$$

$$d: \begin{cases} 2x + 7y + 5z - 3 = 0 \\ 9x + 9 + 12y - 36 + 16z + 16 = 0 \end{cases}$$

$$d: \begin{cases} 2x + 7y + 5z - 3 = 0 \\ 9x + 12y + 16z - 11 = 0 \end{cases}$$

(2). Demonstrați că dreptele $d_1: \frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-5}{4}$ și

$$d_2: \begin{cases} x = 3t + 7 \\ y = 2t + 2 \\ z = -2t + 1 \end{cases}$$

sunt coplanare și determinați ecuația planului.

$M_1(1, -2, 5) \in d_1$ $\quad \overrightarrow{M_1M_2}(6, 4, -4)$

$M_2(7, 2, 1) \in d_2$

$\vec{d_1}(2, -3, 4)$, $\vec{d_2}(3, 2, -2)$.

condiția de coplanaritate este $(\overrightarrow{M_1M_2}, \vec{d_1}, \vec{d_2}) = 0$

$$\Rightarrow \begin{vmatrix} 6 & 4 & -4 \\ 2 & -3 & 4 \\ 3 & 2 & -2 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} 3 & 2 & -2 \\ 2 & -3 & 4 \\ 3 & 2 & -2 \end{vmatrix} = 0$$ Adevărat.

Ecuația planului:

$$\begin{array}{c} \\ \vec{d_1} \\ \vec{d_2} \end{array}\begin{vmatrix} x-1 & y+2 & z-5 \\ 2 & -3 & 4 \\ 3 & 2 & -2 \end{vmatrix} = 0 \iff$$

$$\iff \begin{vmatrix} -3 & 4 \\ 2 & -2 \end{vmatrix} \cdot (x-1) - \begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix} (y+2) + \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} (z-5) = 0$$

$$\iff -2(x-1) + 16(y+2) + 13(z-5) = 0$$

$$\iff -2x + 16y + 13z + 2 + 32 - 65 = 0$$

$$\iff -2x + 16y + 13z - 31 = 0 \iff$$

$$\iff \boxed{2x - 16y - 13z + 31 = 0}.$$

③. Calculați distanța de la punctul $P(1,-1,-2)$ la dreapta

$$d: \frac{x+3}{3} = \frac{y+2}{2} = \frac{z-8}{-2}.$$

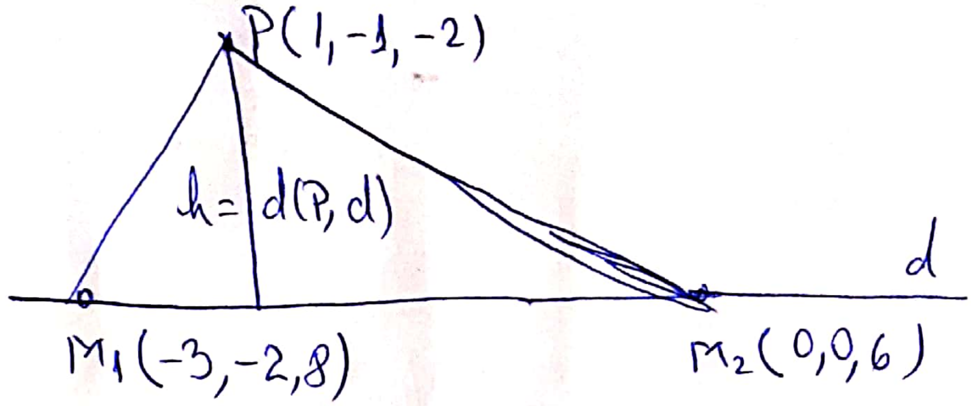
Soluție $\frac{x+3}{3} = \frac{y+2}{2} = \frac{z-8}{-2} = t \iff$

$$\iff \begin{cases} x = 3t-3 \\ y = 2t-2 \\ z = -2t+8 \end{cases}$$

pt $t=0$ avem $M_1(-3,-2,8)$

pt $t=1$ avem $M_2(0,0,6)$.

două puncte pe dreapta d.



$$\mathcal{A}[PM_1M_2] = \frac{1}{2} h \cdot M_1M_2$$

$$\Rightarrow d(P,d) = h = \frac{2 \cdot \mathcal{A}[PM_1M_2]}{M_1M_2}$$

$$\mathcal{A}[PM_1M_2] = \frac{1}{2} \cdot \|\overrightarrow{PM_1} \times \overrightarrow{PM_2}\|.$$

$\overrightarrow{PM_1}\,(-4,-1,10)$

$\overrightarrow{PM_2}\,(-1,1,8)$

$$\overrightarrow{PM_1} \times \overrightarrow{PM_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & -1 & 10 \\ -1 & 1 & 8 \end{vmatrix} =$$

$$= \begin{vmatrix} -1 & 10 \\ 1 & 8 \end{vmatrix} \vec{i} - \begin{vmatrix} -4 & 10 \\ -1 & 8 \end{vmatrix} \vec{j} + \begin{vmatrix} -4 & -1 \\ -1 & 1 \end{vmatrix} \vec{k}$$

$$= -18\vec{i} + 22\vec{j} - 5\vec{k}$$

$$\|\overrightarrow{PM_1} \times \overrightarrow{PM_2}\| = \sqrt{18^2 + 22^2 + 5^2} = \sqrt{324 + 484 + 25} =$$

$$= \sqrt{833} = 7\sqrt{17}.$$

$$\|\overrightarrow{M_1M_2}\| = \sqrt{3^2 + 2^2 + 2^2} = \sqrt{17}$$

$$\Rightarrow d(P,d) = 7.$$