

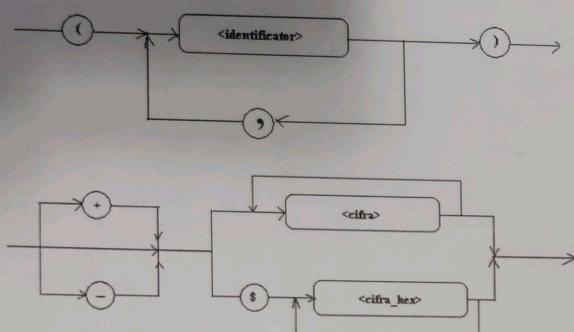
Seminar 1

Minilimbaje de programare. Identificarea si specificarea elementelor lexicale si sintactice.

1. Diagrame de sintaxă

Diagramele de sintaxă sunt reprezentări grafice folosite pentru a descrie sintaxa limbajelor de programare.

1.1. Date cate 2 exemple valide care respectă urmatoarele specificații.



1.2. Descrieți, folosind diagrame de sintaxă, sintaxa secțiunii de declaratii de variabile în Pascal. "Neterminalele" (vor apărea în dreptunghiuri cu colțurile rotunjite și) vor fi: <ID> și <type>.

2. BNF și EBNF

2.1. Date o descriere echivalentă în BNF și EBNF corespunzătoare diagramelelor din secțiunea precedenta.

3. Elemente lexicale și sintactice ale unui limbaj de programare

3.1. Fie o gramatica ce descrie sintaxa unui mini-limbaj de programare, data prin regulile de productie:

$\begin{array}{l} \langle \text{program} \rangle \\ \langle \text{lista_instr} \rangle \\ \langle \text{instr} \rangle \\ \langle \text{atribuire} \rangle \\ \langle \text{expr} \rangle \\ \langle \text{variabila} \rangle \\ \langle \text{instr_if} \rangle \end{array}$	$\rightarrow \text{begin } \langle \text{lista_instr} \rangle \text{ end.}$ $\rightarrow \langle \text{instr} \rangle ; \langle \text{lista_instr} \rangle$ $\rightarrow \langle \text{instr} \rangle$ $\rightarrow \langle \text{atribuire} \rangle$ $\rightarrow \langle \text{expr} \rangle + \langle \text{expr} \rangle$ $\rightarrow \langle \text{variabila} \rangle$ $\rightarrow \text{if } (\langle \text{expr} \rangle) \text{ then } \langle \text{atribuire} \rangle$	<i>neterminale</i> <i>restul terminale</i>
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- 1.1. Identificați terminalele și neterminalele gramaticii
 1.2. Date două "mini-programme" care sunt descrise de specificațiile date.
 ID este un atom lexical. Regulile de formare a acestui atom lexical ID (identificator), specificate folosind expresii regulate, sunt:
 $a (a | b | c)^*$

3.2. Fie următorul exemplu de program Pascal:

```
var f, a1, a2, a3 : integer;
begin
    a1:= 7;
    a2:= 11;
    a3:= a1+a2+3;
    f := 5
end.
```

- 3.3. Identificați elementele lexicale și structurile sintactice.
 3.4. Descrieți sintaxa structurilor sintactice folosind unul dintre mecanismele de specificare: BNF, EBNF sau gramatica independentă de context.
 3.5. Scrieți un program diferit de cel de mai sus care respectă descrierile date.
 3.6. Presupunând că operatorii și cuvintele cheie din exemplul de mai sus au asociate coduri – numere naturale în ordine crescătoare, în ordinea în care ele apar în program, descrieți continutul tabelei FIP, precum și a tabelei de simboluri, atunci când se folosesc 2 tabele de simboluri, una pentru constante, una pentru identificatori, pentru fiecare dintre următoarele 3 organizări:
 i. Tabel sortat lexicographic
 ii. Arbore binar de căutare

iii. Tabela de dispersie
(Alegeți o funcție de dispersie simplu de calculat, alegeți dimensiunea
tabelei ca fiind 11.)

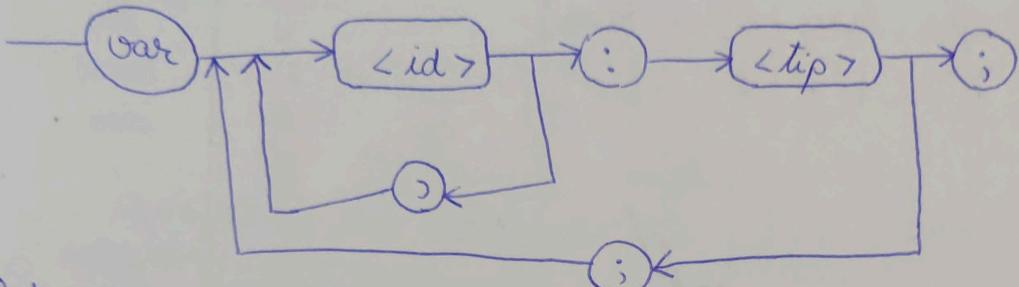
Poate aveți nevoie:
 $ASC('a') = 97$
 $ASC('0') = 48$

Seminar 1

1.1 (a, b) (abc)

+ \$AO 999

1.2 var a, b, c: integer;
x: real;



2.1 BNF

<program> ::= (<lista-id>)

<lista-id> ::= <id> | <id>, <lista-id>

<id> ::= <littera> | <littera> <lista-caractere>

<littera> ::= a | b | ... | z

<lista-caractere> ---

EBNF

program = "(<lista-id>)".

lista-id = id {", id} .

id = littera {lista-caractere} .

BNF

<program> ::= + <numar> | - <numar> | <numar>

<numar> ::= <numar-dcimal> | <numar-hex>

<numar-dcimal> ::= <ufra> | <ufra> <afre>

<ufra> ::= <ufra> | <ufra> <afre>

<ufra> ::= 0 | 1 | ... | 9

<numar-hex> ::= \$ <ufra-hex> | \$ <ufra-hex> <ufra-hex>

<ufra-hex> ::= <ufra-hex> | <ufra-hex> <afre-hex>

<ufra-hex> ::= 0 | 1 | ... | 9 | A | ... | F

EBNF

program = [semn] numar .

semn = "+" | "-" .

numar = numar_decimal | numar_hex .

numar_decimal = cifra { cifra } .

cifra = "0" | "1" | ... | "9" .

numar_hex = \$ cifra_hexa { cifra_hexa } .

cifra_hexa = "0" | ... | "9" | "A" | ... | "F" .

3.1 b) begin

aa = ac

end.

begin

if ac then aa = ac + ab

end.

3.2 a) - identificatori: a1, a2, a3, p

- constante: 3, 5, 7, 11

- cuvinte cheie (rezervate): var, integer, end, begin

- operatori: :=, +

- separatori: ;, :, :, , / or , .

b) <program> → <bloc-decl> <bloc-instr> .

<bloc-decl> → var <lista-decl>

<lista-decl> → <decl-var> <lista-decl>

<lista-decl> → <decl-var> : integer;

<lista-var> → <id>

<lista-var> → <id>, <lista-var>

<bloc-instr> → begin <lista-instr> end

<lista-instr> → <instr>; <lista-instr>

<lista-instr> → <instr>

<instr> → <atribuire>

<atribuire> → id := <expr>

<expr> → id

<expr> → const

<expr> → <expr> + <expr>

c) var a_1 : integer;
 a_2 : integer;
begin
 $a_1 := a_2 + 4 + 5$
end.

Atom lexical	Cod atom
id	0
const	1
var	2
:	3
;	4
integer	5
;	6
begin	7
$:=$	8
+	9
end	10
.	11

FIP		PoZ. TS	Symbol
atom	atom		
1	2		1 a_1
1	0	4, 1, 3	2 a_2
3			3 a_3
a_1	0	1, 2, 9	4 f
3			
a_2	0	2, 3, 10	
3			
a_3	0	3, 4, 0	
4			
5			
6			
a_1	0	1, 2, 9	1 3
8			2 5
4	1	3, 1, 4	3 4
6			4 11
a_2	0	2, 3, 10	
8			
11	1	4, 2, 0	
6			
a_3	0	3, 4, 0	
8			
a_1	0	1, 2, 9	
9			
a_2	0	2, 3, 10	
9			
3	1	1, 3, 3	
6			
f	0	4, 1, 3	
8			
5	1	2, 4, 5	
10			
11			

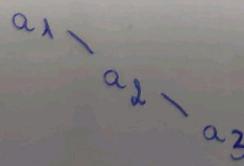
i) TS id

Symbol
1 a_1
2 a_2
3 a_3
4 f

TS Const

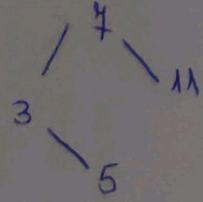
Symbol
1 3
2 5
3 4
4 11

ii)



=>

TS id		St.	Dr.
Symbol			
1 f	2	2	-1
2 a_1	-1	-1	3
3 a_2	-1	-1	4
4 a_3	-1	-1	-1



=>

TS const		St.	Dr.
Symbol			
1 4	3	3	2
2 11	-1	-1	-1
3 3	-1	-1	4
4 5	-1	-1	-1

iii) $\text{hash}(x) = \text{asc}(x) \% 11$

$\Rightarrow T_3 \text{ id}$
symbol
0 ... a_3
1
2
3 ... p
4
5
6
7
8
9 ... a_1
10 ... a_2

$T_3 \text{ const}$
symbol
0 ... 11
1
2
3 ... 3
4
5 ... 5
6
7 ... 4
8
9
10

Seminar 2

Limbaje. Specificari. Gramatici independente de context simple

✓ Multimi si limbaje

Se cere sa se defineasca (folosind multimi si descrierea proprietatilor specifici ale elementelor) urmatoarele limbaje. Se poate folosi concatenare, operatia * - inchiderea reflexiv tranzitiva.

- A. limbajul numerelor naturale in reprezentare binara
- B. limbajul numerelor intregi in reprezentare binara
- C. limbajul numerelor reale pozitive in reprezentare binara
- D. limbajul numerelor naturale in reprezentare zecimala
- E. limbajul numerelor intregi in reprezentare zecimala
- F. limbajul numerelor reale pozitive in reprezentare zecimala

Ex:

$$A: L_A = \{1w \mid w \in \{0, 1\}^*\} \cup \{0\}$$

2 Gramatici independente de context simple

1. Descrieti limbajul generat de urmatoarele gramatici:

- a) $G = (N, \Sigma, S, P)$
 - $N = \{A, B\}$
 - $\Sigma = \{a, b\}$
 - $S = A$
 - P: $\begin{array}{l} A \rightarrow aB \\ A \rightarrow B \\ B \rightarrow b \end{array}$
- b) $G = (N, \Sigma, S, P)$

$$\begin{aligned} N &= \{<\text{propozitie}>, <\text{subiect}>, <\text{predicat-nominal}>, <\text{verb-copulativ}>, \\ &\quad <\text{nume-predicativ}>, <\text{substantiv}>, <\text{adjectiv}>, <\text{verb}>, <\text{determinant}>\} \end{aligned}$$

$$\Sigma = \{o, \text{ orice, functie, derivabila, continua, este}\}$$

$$S = <\text{propozitie}>$$

$$\begin{aligned} P: & \quad <\text{propozitie}> \rightarrow <\text{subiect}> <\text{predicat-nominal}> \quad 1 \\ & <\text{subiect}> \rightarrow <\text{determinant}> <\text{substantiv}> \quad 2 \\ & <\text{predicat-nominal}> \rightarrow <\text{verb-copulativ}> <\text{nume-predicativ}> \quad 3 \\ & <\text{verb-copulativ}> \rightarrow \text{este} \quad 4 \\ & <\text{nume-predicativ}> \rightarrow <\text{adjectiv}> \quad 5 \\ & <\text{adjectiv}> \rightarrow \text{derivabila} \mid \text{continua} \quad 6 \\ & <\text{substantiv}> \rightarrow \text{functie} \quad 7 \\ & <\text{determinant}> \rightarrow o \mid \text{orice} \quad 8 \end{aligned}$$

✓ Dati cate o gramatica care genereaza propozitiile:

- a) ab, ac
- b) abc

3. BNF si EBNF

✓ Dati o descriere echivalenta in BNF si EBNF pentru doua dintre limbajele definite in sectiunile precedente.

4. Descrieri de limbaje folosind mecanisme generative

1. Fie L un limbaj peste alfabetul $\{a, b\}$ definit dupa cum urmeaza:

$$(i) \quad ab \in L$$

$$(ii) \quad \text{Daca } x \in L \text{ atunci } axb \in L$$

$$(iii) \quad \text{Niciun alt cuvant nu apartine lui } L.$$

- a) Descrieti limbajul definit mai sus folosind multimi si descrierea proprietatilor specifici ale elementelor.
- b) Descrieti limbajul definit mai sus folosind o gramatica independenta de context

5. Gramatici independente de context si limbajul generat

✓ Sa se construiasca o gramatica care genereaza limbajul:

$$L = \{x^n y^n \mid n \in \mathbb{N}\}$$

Pentru gramatica construita, demonstrati ca $L(G) = L$.

2. Analog pt. $L = \{a^{2n}bc \mid n \in \mathbb{N}\}$

3. Analog pt. $L = \{a^{2n+1} \mid n \in \mathbb{N}\}$

Seminar 2

$$\Sigma^* = \Sigma^+ \cup \{\epsilon\}$$

* cu ϵ ($0 \dots n$)
 + fără ϵ ($1 \dots n$)

1. A: $L_A = \{ \lambda w \mid w \in \{0,1\}^* \} \cup \{0\}$

B: $L_B = \{ \lambda l w \mid s \in \{+, -, \epsilon\}, w \in \{0,1\}^* \} \cup \{0\}$

C: ~~$L_C = \{ \lambda w \mid w \in \{0,1\}^* \} \cup \{0\}$~~

D: ~~$L_D = \{ m c \mid c \in \{0,1,-,+,\epsilon\}, m \in \{ \}$~~

C: $L_C = L_A \cup \{ \lambda w, v \mid w \in \{0,1\}^*, v \in \{0,1\}^+ \} \cup \{0, w \mid w \in \{0,1\}^+ \}$

D: $L_D = \{ uv \mid u \in \{1,2,\dots,9\}, v \in \{1,2,\dots,9\}^* \} \cup \{0\}$

E: $L_E = \{ suv \mid s \in \{-, +, \epsilon\}, u \in \{1,2,\dots,9\}, v \in \{0,1,\dots,9\}^* \} \cup \{0\}$

F: $L_F = L_D \cup \{ w u v \mid w \in \{1,\dots,9\}, u \in \{0,1,\dots,9\}, v \in \{0,1,\dots,9\}^+ \}$
 $\cup \{0, u \mid u \in \{0,1,\dots,9\}^+ \}$

2.

a) $G = (N, \Sigma, S, P)$

N = {A, B} - meterninale

$\Sigma = \{a, b\}$ - terminale

S = A - ac ce începe "productia"; simbol start

P: $A \xrightarrow{1} aB$ - multimea productiilor

$A \xrightarrow{2} B$

$B \xrightarrow{3} \lambda$

$L(G) = \{ ab, b \} = \{ wb \mid w \in \{a, \epsilon\} \}$

$A \xrightarrow{1} aB \xrightarrow{3} ab$

$A \xrightarrow{2} B \xrightarrow{3} b$

b) $\xleftarrow{\text{propozitie}} \xrightarrow{1} \langle \text{subiect} \rangle \langle \text{PN} \rangle$

$\langle \text{PN} \rangle \xrightarrow{2} \langle S \rangle \langle \text{PN} \rangle \xrightarrow{3} \langle D \rangle \langle \text{Subst} \rangle \langle \text{VC} \rangle \langle \text{NP} \rangle$

$\Rightarrow \langle D \rangle \langle \text{Subst} \rangle \langle \text{VC} \rangle \langle A \rangle$

$\Rightarrow \emptyset$ funcție este derivabilă continuă

4 orice

6

7

8

nu se înlocuiesc
mai multe!

5 \Rightarrow meterninale
diodata!

$\Rightarrow L(G) = \{ u \text{ funcție este } v \mid u \in \{0, \text{orice}\}, v \in \{\text{derivabilă, continuă}\} \}$

2. a) $G = (N, \Sigma, S, P)$

$$N = \{A, B\}$$

$$\Sigma = \{a, b, c\}$$

$$S = A$$

$$P: \begin{array}{l} A \rightarrow aB \\ B \rightarrow b \\ B \rightarrow c \end{array}$$

b) $G = (N, \Sigma, S, P)$

$$N = \{A, B, C\}$$

$$\Sigma = \{a, b, c\}$$

$$S = A$$

$$P: \begin{array}{l} A \rightarrow aB \\ B \rightarrow bC \\ C \rightarrow c \end{array}$$

3.

1. BNF pt. F

Numar = Numar Natural [", "Parte Zecimală"].

Numar Natural = Cifra Nenula {Cifra} | "0".

Cifra Nenula = "1" | "2" | ... | "9".

Cifra = Cifra Nenula | "0".

Parte Zecimală = {Cifra} Cifra Nenula.

L, BNF pt. F

$\langle \text{Numar} \rangle ::= \langle \text{NumarN} \rangle \mid \langle \text{NumarH} \rangle, \langle \text{NumarZ} \rangle$

$\langle \text{NumarN} \rangle ::= \langle \text{CifraNenula} \rangle \mid \langle \text{CifraNenula} \rangle \langle \text{ListaCifre} \rangle$

$\langle \text{CifraNenula} \rangle ::= 1 \mid 2 \mid \dots \mid 9$

$\langle \text{ListaCifre} \rangle ::= \langle \text{Cifra} \rangle \mid \langle \text{Cifra} \rangle \langle \text{ListaCifre} \rangle$

$\langle \text{Cifra} \rangle ::= 0 \mid \langle \text{CifraNenula} \rangle$

$\langle \text{NumarZ} \rangle ::= \langle \text{ListaCifre} \rangle \langle \text{CifraNenula} \rangle \mid \langle \text{CifraNenula} \rangle$

4.

1. $ab \in L$

dacă $x \in L$ atunci $axb \in L$

a) $1 \rightarrow ab \in L, 2 \rightarrow aabb \in L, 3 \rightarrow aaa \quad bb \in L \dots$

$L = \{uvw \mid v \in \{a, b\}^*\} \quad L = \{a^m b^m \mid m \in \mathbb{N}^*\}$

b) $G = (N, \Sigma, S, P)$

$$N = \{A\}$$

$$\Sigma = \{a, b\}$$

$$S = A$$

$$P : \begin{array}{l} A \rightarrow ab \\ A \rightarrow aAb \end{array}$$

5.

i. $L = \{x^m y^m \mid m \in \mathbb{N}\}$ dum. $L = L(G)$

$G = (N, \Sigma, S, P)$

$$N = \{A\}$$

$$\Sigma = \{x, y\}$$

$$S = A$$

$$P : \begin{array}{l} A \xrightarrow{\lambda} E \\ A \xrightarrow{2} xAy \end{array}$$

ii. $L \subseteq L(G)$

$$w = x^m y^m, m \text{ fixat}$$

$A \xrightarrow{2} \underbrace{xx \dots x}_{\text{de mori}} \underbrace{yy \dots y}_{\text{de mori}} \underbrace{y}_{\text{mori}} = x^m A y^m \xrightarrow{\lambda} x^m y^m \in L(G)$

iii. $L(G) \subseteq L$

$$P(m) : \begin{array}{c} x^m A y^m \\ x^{m-1} y^{m-1} \\ \vdots \\ x \\ y \end{array}$$

$$P(\lambda) : \begin{array}{l} A \xrightarrow{\lambda} E \\ A \xrightarrow{2} xAy \end{array}$$

$$\text{pp. RA } P(k) : x^k A y^k \in L(A)$$

$$\frac{}{P(k+1) : x^{k+1} A y^{k+1} \in L} \#$$

$$x^k A y^k \xrightarrow{1} x^k y^k - P(k+1)$$

$$\xrightarrow{2} x x^k A y^k = x^{k+1} A y^{k+1} - P(k+1) \quad (A)$$

$$2. L = \{ a^{2m}bc \mid m \in \mathbb{N} \}$$

$$G = (N, \Sigma, S, P)$$

$$N = \{ A, B \}$$

$$\Sigma = \{ a, b, c \}$$

$$S = A$$

$$3. L = \{ a^{2m+1}bc \mid m \in \mathbb{N} \}$$

$$G = (N, \Sigma, S, P)$$

$$N = \{ A, B \}$$

$$\Sigma = \{ a, b, c \}$$

$$S = A$$

$$P: \begin{array}{l|l} A \rightarrow Bbc & A \xrightarrow{1} bc \\ B \rightarrow \epsilon & A \xrightarrow{2} a^2A \\ B \rightarrow aaB & \end{array}$$

sau

2. d.m.

$$I L \subseteq L(G)$$

für $w = a^{2m}bc \in L$, m fixat

$$A \xrightarrow[2]{ } a^2A \xrightarrow[2]{ } a^4A \xrightarrow[2]{ } \underbrace{\dots}_{\text{durch } m-2 \text{ ori}} \xrightarrow[2]{ } a^{2m}A \xrightarrow[1]{ } a^{2m}bc \in L(G) \Rightarrow$$

$$\Rightarrow w \in L(G) \Rightarrow L \subseteq L(G)$$

$$\bar{u} L(G) \subseteq L$$

$$P(m) : a^{2m}A$$

$$a^{2m}bc$$

$$P(1) : A \xrightarrow[1]{} bc$$

$$A \xrightarrow[2]{} a^2A$$

$$PPP(k) : a^{2k}A \xrightarrow[1]{} a^{2k}bc$$

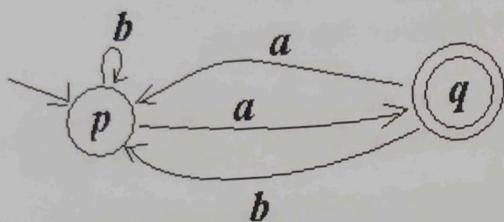
$$\xrightarrow[2]{} a^{2k}a^{2k}A = a^{2(k+1)}A$$

Seminar 3

Automate finite

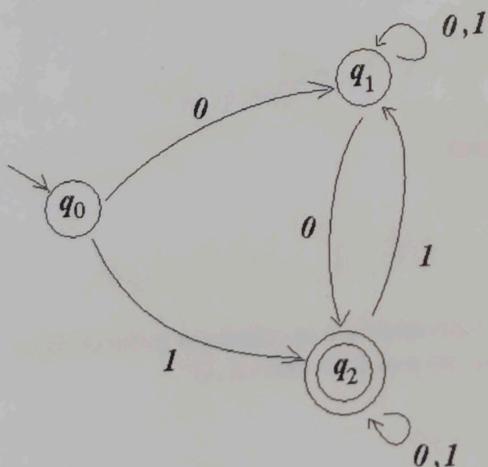
1.1 Probleme cu AF

1. Sa se reprezinte tabelar urmatoarele automate finite.
Sint ele AFD sau AFN?



δ	a	b	
p	q	p	o
q	p	p	1

dt. complet



δ	0	1	
q_0	q_1	q_2	0
q_1	$\{q_1, q_2\}$	q_1	0
q_2	q_2	$\{q_1, q_2\}$	1

nedit.

2. Sa se reprezinte sub forma de graf automatul finit: $M = (Q, \Sigma, \delta, q_0, F)$

	0	1	
q0	q2	q1	1
q1	q3	q0	0
q2	q0	q3	0
q3	q1	q2	0

Verificati apoi, bazandu-vă pe graful obținut, ca:

- a) secvențele 1010, 1100 sunt acceptate de automat
- b) secvența 1011 nu este acceptată de automat

3. Sa se construiasca un AF care accepta

- a) $L = \{ a \text{ a } a \} , \Sigma = \{ a \}$
- b) $L = \{ w_1 a a a w_2 \mid w_1, w_2 \in \{a, b\}^* \}$
- c) cuvinte peste alfabetul $\{0,1\}$ cu proprietatea ca:
orice cuvant al limbajului contine cel putin 2 zerouri consecutive
- d) cuvinte peste alfabetul $\{a, b, c\}$ cu proprietatea ca:
primul simbol al cuvantului este acelasi cu cel cu care se termina cuvantul
- e) cuvinte peste alfabetul $\{a, b, c\}$ cu proprietatea ca:
exista un simbol in cuvant care mai apare cel putin o data in cuvant
- f) $L = \{c^{3n}, n \in \mathbb{N}^*\}$
- g) limbajul ce contine secvente peste $\Sigma = \{a, b\}$ cu nr. par de simb. **a** si nr. par de simb **b**
- h) limbajul ce contine secv. peste $\Sigma = \{a, b\}$ cu nr. impar de simb. **a** si impar de simb **b**
- i) $L = \{ 1^n 0^m 1^u \mid n \geq 0, m \geq 1, u \in \{0,1\}^* \}$
- j) $L = \{ 0 (10)^n 01^m \mid n \geq 0, m \geq 0 \} \cup \{(10)^n 01^m \mid n \geq 1, m \geq 0 \}$ cu cel mult 4 stari
- k) $L = \{ 0^m 1^n \mid m, n \in \mathbb{N} \} \cup \{1^p 0^q \mid p, q \in \mathbb{N}\}$
- l) $L = \{ w_1 a a w_2 \mid w_1 \in \{b, ab\}^*, w_2 \in \{a, b\}^* \}$

1.2 Structuri de date pentru automate finite

Descrieti o modalitate de reprezentare pentru AF
(ganditi cateva modalitati)

ex:

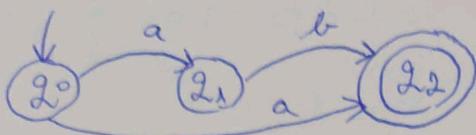
- AF care are ca alfabet multimea caracterelor reprezentabile in calculator (alfabet fixat)
- structura de date care are in vedere ca operatia cea mai frecventa cu AF
este verificare acceptare secventa

```
StateMachine
    description: String          /** A description of the state machine */
    startState : State
    states: Set<State>           /** Can be List, Map, ... */
                                    /** What about redundancy? */

State
    description: String          /** The name of the state */
    isAcceptState: Boolean
    transitions: MultiMap<Symbol, State> /** Can be List, Map ... ? */
```

Seminar 3

$$M_{\text{ex}} = (Q, \Sigma, \delta, q_0, F)$$



$$\begin{aligned}\Sigma &= \{a, b\} \\ Q &= \{q_0, q_1, q_2\} \\ F &= \{q_2\}\end{aligned}$$

δ	a	b	
q_0	$\{q_1, q_2\}$	0	
q_1		q_2	0
q_2			1

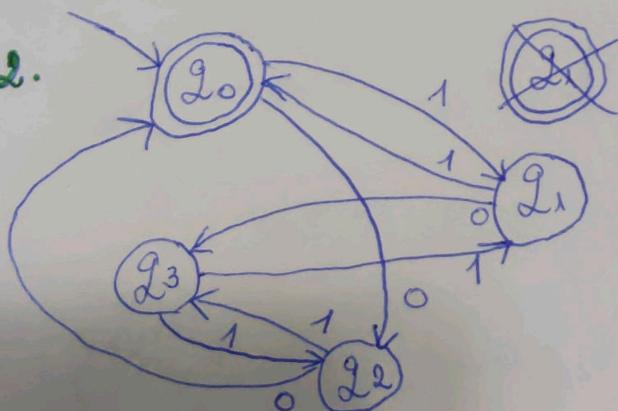
cel puțin o stare în fiecare poz. \Rightarrow det. complet ($| \delta(q, a) | = 1$)
 dacă în cel puțin o poz. sunt mai multe \Rightarrow nedet.

$$(q_0, ab) \vdash (q_1, b) \vdash (q_2, \epsilon)$$

↑
dacă e finală \Rightarrow acceptat
(ϵF) cu ϵ

$$(q_0, abb) \vdash (q_1, ab) \vdash (q_0, b) \Rightarrow \text{refuzat}$$

2.

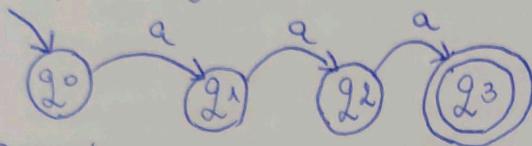


a) $(q_0, 1010) \vdash (q_1, 010) \vdash (q_3, 10) \vdash (q_2, 0) \vdash (q_0, \epsilon)$
 $q_0 \in F \Rightarrow \text{acceptat}$

$(q_0, \lambda, 00) \vdash (q_1, 100) \vdash (q_0, 00) \vdash (q_2, 0) \vdash (q_0, \varepsilon)$
 $q_0 \in F \Rightarrow \text{acceptat}$

b) $(q_0, 1011) \vdash (q_1, 011) \vdash (q_3, 11) \vdash (q_2, 1) \vdash (q_3, \varepsilon)$
 $q_3 \notin F \Rightarrow \text{refuzat}$

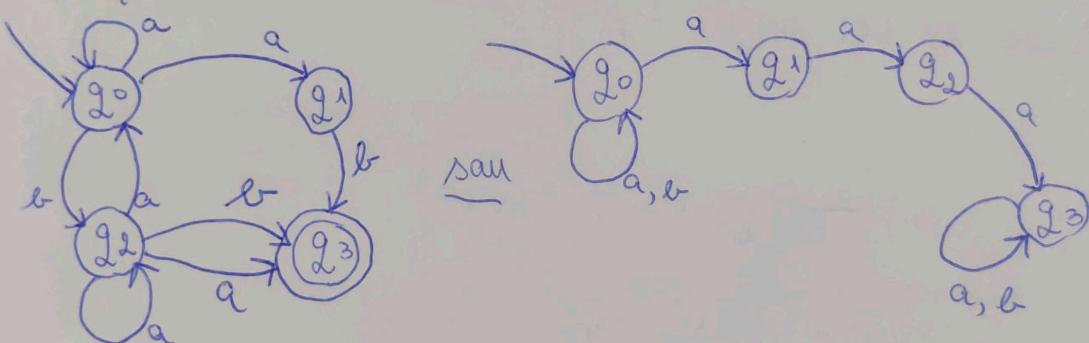
3. a) $L = \{aaa\}, \Sigma = \{a\} \Rightarrow M = (Q, \Sigma, \delta, q_0, F)$



$$Q = \{q_0, q_1, q_2, q_3\}$$

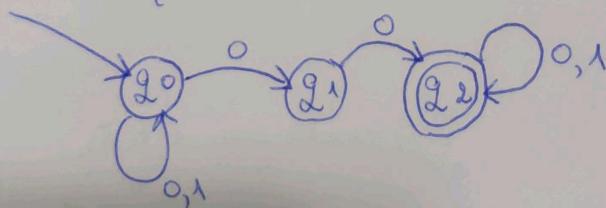
$$F = \{q_3\}$$

b) $L = \{w_1 a a a w_2 \mid w_1, w_2 \in \{a, b\}^*\}$



c) $\Sigma = \{0, 1\}$

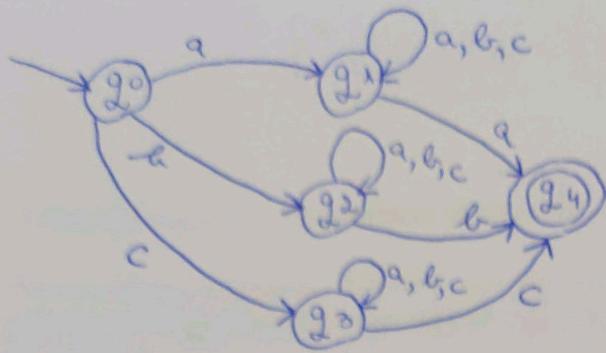
$L = \{w_1 00 w_2 \mid w_1, w_2 \in \{0, 1\}^*\}$



d) $\Sigma = \{a, b, c\}$

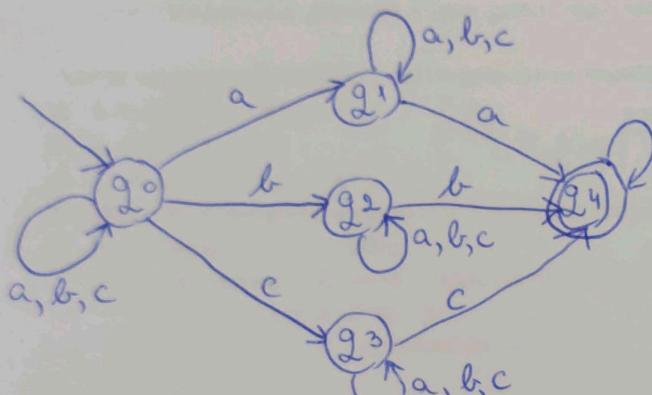
$L = \{awa \mid w \in \{a, b, c\}^*\} \cup \{bab \mid w \in \{a, b, c\}^*\} \cup$

$\{cwc \mid w \in \{a, b, c\}^*\}$

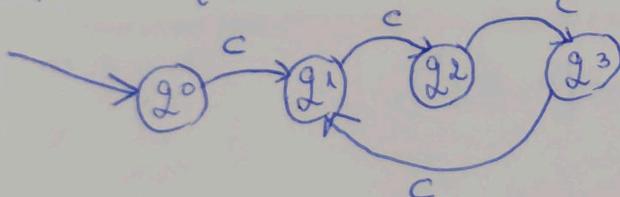


e) $\Sigma = \{a, b, c\}$

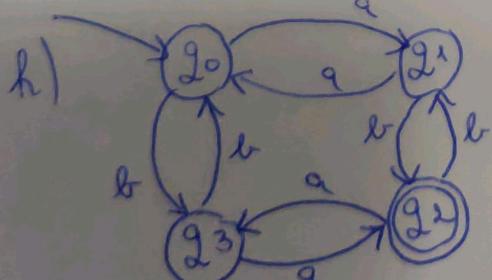
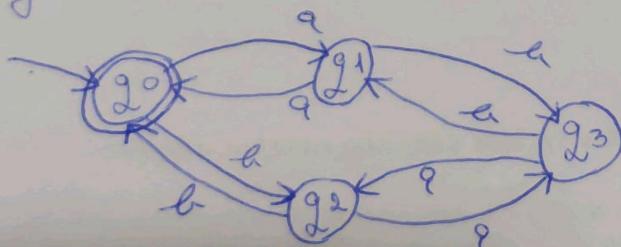
$w_1aw_2aw_3, w_1bw_2bw_3, w_1cw_2cw_3$
 $w_1, w_2, w_3 \in \{a, b, c\}^*$



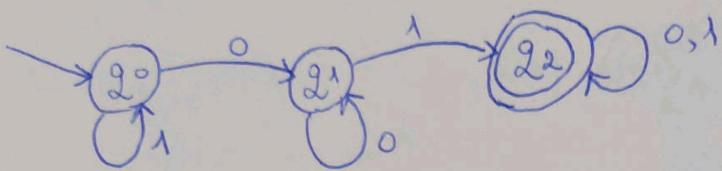
f) $L = \{c^{3m}, m \in \mathbb{N}^*\}$



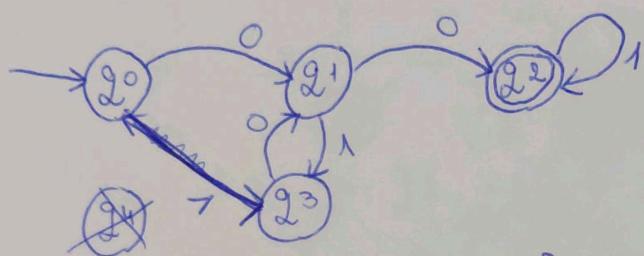
g) $\Sigma = \{a, b\}$



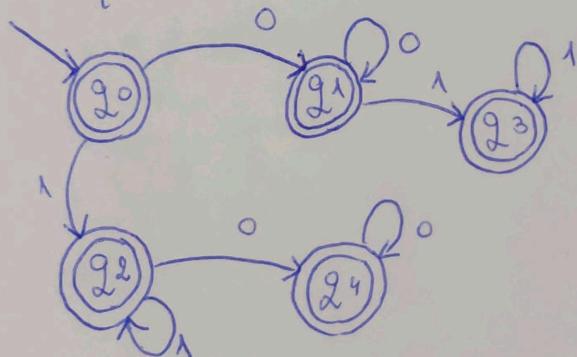
i) $L = \{1^m 0^m 1^n 0^n \mid m \geq 0, n \geq 1, m \in \{0, 1\}^*\}$



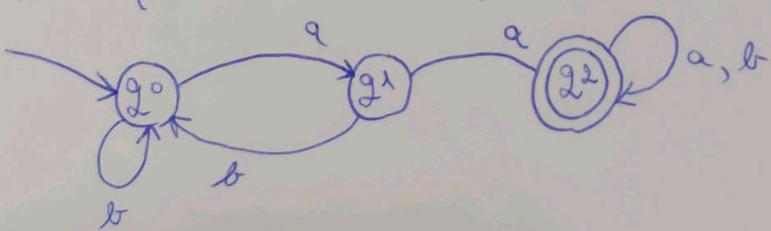
j) $L = \{0^m (10)^n 0^n 1^m \mid m \geq 0, n \geq 0\} \cup \{(10)^m 0^n 1^m \mid m \geq 1, n \geq 0\}$



k) $L = \{0^m 1^n \mid m, n \in \mathbb{N}\} \cup \{1^p 0^q \mid p, q \in \mathbb{N}\}$



l) $L = \{w_1 a w_2 \mid w_1 \in \{b, ab\}^*, w_2 \in \{a, b\}^*\}$



λ.2 State Machine:

description: String

startState: State

states: Set<State>

State

description: String

isAccepted: Boolean

transition: Multimap<Symbol, State>

Seminar 4

Limbaje regulare : gramatici si automate

- ✓ 1. Pentru urmatoarele limbaje, dati AF care le accepta. Apoi dati o gramatica echivalenta. Este regulara? Daca nu, dati gramatica regulara echivalenta.
- $L = \{a\}$
 - $L = \{a^n | n \in \mathbb{N}\}$
 - $L = \{a^n b | n \in \mathbb{N}\}$
 - $L = \{\epsilon\} \cup \{a^n b | n \in \mathbb{N}\}$
 - $L = \{a^m b^n | m, n \in \mathbb{N}, m+n > 0\}$
 - $L = \{ab^n | n \in \mathbb{N}\}$
2. Descrieti constructia generala a unei gram. regulare echivalente cu un AF dat.
- ✓ 3. Pentru urmatoarele limbaje, dati o gramatica regulara care le genereaza.
- $L = \{a^{3n} | n \in \mathbb{N}^*\}$
 - $L = \{a^{3n} | n \in \mathbb{N}\}$
 - $L = \{a^m b^n | m, n \in \mathbb{N}^*\}$
 - limbajul constantelor numerice fara semn reprezentate in baza 2
 - limbajul identificatorilor
- obs.: este permisa scrierea compacta a regulilor de productie folosind $|$ si ...
ex: $S \rightarrow a|...|c$
coresponde la:
 $S \rightarrow a$
 $S \rightarrow b$
 $S \rightarrow c$
unde: a, ..., z – terminale
- ✓ 4. Pentru urmatoarele gram. regulare, descrieti limbajul generat. Dati AF echivalent.
- $A \rightarrow aA$
 $A \rightarrow b$
 - $S \rightarrow \epsilon$
 $S \rightarrow aA$
 $A \rightarrow b$
 - $S \rightarrow \epsilon$
 $S \rightarrow aA$
 $A \rightarrow bA$
 $A \rightarrow c$
- ✓ 5. Descrieti constructia generala a unui AF echivalent cu o gram. regulara data.
- ✓ 6. Pentru urmatoarele limbaje, dati AF care le accepta. Apoi dati gr. regulara echivalenta, aplicand alg. (general) de construire. Apoi dati AF echiv. cu gr. regulara, aplicand alg. general de construire.
- $L = \{a^{2n} | n \in \mathbb{N}\}$
 - $L = \{a^m b^n | m, n \in \mathbb{N}\}$

Seminar 4

$$G = (N, \Sigma, P, S)$$

L-gramatica

Gramatica regulară:

$$- S \rightarrow a$$

$$- A \rightarrow aB, A \rightarrow d$$

- $S \rightarrow E$, atunci S nu mai apare în membrul drept al unei reguli de producție

Exemple care HV sunt regulate:

$$A \rightarrow abC, A \rightarrow AB, AB \rightarrow cB, A \rightarrow Aa$$



$$G = (N, \Sigma, P, S)$$

$$S = A$$

$$N = \{A\}$$

$$\Sigma = \{a\}$$

$$P: A \rightarrow a$$

regulară



$$G = (N, \Sigma, P, S)$$

$$S = A$$

$$N = \{A\}$$

$$\Sigma = \{a\}$$

$$P: A \rightarrow aA$$

$$A \rightarrow E$$

Neregulară

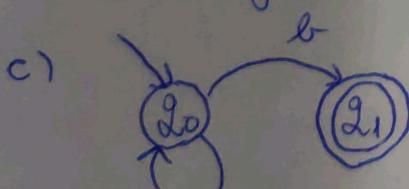
$$S = B$$

$$P: A \rightarrow aA$$

$$A \rightarrow E$$

$$B \rightarrow aA$$

$$B \rightarrow E$$



$$G = (N, \Sigma, P, S)$$

$$S = A$$

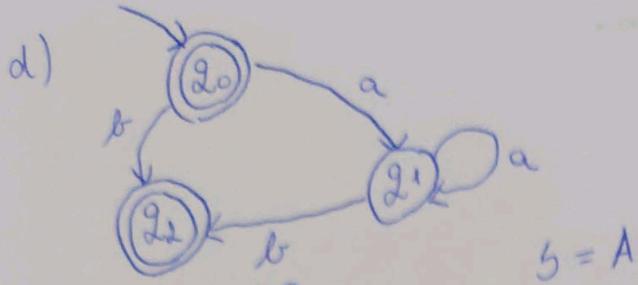
$$N = \{A\}$$

$$\Sigma = \{a, b\}$$

$$P: A \rightarrow b$$

$$A \rightarrow aA$$

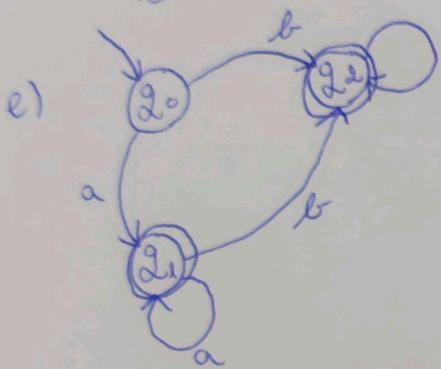
regulară



P:

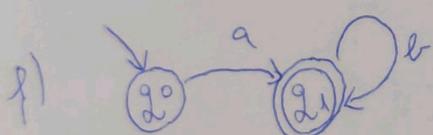
$$\begin{aligned} S &\rightarrow aB \\ A &\rightarrow b \\ A &\rightarrow \epsilon \\ B &\rightarrow aB \\ B &\rightarrow b \end{aligned}$$

$S = A$



$G = (N, \Sigma, P, S)$

P:

$$\begin{aligned} S &\rightarrow a \\ S &\rightarrow b \\ S &\rightarrow aA \\ S &\rightarrow bB \\ A &\rightarrow a \\ A &\rightarrow b \\ A &\rightarrow aA \\ B &\rightarrow b \\ B &\rightarrow bB \end{aligned}$$


P:

$$\begin{aligned} A &\rightarrow a \\ A &\rightarrow aB \\ B &\rightarrow b \\ B &\rightarrow bB \end{aligned}$$

Alg I Algoritmul I de transf. în gramatica regulară

1. copiez toate regulile de producție, mai puțin $S \rightarrow \epsilon$
2. dacă am $S \rightarrow \epsilon$, copiez rez. de prod. care îl conțin pe S , și fară S în membrul drept \hookrightarrow fară $S \rightarrow \epsilon$
3. adaug S' ca simbol de start și copiez regulile cu S în stânga
a.i. să fie S' în stânga + $S \rightarrow \epsilon$

2. $M = (Q, \Sigma, \delta, q_0, F) \Rightarrow G = (H, \Sigma, P, S)$

Alg. II transformare

$$S = q_0, \Sigma = \Sigma, H = Q$$

$S \rightarrow aB$, dacă $B \in \delta(A, a)$

$P: \left\{ \begin{array}{l} A \rightarrow aB, \text{ dacă } B \in \delta(A, a) \text{ și } B \in F \\ A \rightarrow a, \text{ dacă } B \in \delta(A, a) \text{ și } B \in F \end{array} \right.$

dacă $B \in F$ și $\delta(B, a), \forall a \in \Sigma = \emptyset$

atunci nu mai scriu regula $A \rightarrow aB$
scriu doar $A \rightarrow a$

dacă $S(q_0) \in F$ adaug $S \rightarrow \epsilon$ (în eventual aplic alg. I).

3. a) $A \rightarrow aB$

$$B \rightarrow ac$$

$$C \rightarrow aA$$

$$C \rightarrow a$$

b) $\left. \begin{array}{l} A \rightarrow \epsilon \\ A \rightarrow aB \\ B \rightarrow ac \\ C \rightarrow a \\ C \rightarrow aA \end{array} \right|_{S=A}$

\Rightarrow

$$S \rightarrow \epsilon$$

$$S \rightarrow aB$$

$$A \rightarrow \epsilon$$

$$A \rightarrow aB$$

$$B \rightarrow ac$$

$$C \rightarrow a$$

$$C \rightarrow aA$$

regulară ✓

c) $S \rightarrow aA$

$$A \rightarrow aA$$

$$A \rightarrow aB$$

$$B \rightarrow bB$$

$$B \rightarrow b$$

d) $L = \{0\} \cup \{\lambda w \mid w \in \{0, 1\}^*\}$

$$A \rightarrow \lambda$$

$$A \rightarrow 0$$

$$A \rightarrow \lambda B$$

$$B \rightarrow 0B$$

$$B \rightarrow \lambda B$$

$$B \rightarrow 0$$

$$B \rightarrow \lambda$$

e) id $(a \dots z) \{ a \dots \# 0 \dots 9 \}$

$$S \rightarrow aAb \dots \#$$

$$S \rightarrow aA|bA| \dots | \# A$$

$$A \rightarrow aAb \dots \#$$

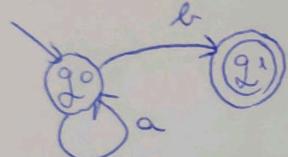
$$A \rightarrow 0Ab \dots \#$$

$$A \rightarrow 0A|1A| \dots | 9A$$

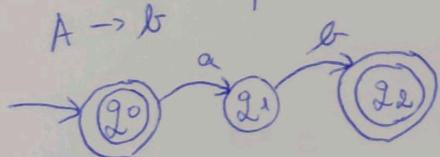
$$A \rightarrow aA|bA| \dots | \# A$$

4. a) $A \rightarrow aA \quad | \Rightarrow L = \{ a^m b \mid m \in \mathbb{N} \}$

$$A \rightarrow b$$



b) $S \rightarrow \epsilon \quad | \quad S \rightarrow aA \quad | \quad A \rightarrow b \Rightarrow L = \{ \epsilon, ab \}$



c) $S \rightarrow \epsilon \quad | \quad S \rightarrow aA \quad | \quad A \rightarrow bA \quad | \quad A \rightarrow c \Rightarrow L = \{ \epsilon \} \cup \{ a^m b^m c \mid m \in \mathbb{N} \}$



5. $G = (N, \Sigma, P, S) \Rightarrow M = (Q, \Sigma, \delta, q_0, F)$

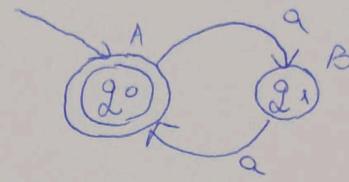
Alg. III $Q = N \cup \{ K \}, K \in F$

$$\Sigma = \Sigma$$

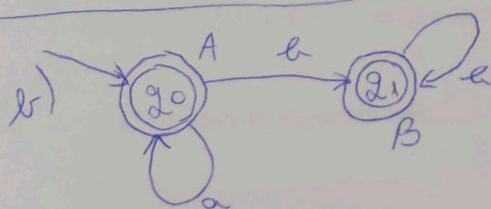
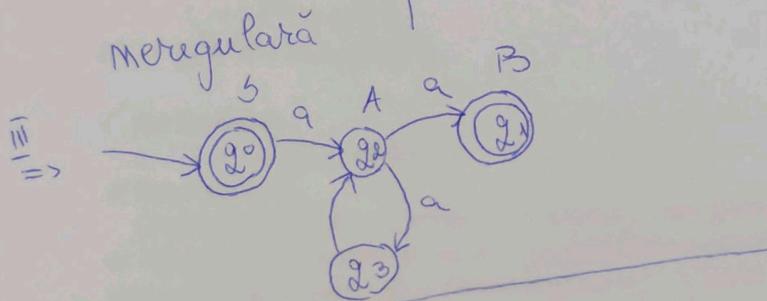
$$q_0 = S$$

$$F = \{ K \} \text{ sau } \{ K, S \} \text{ dacă am } S \rightarrow \epsilon \in P$$

$$\delta: \left\{ \begin{array}{l} \delta(A, a) = \{B \mid A \xrightarrow{} aB \in P\} \cup \{K \mid A \xrightarrow{} a \in P\} \\ \delta(K, a) = \emptyset, \forall a \in \Sigma \end{array} \right.$$

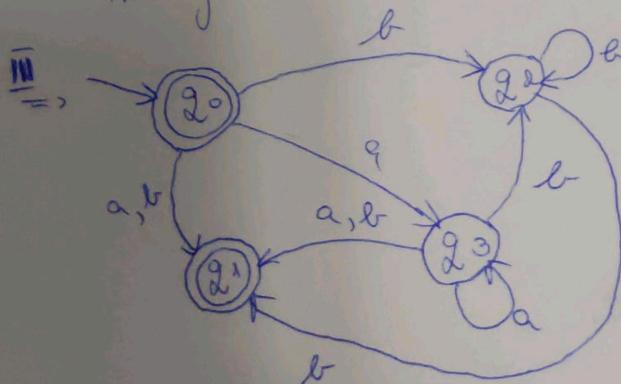


$$\stackrel{\text{II}}{\Rightarrow} P: \begin{array}{l} A \xrightarrow{} aB \\ B \xrightarrow{} aA \\ B \xrightarrow{} a \\ A \xrightarrow{} \epsilon \end{array} \quad \left| \begin{array}{c} \stackrel{\text{I}}{\Rightarrow} \\ S \xrightarrow{} E \\ S \xrightarrow{} aB \\ A \xrightarrow{} aB \\ B \xrightarrow{} a \\ B \xrightarrow{} aA \end{array} \right. \Rightarrow \begin{array}{l} S \xrightarrow{} E \\ S \xrightarrow{} aB \\ A \xrightarrow{} aB \\ B \xrightarrow{} a \\ B \xrightarrow{} aA \end{array}$$



$$\stackrel{\text{II}}{\Rightarrow} P: \begin{array}{l} A \xrightarrow{} \epsilon \\ A \xrightarrow{} a \\ A \xrightarrow{} b \\ A \xrightarrow{} aA \\ A \xrightarrow{} bB \\ \hline B \xrightarrow{} b \\ B \xrightarrow{} bB \end{array} \quad \left| \begin{array}{c} \stackrel{\text{I}}{\Rightarrow} \\ S \xrightarrow{} E \\ S \xrightarrow{} a \\ S \xrightarrow{} b \\ S \xrightarrow{} aA \\ A \xrightarrow{} a \\ A \xrightarrow{} b \\ A \xrightarrow{} aA \\ A \xrightarrow{} bB \\ B \xrightarrow{} b \\ B \xrightarrow{} bB \end{array} \right. \quad \begin{array}{l} S \xrightarrow{} bB \\ \swarrow \quad \nearrow \\ aA \end{array}$$

mergulară X



regulară ✓

Seminar 5

1.1 limbaje regulare – a fi sau anu fi

1. Sa se verifice daca urmatoarele limbaje sint regulare. Daca nu sint – demonstrati! Daca sint, doar argumentati. (Puteti indica un AF care le accepta. Puteti argumenta si altfel? ☺)

- a. $L = \{a^n b^{2n} \mid n \in \mathbb{N}^*\}$
- b. $L = \{a^k \mid k - \text{nr.prim}\}$
- c. $L = \{a^n n^2 \mid n \in \mathbb{N}^*\}$
- d. $L = \{a^{2^n} \mid n \in \mathbb{N}^*\}$
- e. Fie $k - \text{un nr. natural fixat}$
 $L = \{a^{kn} \mid n \in \mathbb{N}\}$
- f. $L = \{a^m b^n c^p \mid m, n, p \in \mathbb{N}\}$

2. Fie limbajul:

$$L = \{a^k \mid k - \text{neprim}\}$$

a) Este regular?

(Indicatie: folositi proprietatile de inchidere ale limbajelor regulare)

b) Alegeti un nr. natural p astfel incat, alegand un cuvant din limbaj, de lungime mai mare decat p , sa puteti da o descompunere $w=xyz$ astfel incat xy^iz sa fie tot un cuvant din limbaj, pentru orice $i - \text{numar natural nenul}$.

1.2 Limbaje de toate felurile

1. a) Dati un limbaj regular. Dati AF care il accepta. Dati o gram. regulara ce il genereaza. Dati o gramatica i.c. care nu e regulara ce il genereaza.
- b) Dati un limbaj i.c. care nu este regular. Dati g.i.c. ce il genereaza.

1.3 Gramatici regulare echivalente cu o gramatica data (dar care genereaza un limbaj regular)

1. Dati o gramatica regulara echivalenta cu gram. data prin urmatoarele r.p.:

- a) S->abS
- S->ab
- b) S->Sa
- S->b

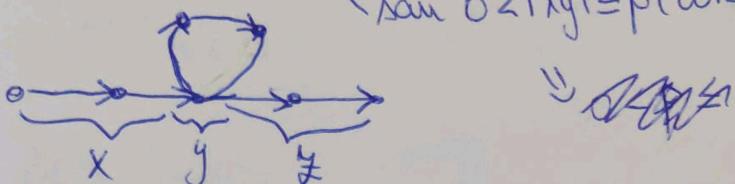
Seminar 5

Lema de pompare pentru limbi regulate

L regular atunci $\exists p \geq 1$ pentru care $\forall w \in L, |w| \geq p$

$\exists w = xyz, 0 < |y| \leq p, w_i = xy^i z \in L, \forall i \in \mathbb{N}$

sun $0 < |xy| \leq p$ (conditie mai tare)



Negatia: $\nexists p \geq 1, \exists w \in L, \exists w = xyz, 0 < |y| \leq p, |w| \geq p$

$w_i = xy^i z \quad \exists i \in \mathbb{N}$ pt. care $w_i \notin L$

1.1

a) $L = \{a^m b^{dm}\}$ (deobicei, cand avem două cicluri care depind unul de altul, nu e regular)

presupunem RA ca L - regulat

$\exists p \geq 1$ a.i. $\forall w \in L, |w| \geq p$

$\exists w = xyz; 0 < |y| \leq p$

$w_i = xy^i z, w_i \in L, \forall i \in \mathbb{N}$

$$\begin{array}{l} w = a^p b^{2p} \\ x = a^{p-k} \\ y = a^k \\ z = b^{2p} \end{array} \Rightarrow |w| = 3p \geq p \quad \checkmark$$

$$w_i = a^{p-k} (a^k)^i b^{2p}$$

$$w_i = a^{p-k+i k} b^{2p}$$

$$w_i = a^{p-k+i k} b^{2p}$$

$$\text{de ex: } w_2 = a^{p+k} b^{2p} \quad \left| \Rightarrow w_2 \notin L \Rightarrow L \text{ nu este regular} \right.$$

dar $k \neq p$

b) $L = \{a^k \mid k - \text{nr prim}\}$

pp. RA, L-regular, conf. bunei de pompate

$\Rightarrow \exists p \geq 1 \text{ a.t. } \forall w \in L \text{ cu } |w| \leq p$

$\exists w = xy^z, 0 < |y| \leq p$

$w_i = x y^{i-1}, w_i \in L, \forall i \in \mathbb{N}$

$w = a^p \Rightarrow |w| = p \leq p$

$y = a^j \Rightarrow |y| \leq p$

$\Rightarrow w = a^{p-j} a^j$

$$w_i = a^{p-j} (a^j)^i = a^{p-j+ij}$$

$$\text{ex: } w_{p+i} = a^{p-j+(p+1)j} = a^{p+pj} = a^{p(j+1)}$$

dar $p(j+1)$ nu este prim $\Rightarrow w_{p+i} \notin L$ $\Rightarrow L$ nu este regular

c) $L = \{a^{m^2} \mid m \in \mathbb{N}^*\}$

$w = a^{p^2} \text{ cu } j \leq p \text{ (} 0 < j \leq p \text{)}$

$$y = a^j \Rightarrow w_i = a^{p^2-j} (a^j)^i = a^{p^2-j} \cdot a^{ij} = a^{p^2+j(i-1)}$$

$$\text{ex: } w_2 = a^{p^2}$$

$$p^2 < p^2 + j < (p+1)^2 = p^2 + 2p + 1 \quad / -p^2$$

$$0 < j < 2p+1 \quad (\text{A}) \quad (\text{disparece } 0 < j \leq p)$$

d) $L = \{a^{2^m} \mid m \in \mathbb{N}^*\}$

$$w = a^{2^p}$$

$$y = a^j \Rightarrow 0 < j \leq p$$

$$\Rightarrow w = a^{2^p-j} \cdot a^j$$

$$w_i = a^{2^p-j} \cdot (a^j)^i = a^{2^p-j+i j} = a^{2^p+j(i-1)}$$

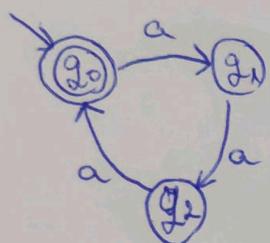
$$w_2 = a^{2^p+j}$$

$$0 < j < p \Leftrightarrow \mathcal{L}^P < \mathcal{L}^{j+1} \subset \underbrace{\mathcal{L}^P + \mathcal{L}^P}_{\leq 2} \Rightarrow P \geq 2$$

$\Rightarrow \mathcal{L}^P + j$ nu este de forma $\mathcal{L}^m \Rightarrow w_2 \notin L \Rightarrow L$ nu este regular

e) k-mr. mat. fixat

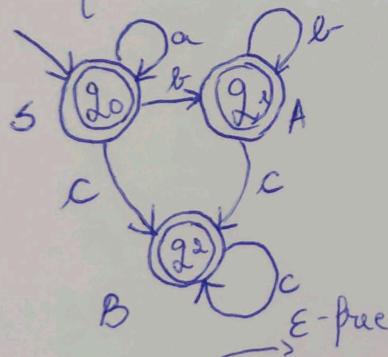
$$L = \{a^{km} \mid m \in \mathbb{N}\}$$



$$\text{ex: } a^{2m}, a^{3m}$$

Limbaj regular \Leftrightarrow pot desena un AF \Leftrightarrow
 \Leftrightarrow gramatica regulara \Leftrightarrow expr. reg.

p) $L = \{a^m b^m c^p \mid m, m, p \in \mathbb{N}\}$



$$\text{expr. reg. } a^* b^* c^*$$

$\xrightarrow{s \rightarrow E}$

$s \rightarrow E$

$s \rightarrow a$

$s \rightarrow a s$

$s \rightarrow b$

$s \rightarrow b A$

$s \rightarrow c$

$s \rightarrow c B$

$A \rightarrow b$

$A \rightarrow c B$

$A \rightarrow c$

$B \rightarrow c$

$B \rightarrow c B$

$A \rightarrow b A$

$$\begin{cases} s \rightarrow E \\ s \rightarrow a \\ s \rightarrow b \\ s \rightarrow c \\ s \rightarrow a s \\ s \rightarrow b A \\ s \rightarrow c B \end{cases}$$

$$\begin{cases} s \rightarrow c \\ s \rightarrow a s \\ s \rightarrow b A \\ s \rightarrow c B \end{cases}$$

$A \rightarrow b$

$A \rightarrow c$

$A \rightarrow b A$

$A \rightarrow c B$

$B \rightarrow c$

$B \rightarrow c B$

$$2. L = \{a^k \mid k \text{-nprim}\}$$

Proprietăți de închidere

L_1, L_2 reg. $\Rightarrow L_1 \cup L_2, L_1 \cap L_2, L_1 L_2, L_1^*, \text{compl}(L_1) \Leftrightarrow \bar{L}_1$, toate sunt regulare

a) p.p. L reg. \Rightarrow (prop. închidere) $\text{compl}(L)$ regular

$$\bar{L} = \{a^k \mid k \text{-nprim}\} \quad \Rightarrow \bar{L} \text{ nu e regular}$$

la b) am arătat că nu e reg.

$$w = a^p, |w| \leq p$$

$$y = a^j$$

$$w_i = a^{p-j} \cdot a^{ji} = a^{p+j(p-i)}$$

$$w_{p+1} = a^{p+pj} = a^{p(j+1)}$$

$p(j+1)$ nu e prim ---

b) p -natural ≥ 1

$$p = ? \text{ a. i. } \nexists w \in L, \exists w = xy \neq a \cdot i$$

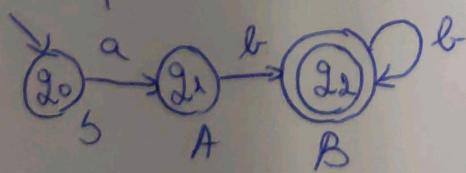
$$w_i = x y^i z \in L, \forall i \in \mathbb{N}^*$$

$$p=3 \quad |w| > 3$$

$$w = a^3 \quad \Rightarrow w = a^2 \cdot a^2 \\ w_i = a^2 \cdot a^{2i} = a^{2(i+1)}, \forall i \in \mathbb{N}^*$$

1.2

$$1.a) L = \{ab^n \mid n \in \mathbb{N}^*\}$$



$$S \rightarrow aA$$

$$A \rightarrow b$$

$$A \rightarrow bB$$

$$B \rightarrow bB$$

$$B \rightarrow b$$

$$S \rightarrow abB$$

$$B \rightarrow bB$$

$$B \rightarrow E$$

b) $L = \{a^m b^{m+\lambda} \mid m \in \mathbb{N}\}$

$S \rightarrow b$

$S \rightarrow aA bb$

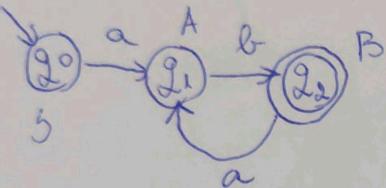
$A \rightarrow aAb$

$A \rightarrow E$

1.3

a) $S \rightarrow abS \quad | \Rightarrow (ab)^m, m \geq 1$

$S \rightarrow ab$



$S \rightarrow aA$

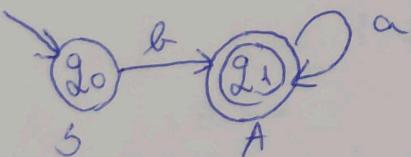
$A \rightarrow bB$

$A \rightarrow b$

$B \rightarrow aA$

b) $S \rightarrow Sa \quad | \Rightarrow ba^m, m \geq 0$

$S \rightarrow b$



$S \rightarrow bA$

$A \rightarrow E$

$A \rightarrow aA$

Seminar 6

a) $\underbrace{1 \cdot 0}_{E} \underbrace{1110111}_{\lambda}$ $(1^*01)^*(11+0)^*$ $+ = sau$
 $(1^*01) \quad (1^*01) \quad (11+0)^*$ $\Rightarrow \text{acceptata da}$

b) $\underbrace{11100}_{\lambda} \underbrace{111}_{\lambda}$ $(1^*0) + (0^*11)$
 $\underbrace{(1^*0)}_{\lambda} \quad \underbrace{(1^*0)}_{E} \quad \downarrow m\mu \Rightarrow m\mu$

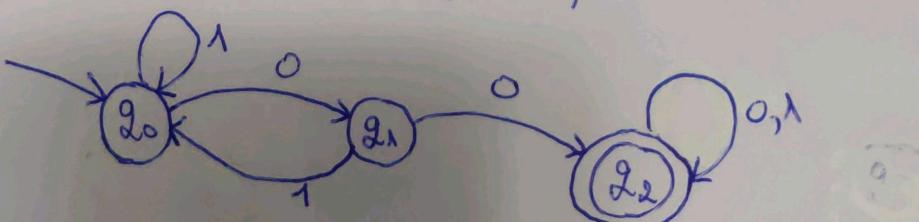
c) $\underbrace{11100}_{\lambda} \underbrace{11}_{\lambda}$ $(1^*0) + (0^*11)$
 $\underbrace{(1^*0)}_{\lambda} \quad \downarrow m\mu \Rightarrow m\mu$

d) $\underbrace{11100}_{\lambda} \underbrace{11}_{\lambda}$ $(1^*0)^*(0^*11)$
 $\underbrace{(1^*0)}_{\lambda} \quad \underbrace{(0^*11)}_{0} \quad \Rightarrow da$

e) $\underbrace{011100010}_{\lambda} \underbrace{\lambda}_{\lambda}$ $01^*01^*(11^*0)^*$
 $\underbrace{01^*}_{\lambda} \quad \underbrace{01^*}_{E} \quad ? \Rightarrow m\mu$

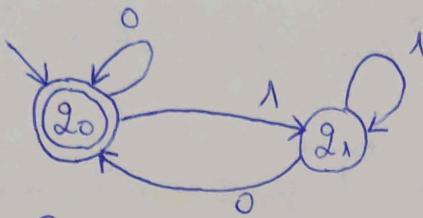
f) $\underbrace{10000}_{0000} \underbrace{011}_{\lambda}$ $(10^*+11)^*(0^*1)^*$
 $\underbrace{(10^*+11)}_{0000} \quad \underbrace{(0^*1)^*}_{11} \quad \Rightarrow da$

2. a) $(01+\lambda)^* 00 (0+\lambda)^*$

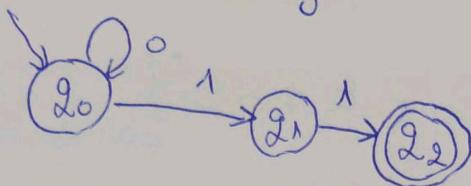


$$b) (\lambda^* \circ)^* + \circ^* \lambda \lambda$$

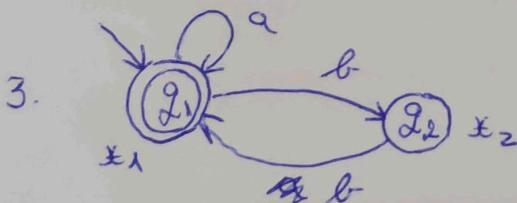
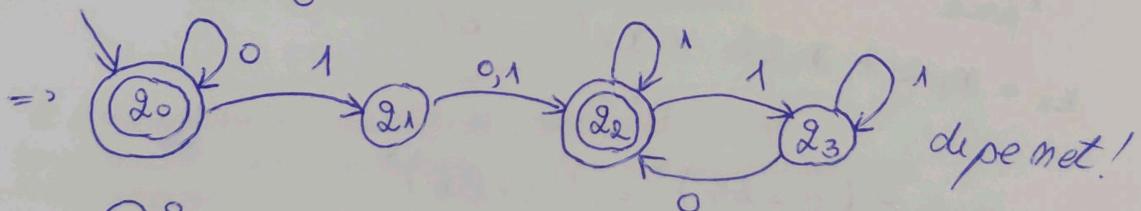
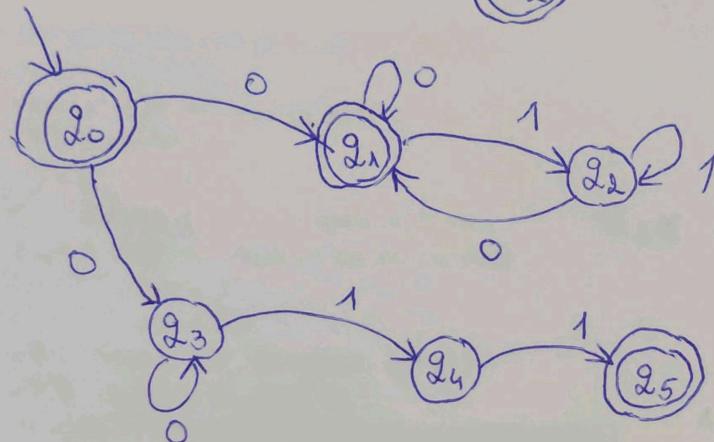
$$(\lambda^* \circ)^*:$$



$$\circ^* \lambda \lambda:$$



total:



	$k=0$	$k=\lambda$	$k=\lambda^*$
r_{11}^k	$\epsilon + a$	a^*	
r_{12}^k	b	$a^* b$	
r_{21}^k	b	$b a^*$	
r_{22}^k	ϵ	$\epsilon + b a^* b$	

$$r_{12}^\lambda = r_{12}^0 + r_{11}^0 (r_{11}^0)^* r_{12}^0 =$$

$$= b + (\epsilon + a)(\epsilon + a)^* b =$$

$$= b + (\epsilon + a)^+ b = (\epsilon + (\epsilon + a)^+) b =$$

$$= (\epsilon + a)^* b = a^* b$$

$$r_{21}^* = r_{21}^o + r_{21}^o (r_{11}^o)^* r_{11}^o = b + b(E+a)^*(E+a) = ba^*$$

$$\begin{aligned} r_{22}^* &= r_{22}^o + r_{21}^o (r_{11}^o)^* r_{12}^o = \\ &= E + b \underbrace{(E+a)^*}_a b = E + ba^*b \end{aligned}$$

Metoda 2

$$\begin{cases} x_1 = x_1 a + x_2 b \\ x_2 = x_1 b \end{cases}$$

$$\text{regulă } X = \beta + \alpha X \\ \Rightarrow X = \beta \alpha^*$$

$$\begin{aligned} \cup x_{\text{final}} &= x_1 \\ x_1 &= x_1 a + x_1 b b = x_1 \underbrace{(a + b b)}_{\alpha} \\ &\quad \beta = 0 \\ \Rightarrow x_1 &= E(a + b b)^* = (a + b b)^* \end{aligned}$$

Seminar 7

1 Limbaje si gramatici de toate felurile

X. Definiti (folosind multimi) limbajul sevenelor de simb. a care apar in numar par. Dati o gramatica regulara, una independenta de context si una ce nu e independenta de context care genereaza acest limbaj.

✓. Sa se dea cate o gramatica care genereaza limbajele:

1. $L = \{ww \mid w \in \{a, b\}^*\}$
2. $L = \{wxw \mid w \in \{a, b\}^+, x \in \{a, b\}^*\}$
3. $L = \{a^n b^n c^n \mid n \in \mathbb{N}^*\}$
4. $L = \{a^n b^n c^n d^n \mid n \in \mathbb{N}^*\}$
5. $L = \{a^{2^n} \mid n \in \mathbb{N}\}$ a apare de 2^n ori
6. $L = \{w \mid w \in \{a, b, c\}^*, nr_a(w) = nr_b(w) = nr_c(w)\}$
7. $L = \{a^n b^n c^m d^m \mid n, m \in \mathbb{N}\}$
8. $L = \{a^n b^m c^m d^n \mid n, m \in \mathbb{N}\}$
9. $L = \{a^n b^m c^k \mid n, m, k \in \mathbb{N}, (n=m) \text{ sau } (m=k)\}$
10. $L = \{a^n b^m c^k \mid n, m, k \in \mathbb{N}, m+n = k\}$
11. $L = \{w \in \{a, b\}^* \mid w \text{ incepe si se termina cu acelasi simbol}\}$

2 Ambiguitate in gramatici independente de context

1. Sa arate ca gramaticile urmatoare sunt ambigue si sa se gaseasca o gramatica echivalenta neambigua.

- a) $S \rightarrow aS \mid Sb \mid c$
- b) $S \rightarrow \text{if } b \text{ then } S \text{ else } S \mid \text{if } b \text{ then } S \mid \text{stmt}$
- c) $S \rightarrow (S \mid S) \mid (S) \mid 1$

3. Reprezentari pentru gramatici independente de context

Fie gramatica:

$$G = (\{E, T, F\}, \{a, +, *, (,)\}, P, E)$$

$$\begin{aligned} P: \quad & E \rightarrow E + T \\ & E \rightarrow T \\ & T \rightarrow T * F \\ & T \rightarrow F \\ & F \rightarrow (E) \\ & F \rightarrow a \end{aligned}$$

Pentru gramatica data, ilustrati modurile de reprezentare:

- a) Folosind liste liniare (de exemplu: vectoriala)
- b) reprezentarea cu ajutorul unor liste inlantuite ramificate
 - Liste dublu inlantuite
 - Liste triplu inlantuite

Seminar 4

Gramatici regulare

$$\begin{aligned} S &\rightarrow \epsilon \\ S &\rightarrow aA \\ S &\rightarrow a \end{aligned}$$

Gramatici independente de context
(Gic)

$$\begin{aligned} S &\rightarrow \alpha \\ | \\ H \cup \Sigma \end{aligned}$$

ex. $S \rightarrow ABC$
 $S \rightarrow aaBbb$

Gramatici dependente de context (Gdc)

$$\alpha A\beta \rightarrow \alpha \delta \beta$$

1. $L = \{a^{2m} \mid m \in \mathbb{N}\}$

$A \rightarrow \epsilon$ G. reg.

$$A \rightarrow aB$$

$$B \rightarrow a$$

$$B \rightarrow aC$$

$$C \rightarrow aB$$

Gic - $A \rightarrow \epsilon$
 $A \rightarrow aaA$

Gdc $A \rightarrow \epsilon$

$$A \rightarrow aB$$

$$aB \rightarrow aa$$

$$aB \rightarrow aaaB$$

2. 1. $L = \{ww \mid w \in \{a, b\}^*\}$

ex. $\frac{aba}{w} \frac{aba}{w}$

$$\begin{array}{l|l} \begin{aligned} S &\rightarrow \epsilon \\ S &\rightarrow aS \\ S &\rightarrow bS \\ aS &\rightarrow ab \\ bS &\rightarrow bb \\ DS &\rightarrow D \\ DS &\rightarrow DS \end{aligned} & \begin{aligned} DA &\rightarrow Da \\ aA &\rightarrow Aa \\ bA &\rightarrow A b \\ aB &\rightarrow Ba \\ bB &\rightarrow Bb \\ D &\rightarrow \epsilon \end{aligned} \end{array}$$

2. $L = \{wxw \mid w \in \{a,b\}^+, x \in \{a,b\}^*\}$

$S \rightarrow aS A$

$S \rightarrow bS B$

~~$AB \rightarrow aAb$~~

$\{D \rightarrow X\} - \text{sauf } \begin{cases} D \rightarrow \epsilon \\ aS \rightarrow aXD \\ bS \rightarrow bXD \end{cases}$

$X \rightarrow E$

$X \rightarrow aX$

$X \rightarrow bX$

$DB \rightarrow Db$

$DA \rightarrow Da$

$aA \rightarrow Aa$

$bB \rightarrow Bb$

$aB \rightarrow Ba$

$bA \rightarrow Ab$

- "delimitator"?

$\rightarrow D$

3. $L = \{a^m b^m c^m \mid m \in \mathbb{N}^*\}$

$S \rightarrow abc$

$S \rightarrow aAbc$

$Ab \rightarrow bAc$

$Ac \rightarrow Bbcc$

$B \rightarrow Bl$

$aB \rightarrow aa$

(*) $aB \rightarrow aaA$

4. $L = \{a^m b^m c^m d^m \mid m \in \mathbb{N}^*\}$

$S \rightarrow abcd$

$S \rightarrow aAbcd$

$Ab \rightarrow bA$

$Ac \rightarrow cA$

$Ad \rightarrow cBdd$

$abcd$

\uparrow

~~puncte cheie~~

$a\underline{bcd}$

\downarrow

$a\underline{bAgd}$

\downarrow

$a\underline{bcAd}$

\downarrow

$a\underline{bcBdd}$

\downarrow

$a\underline{bgPccdd}$

\downarrow

"cursor"?

$\downarrow_{A,B}$

$m(*)$

$CB \rightarrow BC$ $\& B \rightarrow Bb$ $aB \rightarrow aab$ $aB \rightarrow aaAb$ $\underline{aB} bccdd \Rightarrow$ aab $aaAb bccdd \Rightarrow$ aaaaaa

$$5. L = \{a^m \mid m \in \mathbb{N}\}$$

 $S \rightarrow LaR$ $L \rightarrow E$ $R \rightarrow E$ $L \rightarrow LD$ $Da \rightarrow aad$ $DR \rightarrow R$ cursor Δ

$$6. L = \{w \mid w \in \{a, b, c\}^*, m_a(w) = m_b(w) = m_c(w)\}$$

 $S \rightarrow E$ $S \rightarrow ABCS$ $AB \rightarrow BA$ $AC \rightarrow CA$ $BA \rightarrow AB$ $BC \rightarrow CB$ $CA \rightarrow AC$ $CB \rightarrow BC$ $A \rightarrow a$ $B \rightarrow b$ $C \rightarrow c$

prea mult pentru capacitatea mea...

$$4. L = \{a^m b^n c^m d^m \mid m, n \in \mathbb{N}\}$$

cicluri 3 \geq GDC, altfel GIC $S \rightarrow E$ $S \rightarrow aAb$ $A \rightarrow aAb$ $A \rightarrow E$ $S \rightarrow cDd$ $D \rightarrow cDd$ $D \rightarrow E$ $S \rightarrow aAbcDd \Leftrightarrow S \rightarrow AD$

8. $L = \{a^m b^m c^m d^m \mid m, n \in \mathbb{N}\}$

$S \rightarrow E$
 $S \rightarrow aAd$

$A \rightarrow aAd$

$A \rightarrow E$

$A \rightarrow bBc$

$B \rightarrow bBc$

$B \rightarrow E$

9. $L = \{a^m b^m c^k \mid m, m, k \in \mathbb{N}, (m=m) \text{ sau } (m=k)\}$

$S \rightarrow E$
 $S \rightarrow aABC$

$A \rightarrow aAb$

$A \rightarrow E$

$C \rightarrow cc$

$C \rightarrow E$

$S \rightarrow D bEc$

$E \rightarrow bEc$

$E \rightarrow E$

$D \rightarrow aD$

$D \rightarrow E$

10. $L = \{a^m b^m c^k \mid m+m=k, m, m, k \in \mathbb{N}\}$ peste puteri

$k=0 S \rightarrow E$

$m=0 S \rightarrow bDc$

$D \rightarrow E$

$D \rightarrow bDc$

$m=0 S \rightarrow aEc$

$E \rightarrow E$

$E \rightarrow aEc$

altfel $S \rightarrow aAbcc$

$Ab \rightarrow bA$

$Ac \rightarrow bBcc$

$Bc \rightarrow c$

$Bc \rightarrow bBcc$

$bB \rightarrow Bb$

$aB \rightarrow aaC$

$Cb \rightarrow bc$

$cc \rightarrow cc$

$cc \rightarrow Bcc$

$aA \rightarrow aac$

11. $L = \{w \in \{a, b\}^* \mid w \text{ inceps si sf. un accesi}\}$

$S \rightarrow E$
 $S \rightarrow aAa$
 $S \rightarrow bAb$
 $A \rightarrow E$
 ~~$A \rightarrow b$~~
 $A \rightarrow aA$
 $A \rightarrow bA$

2.

a) $S \rightarrow aS$
 $S \rightarrow Sb$ ambigua

$S \rightarrow c$

$S \rightarrow aS$
 $S \rightarrow cB$
 $B \rightarrow E$
 $B \rightarrow bB$

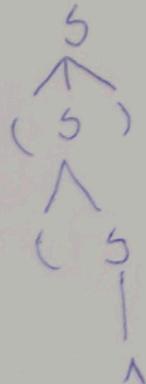
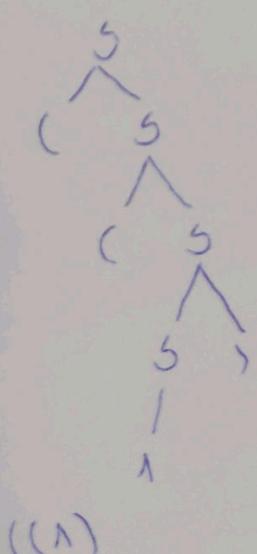
b) $S \rightarrow \text{if } b \text{ then } S$

$S \rightarrow \text{if } b \text{ then } S \text{ else } S$

$S \rightarrow \text{stmt}$

$S \stackrel{\text{st}}{\Rightarrow} \text{if } b \text{ then } S \stackrel{\text{st}}{\Rightarrow} \text{if } b \text{ then if } b \text{ then } S \text{ else } S \Rightarrow$

c) $S \rightarrow (S)$
 $S \rightarrow S)$
 $S \rightarrow (S)$
 $S \rightarrow \lambda$



$((\lambda)) \Rightarrow \text{ambigua}$

$\Rightarrow S \rightarrow (S)$

$S \rightarrow \lambda A$

$A \rightarrow A)$

$A \rightarrow \epsilon$

3. $G = (N, \Sigma, P, S)$

CFGrammar

startSymbol : Symbol

nonTerminals : Set <Symbol>

terminals : Set <Symbol>

rules : List <Rule>

see
Multimap <Symbol, List <Symbol>>

Symbol

value : String

isTerminal : boolean

Rule :

LHS : Symbol

RHS : ~~Symbol~~ List <Symbol>

Seminar 8

Automate push-down (APD)

1. Construiti APD care accepta urmatoarele limbaje dupa criteriul stivei vide:

- a) $L = \{a^n b^{2n} \mid n \geq 0\}$
- b) $L = \{a^n b^m \mid m, n \geq 0\}$
- c) $L = \{a^n b^m \mid n \geq m \geq 0\}$
- d) $L = \{a^m b^n \mid n \geq m \geq 0\}$
- e) $L = \{ww^{\text{tilde}} \mid w \in \{a,b\}^*, w^{\text{tilde}} \text{ este inversul lui } w\}$
- f) $L = \{w \mid w \in \{a, b\}^*, nr_a(w) = nr_b(w)\}$
- g) $L = \{a^{2n} b^{2n} \mid n \geq 0\}$

- h) $L = \{a^n b^n \mid n \geq 0\} \cup \{b^n a^n \mid n \geq 0\}$
- i) $L = \{a^n b^n \mid n \geq 0\} \cup \{a^n b^{2n} \mid n \geq 1\}$
- j) $\{w \mid w^{\text{tilde}} \text{ is a substring of } x, \text{ where } x \in \{a, b\}^*, w \in \{a, b\}^*, |w| \geq 1\}$

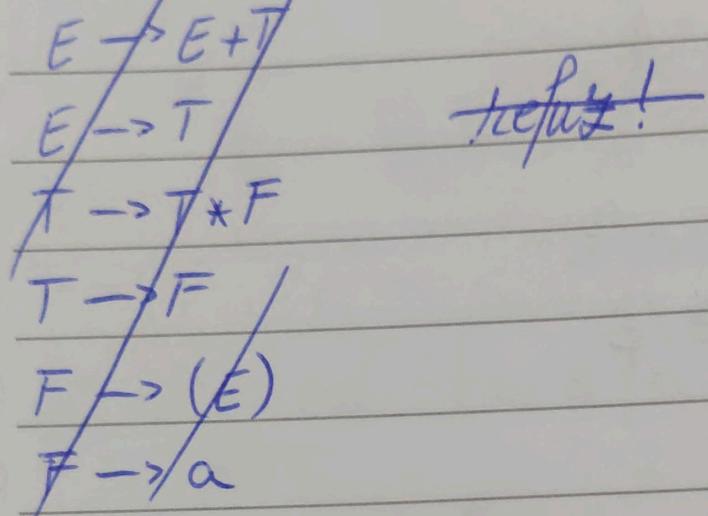
2. Pentru limbajul de la punctul f, dati o gramatica independenta de context (GIC) ce il genereaza. Construit APD echivalent cu GIC data (aplicand algoritmul de constructie).

3. Pentru APD de la punctele e si f, dati APD care accepta acelasi limbaj dupa criteriul starii finale. >

Seminarul 8

5em 4

3.b)



$$M_F = (Q, \Sigma, \delta, q_0, F)$$

$$M_{PD} = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

PD - push down
 ↓ alfabetul stivei stare initială
 'stivă'

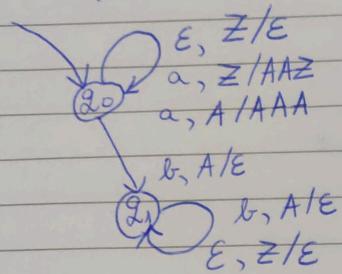
$$\delta(q_0, w, z_0) \vdash (q_1, \underbrace{A}_{\text{dun } \Gamma^*})$$

$$L_\varepsilon = (\overset{q \in Q}{\overleftarrow{q}}, \varepsilon, \varepsilon) \quad - \text{stiva vidă}$$

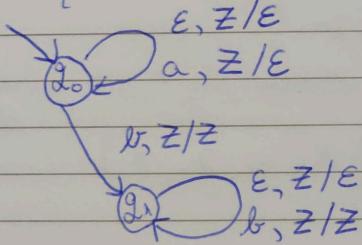
$$L_f = (q_f, \varepsilon, \text{ava})$$

↳ $q_f \in F$

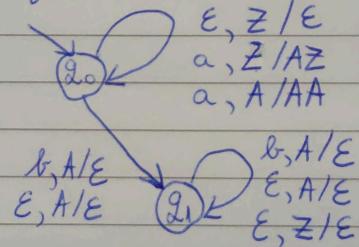
1.a) $L = \{a^m b^m \mid m \geq 0\}$



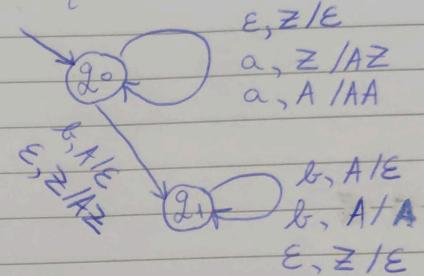
b) $L = \{a^m b^m \mid m, n \geq 0\}$



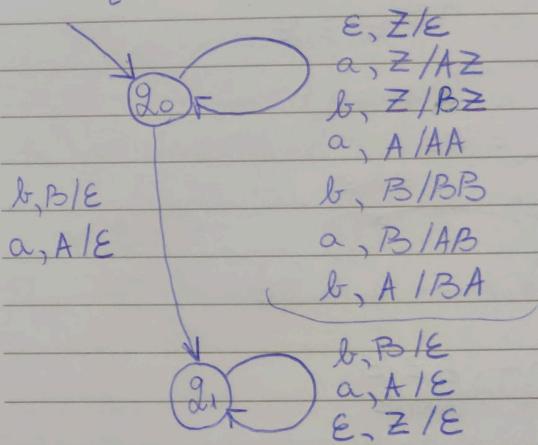
c) $L = \{a^m b^m \mid m \geq m \geq 0\}$



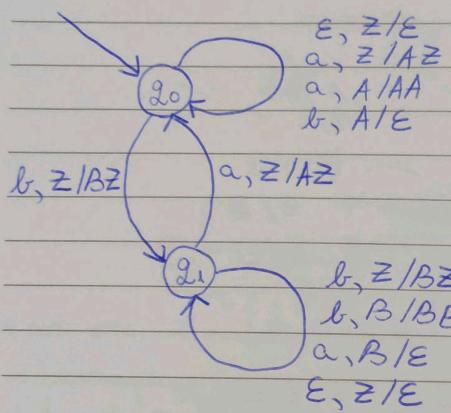
d) $L = \{a^m b^m \mid m \geq m \geq 0\}$



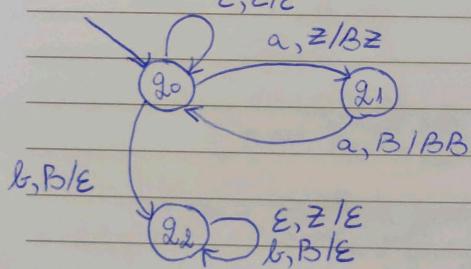
e) $L = \{w w^* \mid w \in \{a, b\}^*, w^* - \text{inv lui } w\}$



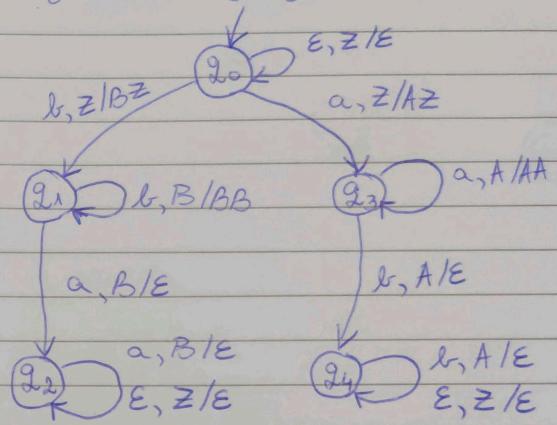
f) $L = \{w \mid w \in \{a, b\}^*, m_{\alpha}(w) = m_{\beta}(w)\}$



g) $L = \{a^m b^m \mid m \geq 0\}$



h) $L = \{a^m b^m \mid m \geq 0\} \cup \{b^m a^m \mid m \geq 0\}$



2. $w, m_{\alpha}(w) = m_{\beta}(w)$

$$\begin{aligned} s &\rightarrow \epsilon \\ s &\rightarrow a s b \\ s &\rightarrow b s a \\ s &\rightarrow s s \end{aligned}$$

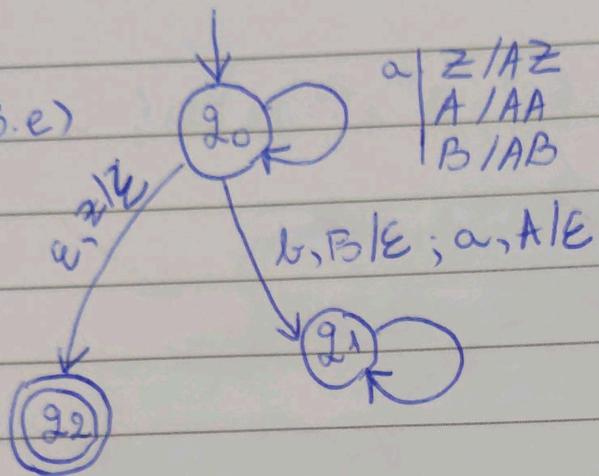
$$A \rightarrow \alpha$$

$$\delta(g, \epsilon, A) = \{(g, \alpha)\}$$

$$\delta(g, a, a) = \{(g, \epsilon)\}$$

	a	b	ϵ
g	$\begin{cases} g \\ a \end{cases}$	$\begin{cases} g \\ \epsilon \end{cases}$	$\begin{cases} (g, a^* b), (g, \epsilon) \\ (g, b^* g), (g, s s) \end{cases}$
g	$\begin{cases} g \\ b \end{math>$	$\begin{cases} g \\ \epsilon \end{cases}$	

3.e)



$\alpha | Z/AZ$
 A/AA
 B/AB

$\beta | Z/BZ$
 A/BA
 B/BB

Analiza sintactica

Seminar saptamanile 9, 10, 11, 12

Continut:

1	Analizorul dependent cu reveniri	1
2	Functiile FIRST ₁ , FOLLOW ₁	1
3	Analiza sintactica dependenta LL(1).....	1
4	Analiza sintactica LR* (LR(0), SLR, LR(1), LALR).....	2
5	Gramatica de precedenta simpla. Exemplu	3

1 Analizorul dependent cu reveniri

1. Fie gramatica:

$$\begin{aligned} S &\rightarrow aSbS \\ S &\rightarrow aS \\ S &\rightarrow c \end{aligned}$$

Folosind analizorul dependent cu reveniri verificati daca :

- $acbc \in L(G)$ (?)
- $cb \in L(G)$ (?)

2. Analog pentru gramatica:

$$\begin{aligned} S &\rightarrow +SS \\ S &\rightarrow -SS \\ S &\rightarrow a \end{aligned}$$

si secenta: $+a-aa$.

2 Functiile FIRST₁, FOLLOW₁

1. Determinati FIRST₁ si FOLLOW₁ pentru neterminalele urmatoarei gramatici:

$$\begin{aligned} S &\rightarrow abA \\ S &\rightarrow \epsilon \\ A &\rightarrow Sa \\ A &\rightarrow b \end{aligned}$$

3 Analiza sintactica dependenta LL(1)

1. Fie gramatica:

$$\begin{aligned} S &\rightarrow \text{if } c \text{ then } S \text{ endif} \\ S &\rightarrow \text{if } c \text{ then } S \text{ else } S \text{ endif} \\ S &\rightarrow \text{stmt} \end{aligned}$$

Daca inlocuim: *if c then cu a, else cu b, endif cu c, si stmt cu i* avem:

$$\begin{aligned} S &\rightarrow aS \\ S &\rightarrow aSbS \\ S &\rightarrow i \end{aligned}$$

Pentru una dintre cele 2 gramatici de mai sus:

- Verificati daca gramatica este LL(1).
- Incercati sa transformati gramatica in una echivalenta LL(1) aplicand factorizarea la stanga. Verificati daca noua gram. este LL(1).
- Folosind un analizor dependent verificati daca secenta:
 $\text{if } c \text{ then if } c \text{ then stmt else stmt endif endif}$
(sau echivalenta ei scrisa cu a,b,c,i)
apartine limbajului generat de gramatica.

2. Fie gramatica ambigua:

$$S \rightarrow \text{if } c \text{ then } S \text{ else } S \mid \text{if } c \text{ then } S \mid \text{stmt}$$

Daca inlocuim: *if c then cu a, else cu b, si stmt cu i* avem:

$$\begin{aligned} S &\rightarrow aS \\ S &\rightarrow aSbS \\ S &\rightarrow i \end{aligned}$$

Pentru una dintre cele 2 gramatici de mai sus:

- Verificati daca gramatica este LL(1).
- Incercati sa transformati gramatica in una echivalenta LL(1) aplicand factorizarea la stanga. Verificati daca gramatica obtinuta este LL(1).
- Discutati, impreuna cu cadrul didactic, cum se poate modifica tabelul de analiza astfel incat sa se eliminate conflictele.
- Folosind analizorul LL(1), verificati daca secenta:
 $\text{if } c \text{ then if } c \text{ then stmt else stmt}$
(sau echivalenta ei scrisa cu a,b,c,i)
apartine limbajului generat de gramatica.

3. Fie gramatica:

$$\begin{aligned} E &\rightarrow T + E \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow (E) \mid a \end{aligned}$$

- a) Verificati daca gramatica este LL(1).
b) Incercati sa transformati gramatica in una echivalenta LL(1) aplicand factorizarea la stanga. Verificati daca gramatica obtinuta este LL(1).
c) Folosind un analizor descendant verificati daca seceventa:
 $a + a$
apartine limbajului generat de gramatica.

4. Fie gramatica:

$$\begin{aligned} \text{List} &\rightarrow \text{id} \\ \text{List} &\rightarrow \text{id} \text{ sep List} \end{aligned}$$

- a) Verificati daca gramatica este LL(1).
b) Incercati sa transformati gramatica in una echivalenta LL(1) aplicand factorizarea la stanga. Verificati daca gramatica obtinuta este LL(1).

5. Fie gramatica:

$$\begin{aligned} S &\rightarrow \text{begin Slist end} \\ S &\rightarrow \text{stmt} \\ \text{SList} &\rightarrow S \\ \text{SList} &\rightarrow S ; \text{SList} \end{aligned}$$

- a) Verificati daca gramatica este LL(1).
b) Incercati sa transformati gramatica in una echivalenta LL(1) aplicand factorizarea la stanga. Verificati daca gramatica obtinuta este LL(1).

comparare

notiuni de gramatica

4. Dati gramatica echivalenta neambigua a gramaticii pentru structura if-then-else (gram. ambigua data in problema 1)

4 Analiza sintactica LR* (LR(0), SLR, LR(1), LALR)

1. Fie gramatica:

$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow aA \\ A &\rightarrow b \end{aligned}$$

- a) Verificati daca este LR(0)
b) Verificati daca este LR(1)
c) Verificati daca este LALR
d) Folosind un analizor de tip LR(K), verificati daca seceventa: abab apartine limbajului generat de gramatica.

Analizorul va fi ales in functie de raspunsul la intrebarile de mai sus.

2. Fie gramatica:

$$\begin{aligned} S &\rightarrow \text{if } c \text{ then } S \text{ endif} \\ S &\rightarrow \text{if } c \text{ then } S \text{ else } S \text{ endif} \\ S &\rightarrow \text{stmt} \end{aligned}$$

Daca inlocuim: *if c then cu a, else cu b, endif cu c, si stmt cu i* avem:

$$\begin{aligned} S &\rightarrow a S c \\ S &\rightarrow a S b S c \\ S &\rightarrow i \end{aligned}$$

Pentru una dintre cele 2 gramatici de mai sus:

- a) Verificati daca gramatica este LR(0).
b) Verificati daca este SLR.
c) Este LR(1)?
d) Folosind un analizor de tip LR(K), verificati daca seceventa:
if c then if c then stmt else stmt endif endif
(sau echivalenta ei) apartine limbajului generat de gramatica.
Analizorul va fi ales in functie de raspunsul la intrebarile de mai sus.

3. Fie gramatica ambigua:

$$S \rightarrow \text{if } c \text{ then } S \text{ else } S \mid \text{if } c \text{ then } S \mid \text{stmt}$$

Daca inlocuim: *if c then cu a, else cu b si stmt cu i* avem:

$$\begin{aligned} S &\rightarrow a S \\ S &\rightarrow a S b S \\ S &\rightarrow i \end{aligned}$$

Pentru una dintre cele 2 gramatici de mai sus, verificati daca este LR(1).

4. Dati gramatica echivalenta neambigua a gramaticii pentru structura if-then-else (gram. ambigua data in problema anterioara)

Dati un cuvant care, in gramatica originala ambigua, poate sa corespunda la doua structuri if cu proprietatea:

a) cel mai interior if **contine** ramura "else", iar cel exterior **nu contine** ramura "else"

b) cel mai interior if **nu contine** ramura "else", iar cel exterior **contine** ramura "else"

Verificati, folosind gramatica neambigua echivalenta si analiza LR(1) ca acel cuvant dat anterior este generat de gramatica echivalenta neambigua.

5. Fie gramatica:

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow id$$

$$F \rightarrow (E)$$

Verificati daca gramatica este LR(1)

6. Fie gramatica:

$$S \rightarrow begin SL end$$

$$S \rightarrow stmt$$

$$SL \rightarrow S$$

$$SL \rightarrow S ; semicolon SL$$

a) Verificati daca gramatica este LR(0).

b) Este SLR?

c) Folosind un analizor de tip LR(K), verificati daca se poate:

begin stmt semicolon stmt end

Analizorul va fi ales in functie de raspunsul la intrebarile de mai sus.

5 Gramatica de precedenta simpla. Exemplu

Mai exista (si vor fi studiate doar la seminar) si alte tipuri de analiza sintactica ascendenta. Dintre acestea, vom vedea doar cum se lucreaza cu (/un exemplu de) gramatici de precedenta simpla.

- Analiza ascendenta
- Depisteaza limita dreapta si a celei stanga pentru a face o reducere
Se folosesc relatiile \leftarrow , \rightarrow , $=\bullet$ (relatii de precedenta)

Relatii de precedenta Wirth-Weber

$$R_{\leftarrow} \subset (N \cup \Sigma \cup \{\$\}) \times (N \cup \Sigma \cup \{\$\})$$

$$R_{\rightarrow} \subset (N \cup \Sigma \cup \{\$\}) \times (N \cup \Sigma \cup \{\$\})$$

$$R_{\Rightarrow} \subset (N \cup \Sigma \cup \{\$\}) \times (\Sigma \cup \{\$\})$$

$$X =\bullet Y : A \rightarrow \alpha XYg \in P$$

$$X \leftarrow \bullet Y : A \rightarrow \alpha XBy \in P, B = \Rightarrow^+ Y\gamma$$

$$X \rightarrow \bullet a : A \rightarrow \alpha BY\gamma \in P, B = \Rightarrow^+ \gamma X, Y = \Rightarrow^* a\delta$$

$$\$ \leftarrow \bullet X : S = \Rightarrow^+ X\alpha$$

$$X \rightarrow \bullet \$: S = \Rightarrow^+ \alpha X$$

Definitie:

gramatica de precedenta simpla

este o gramatica indep. de context propriu

- unic invertibila:
nu exista 2 reguli de productie cu acelasi membru drept
- intre oricare 2 simboluri exista cel mult o relatie de precedenta

Analizorul de precedenta simpla

- construieste tabelul de precedenta a operatorilor

- analizeaza o secenta de terminale

modelul stivei $\sim LR$

$\leftarrow \bullet$ si $=\bullet$ - deplasare

$\rightarrow \bullet$ - reducere $Y \leftarrow \bullet X_1 = \bullet \dots = \bullet X_i \rightarrow Z$

$A \rightarrow X_1 \dots X_i$

Exemplu:

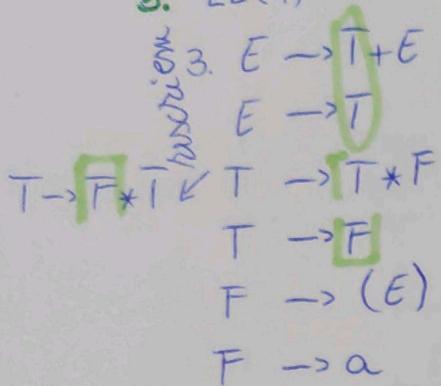
$$\begin{aligned} S &\rightarrow aSSb \\ S &\rightarrow c \end{aligned}$$

	S	a	b	c	\$
S	='	<	='	<	
a	='	<		<	
b		>	>	>	>
c		>	>	>	>
\$		<		<	

Cuvantul: accb ? $\in L(G)$

Seminar 10

3. LL(1)



	First _λ	Follow _λ
E	(, a	\$,)
T	(, a	+ \$,), *
F	(, a	+ \$,), *

$$\text{First}_\lambda(T+E) \cap \text{First}_\lambda(T) = \{ (, a \} \neq \emptyset \Rightarrow \text{nu e } LL(1)$$

- b) $E \rightarrow TE'$ (1)
 $E' \rightarrow +E$ (2)
 $E' \rightarrow E$ (3)
 $T \rightarrow FT'$ (4)
 $T' \rightarrow *T$ (5)
 $T' \rightarrow E$ (6)
 $F \rightarrow (E)$ (7)
 $F \rightarrow a$ (8)

	First _λ	Follow _λ
E	(, a	\$,)
E'	+ , E	\$,)
T	(, a	+ , \$,)
T'	* , E	+ , \$,)
F	(, a	* , + , \$,)

$$\text{First}_\lambda(+E) \cap \text{First} \text{Follow}_\lambda(E') = \emptyset \quad \Rightarrow \text{este } LL(1)$$

$$\text{First}_\lambda(*T) \cap \text{Follow}_\lambda(T') = \emptyset$$

$$\text{First}_\lambda((E)) \cap \text{First}_\lambda(a) = \emptyset$$

sau

cu tabel de analiză

	+	*	(a)	\$
E			TE', λ	TE', λ		
E'	+E, 2				E, 3	E, 3
T			FT', 4	FT', 4		
T'	E, 6		*T, 5		E, 6	E, 6
F			(E), 4	a, 8		
+	pop					
*		pop				
(pop			
a				pop		
)					pop	acc
\$						

\$ - marcator de sf.

- c) $(w\$, \$\$, \$_{11})$
- ↑ stiva de intrare (\$, simbol start)
- secreta
- $(a+a\$, E\$, E) \vdash (a+a\$, TE'\$, \lambda) \vdash$
 $(a+a\$, FT'E'\$, \$_{14}) \vdash (a+a\$, aT'E'\$, \$_{148}) \vdash$
 $(a+a\$, T'E'\$, \$_{148}) \vdash (+a\$, E'\$, \$_{1486}) \vdash$
 $(+a\$, +E\$, \$_{14862}) \vdash (a\$, E\$, \$_{1486214}) \vdash$
 $(a\$, FT'E'\$, \$_{1486214}) \vdash (a\$, T'E'\$, \$_{14862148}) \vdash$
 $(a\$, aT'E'\$, \$_{14862148}) \vdash (\$, T'E'\$, \$_{148621483}) \vdash acc$
 $(\$, E'\$, \$_{148621486}) \vdash (\$, \$, \$_{1486214863}) \vdash acc$
4. $\$ \rightarrow begin$ ~~list end~~
- List $\rightarrow id^{(1)}$
- List $\rightarrow id \ sep$ List (2)

	First _λ	Follow _λ
List	id	\$

	id	sep	\$
List	id ¹ id sep list ²		
id	pop		
sep		pop	
\$			acc

conflict \Rightarrow no es LL(1)!

$$\text{b) } \begin{aligned} \text{List} &\rightarrow id L' \quad (1) \\ L' &\rightarrow sep \text{ List} \quad (2) \\ L' &\rightarrow E \end{aligned}$$

	First _λ	Follow _λ
List	id	\$
L'	sep, E	\$

	id	sep	\$
List	id L'(1)		
L'		sep List(2)	E, 3
id	pop		
sep		pop	
\$			acc

\Rightarrow no existe conflicto
 \Rightarrow este LL(1)

$$\begin{aligned} 5 &\rightarrow b \text{ } SL \text{ } e \\ 5 &\rightarrow \Delta \\ SL &\rightarrow 5 \\ SL &\rightarrow 5 ; SL \end{aligned}$$

	First _λ	Follow _λ
5	b, Δ	\$, ;, e
SL	b, Δ	e

$$\text{a) } \text{First}_\lambda(5) \cap \text{First}_\lambda(5; SL) = \{b, \Delta\} \neq \emptyset \Rightarrow \text{no es LL(1)}$$

$$\begin{aligned} \text{b) } 5 &\rightarrow b \text{ } SL \text{ } e \quad (1) \\ 5 &\rightarrow \Delta \quad (2) \\ SL &\rightarrow 5 S' \quad (3) \\ S' &\rightarrow ; \text{ } SL \quad (4) \\ S' &\rightarrow E \quad (5) \end{aligned}$$

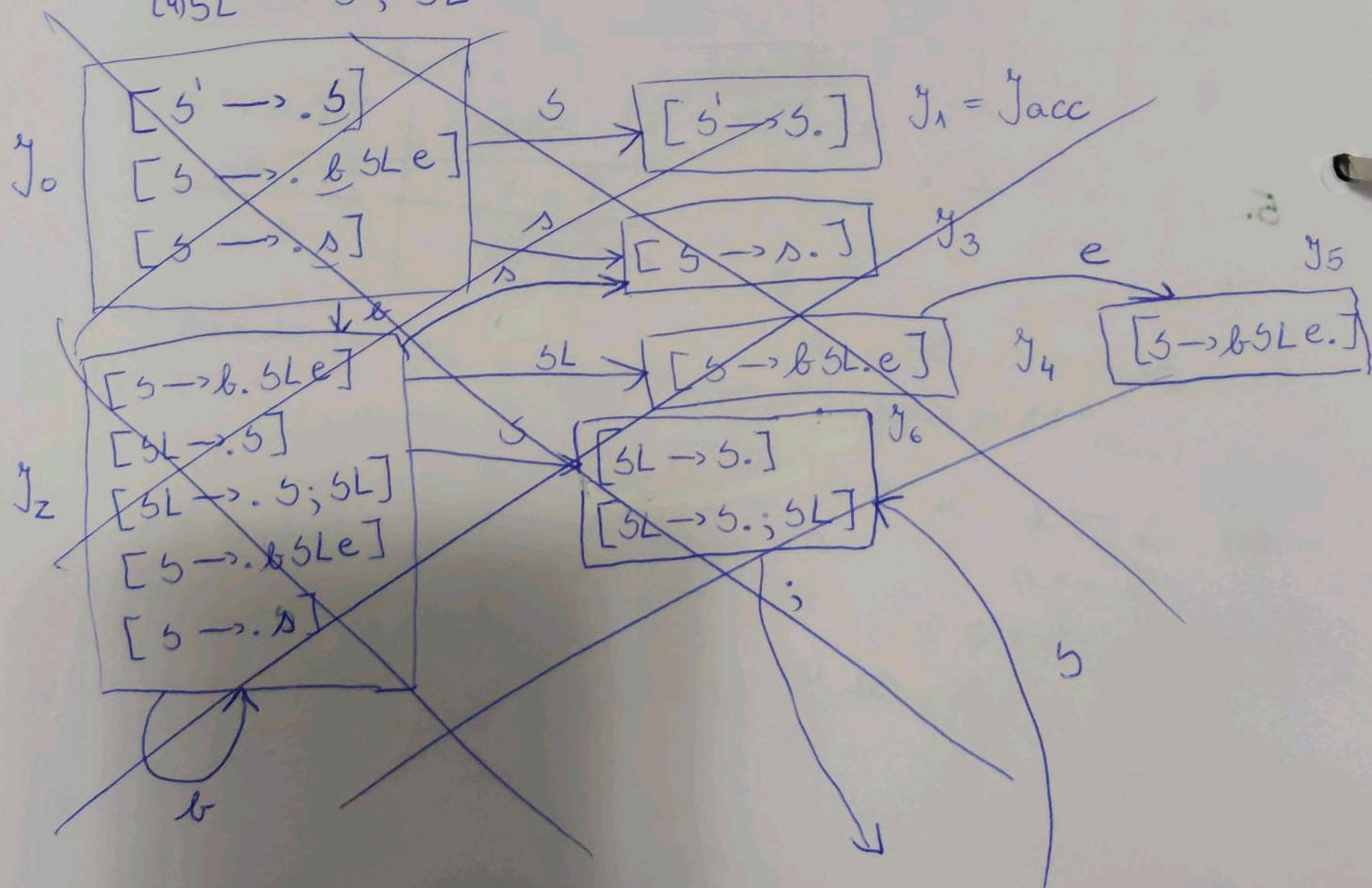
	First _λ	Follow _λ
5	b, Δ	\$, ;, e
SL	b, Δ	e
S'	;	e

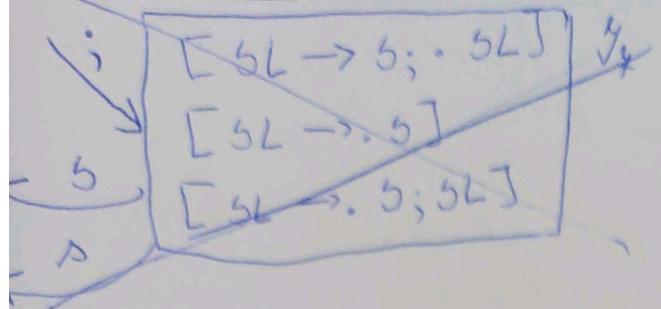
	b	e	s	;	\$
S	BSLe 1		S &		
SL	SS' 3		SS' 3		
S'		SL 4		SL 4	
b	pop				
e		pop			
s			pop		
;				pop	
\$					acc

=> nu are conflicte
=> e LL(1)

4. LR

- (1) $S' \rightarrow S$ ← adăugăm asta ca să obtinem o gramatică îmbogățită
6. (1) $S \rightarrow b \text{ } SL \text{ } e$
 (2) $SL \rightarrow s$
 (3) $SL \rightarrow S$
 (4) $SL \rightarrow S; \text{ } SL$





s-shift
üfră-üfră regulii (reduce)

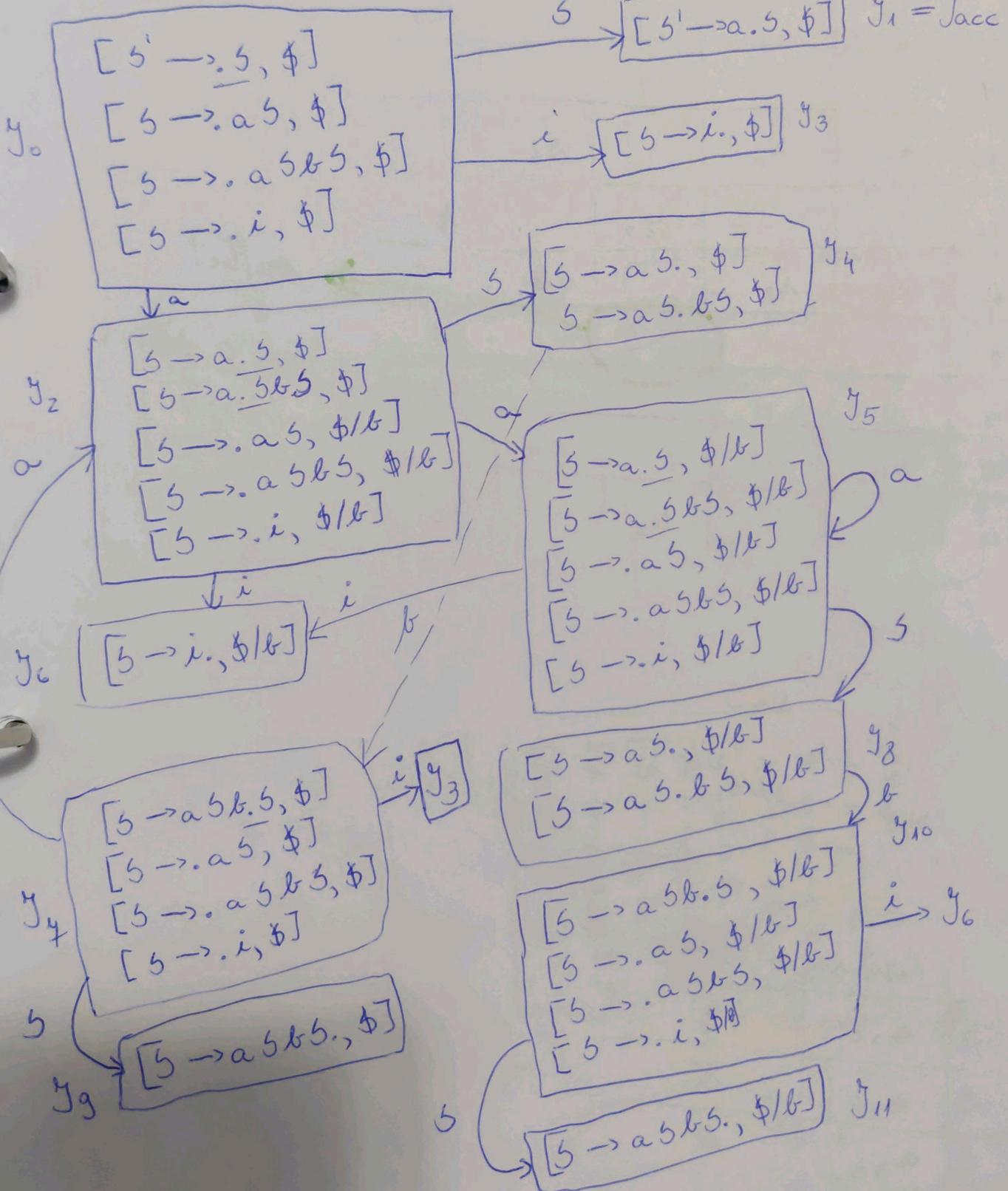
	actiune	5	5L	b	e	s	;
y_0	Δ	y_1		y_2		y_3	
y_1	acc						
y_2	Δ	y_6	y_4	y_2		y_3	
y_3	λ						
y_4	Δ					y_5	
y_5	λ						
y_6	3	Δ					y_4
y_7							
y_8	Δ	y_6	y_8	y_2		y_3	
y_9	4						

conflict \Rightarrow nu e LR(0)!

Seminar 12

4. 3. (0) $S' \rightarrow S$
 (1) $S \rightarrow aS$
 (2) $S \rightarrow aSbS$
 (3) $S \rightarrow i$

	First ₁	Follow ₁
S'	a, i	\$
S	a, i	\$, b



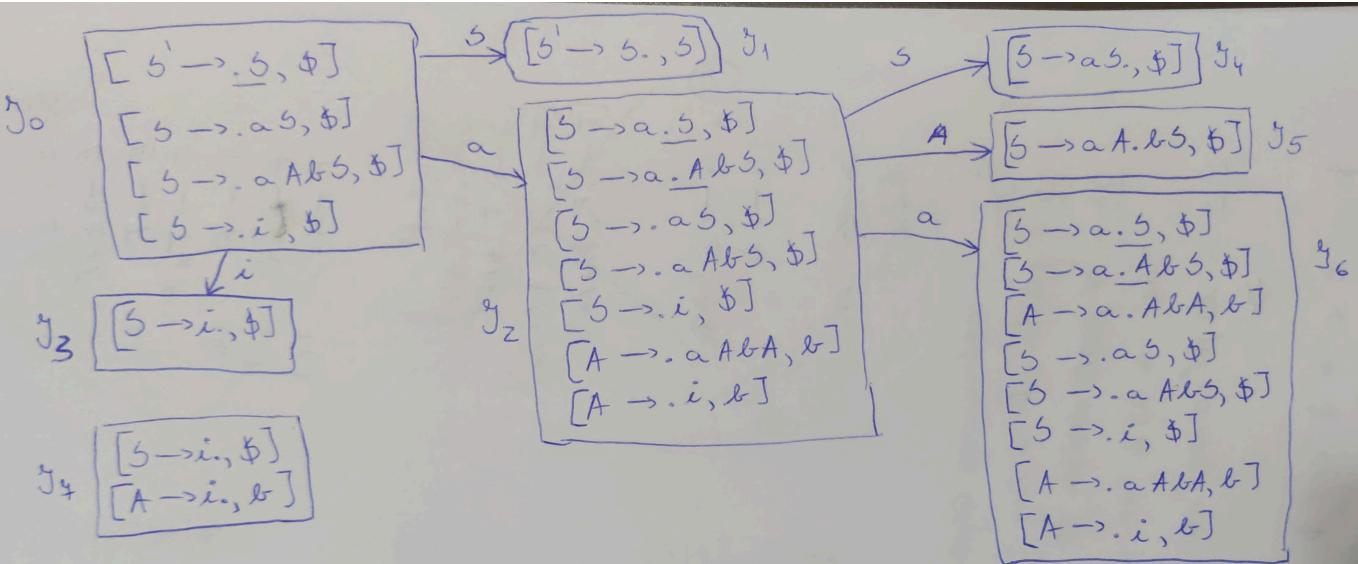
	s	a	b	i	$\$$
y_0	s_1	s_2		s_3	
y_1					acc
y_2	s_4	s_5		s_6	
y_3					r_3
y_4			s_4		
y_5	s_8	s_5		s_6	
y_6			r_3		r_3
y_7	s_9	s_2		$s_3 \cup r_1$	
y_8			r_1		r_1
y_9					r_2
y_{10}	s_{11}	s_5		s_6	
y_{11}			r_2		r_2

conflict

\Downarrow
 $r_1 \in LR(1)$

- (0) $s' \rightarrow s$
- (1) $s \rightarrow a s$
- (2) $s \rightarrow a A b s$
- (3) $s \rightarrow i$
- (4) $A \rightarrow a A b A$
- (5) $A \rightarrow i$

	First	Follow
s'	a, i	$\$$
s	a, i	$\$$
A	a, i	b



- $S' \rightarrow E \quad (0)$
 $E \rightarrow E + T \quad (1)$
 $E \rightarrow T \quad (2)$
 $T \rightarrow T * F \quad (3)$
 $T \rightarrow F \quad (4)$
 $F \rightarrow id \quad (5)$
 $F \rightarrow (E) \quad (6)$

	First,	Follow
S'	id, (\$
E	id, (\$, +,)
T	id, (\$, +,), *
F	id, (\$, +,), *

~~$[S' \rightarrow .E, \$]$~~
 ~~$[E \rightarrow .E + T, \$ / +]$~~
 ~~$[E \rightarrow .T, \$ / +]$~~
 ~~$[T \rightarrow .T * F, \$ / + / *]$~~
 ~~$[T \rightarrow .F, \$ / + / *]$~~

- $\textcircled{5} \quad 1. \quad S \rightarrow a \underline{S} S b$
 $S \rightarrow c$
 $\therefore \leftarrow \Rightarrow$
 rune?

	S	a	b	c	\$
S	\vdash	\lhd	\vdash	\lhd	
a	\vdash	\lhd		\lhd	
b		\Rightarrow	\Rightarrow	\Rightarrow	
c		\Rightarrow	\Rightarrow	\Rightarrow	
\$		\lhd		\lhd	

} dear \lhd
 dear \Rightarrow

$\textcircled{L} \dots \Rightarrow$ când ai aşa ceva, trebuie reduce
 $\underline{S} \underline{S}$ - există într-o reg. $\Rightarrow \vdash$
 $\underline{S} a$ - se poate deriva la dr. ceva ca să obținem?

$\underline{S} a$ - se poate deriva la dr. ceva ca să obținem?
 $\underline{S} a$ - da $a \underline{S} b \Rightarrow a \underline{S} a \underline{S} b b$ \lhd

$\underline{S} a$ - la fel ca mai sus, dar în partea dreaptă
 obligatoriu terminal
 \Rightarrow

$\$ a \underline{c} c b \$ \Leftrightarrow \$ a \vdash \underline{S} c b \$ \Leftrightarrow$
 $\lhd \lhd \lhd \Rightarrow \Rightarrow \Rightarrow$
 $\Leftrightarrow \$ a \vdash \underline{S} \underline{S} b \$ \Leftrightarrow \$ \$ \$$
 $\lhd \lhd \lhd \vdash \vdash \vdash \Rightarrow$
 $\Leftrightarrow \$ a \vdash \vdash \vdash \vdash \Rightarrow$ acc

Seminar 13

Gramatici de atribute

1. Fie limbajul: $L = \{a^n b^n c^n \mid n - \text{natural}\}$
Dati o gramatica de atribute care il genereaza.
2. Descrieti o gramatica de atribute care determina valorile expresiilor aritmetice in forma postfixata. EX: $5\ 6 + 5 *$
3. Descrieti o gramatica de atribute care, pentru o expresie aritmetica data in forma infixata, determina expresia aritmetica in forma postfixata. EX: $a + b * c$
4. Dati o gramatica care genereaza secvente de 0 si 1 in care nr de 0 este egal cu nr. de 1.
Atributati gramatica.
Folositi atributul nr (asociat radacinii arborelui de derivare) cu semnificatia: numarul de 0 dintr-o secventa data.
Aratati cum se evaluateaza atributul pentru secventa 0101.
(Puteti adauga si alte atribute daca este necesar.)
5. a) Dati o gramatica care genereaza secvente de 0 si 1 in care nr de 0 este par si nr. de 1 este par.
b) Fie atributele cu semnificatia: nr0 este numarul de 0 si nr1 este numarul de 1 dintr-o secventa data. Asociati atributele simbolurilor gramaticii si definitii regulile de evaluare.
c) Aratati cum se evaluateaza atributul pentru secventa 0101

Cod intermediu

1. a) Fie secventa de instructiuni

```
A := B + C*D  
B := B + C*D  
D := B + C*D
```

Traduceti in cod intermediu cu 3 adrese, reprezentare cvadruple si apoi in cod intermediu cu 3 adrese reprezentare triplete.

2. a) Traduceti in cod intermediu :

```
a := 0  
for i := 1 to 5 do begin  
    a := a+1;  
    i := i+1  
end
```

b) Care este valoarea lui a la iesirea din secventa de instructiuni?
Dar valoarea lui i?

Seminar 14

TRANSLATARE SI TRANSLATOARE

1. Definiți un translator finit M a.i.: $T(M) = \{ (a^n, b^n) \mid n \geq 1 \}$
2. Definiți un translator finit M a.i.: $T(M) = \{ (a^n, (ab)^n) \mid n \geq 1 \}$
3. Definiți un translator finit M a.i.: $T(M) = \{ (a^m, b^n) \mid n \geq m \geq 1 \}$
4. Să se construiască un translator push-down care transformă o expresie aritmetică de la forma poloneză prefixată în forma poloneză postfixată. Presupunem că expresia aritmetică conține operatorii binari $+$, $*$, și operanții simbolizați prin a .
5. Să se construiască un translator push-down care translatează limbajul $\{a^n \mid n \geq 1\}$ în $\{a^n b^n \mid n \geq 1\}$, după criteriul stivei vide.

Translator finit

- $$M = (Q, \Sigma, D, \delta, q_0, F)$$
- Q alfabetul stărilor;
 - Σ alfabetul de intrare;
 - D alfabetul de ieșire;
 - $q_0 \in Q$ stare initială;
 - $F \subseteq Q$ mulțimea stărilor finale;
 - $\delta: Q \times (\Sigma \cup \{\epsilon\}) \rightarrow \mathcal{P}_0(Q \times D^*)$ multimea partilor finite

Translatare definită de M :

$$T(M) = \{(x, y) \mid x \in \Sigma^*, y \in D^*, (q_0, x, \epsilon) \xrightarrow{*} (q, \epsilon, y), q \in F\}$$