Băcănulescu Raul

grupa 311

A

Exercițiul 1

$f:(-1,1)\to\mathbb{R}$, $f(x)=\arccos x$ $\left(\frac{\pi}{2}\ \text{în}\ 0\right)$

$$f'(x)=\frac{-1}{\sqrt{1-x^2}} \Rightarrow f'(0)=\frac{-1}{\sqrt{1-0}}=-1$$

$$f''(x)=\frac{(-1)'\sqrt{1-x^2}-(-1)\left(\sqrt{1-x^2}\right)'}{(1-x^2)}$$

$$=\frac{(\sqrt{1-x^2})'}{1-x^2}=\frac{-2x}{2\sqrt{1-x^2}\,(1-x^2)}$$

$$=\frac{-x}{(1-x^2)^{\frac{1}{2}}(1-x^2)}=\frac{-x}{(1-x^2)^{\frac{3}{2}}}$$

$$f'''(x)=\left(\frac{-x}{(1-x^2)^{\frac{3}{2}}}\right)'=\frac{-(1-x^2)^{\frac{3}{2}}-(-x)\frac{3}{2}(1-x^2)^{\frac{3}{2}-1=\frac{1}{2}}(-2x)}{(1-x^2)^3}$$

$$=\frac{\cancel{(1-x^2)^{\frac{1}{2}}}\left(x^2-1-3x\right)}{\cancel{(1-x^2)^3}\,(1-x^2)^{\frac{5}{2}}}$$

Formula lui Maclaurin $\exists c$ între 0 și x.

$$f(x)=\sum_{k=0}^{3}\frac{f^{(k)}(0)}{k!}x^k+(R_{3;0}f)(x),\quad (R_{3;0}f)(x)=\frac{f^{4}(c)}{4!}x^4$$

$=$

$f''(0) = \frac{0}{(1-0)^{\frac{3}{2}}} = 0.$ $\quad f'''(0) = \frac{0-1-0}{1} = -1$

~~f(x) = π/2~~ $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + (R_{3,0}f)$

$$f(x) = \frac{\pi}{2} - x - \frac{x^3}{3!} + \frac{f^{iv}(c)}{4!}x^4$$

$$f^{iv}(x) = \left(\frac{x^2-1-3x}{(1-x^2)^{5/2}}\right)' = \frac{(2x-3)(1-x^2)^{5/2} - (x^2-1-3x)\frac{5}{2}\cdot}{(1-x^2)^{5/2\cdot 2}}$$

$$\cdot\ (1-x^2)(-2x)$$

$$\cancel{f^{iv}(0) = \frac{-3-(-1)\cdot 5\cdot 0}{1} = -3}$$

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Exercițiul 2

(u_n) un șir ↓ șir pozitiv.

$\sum u_n$ e C Natura $\sum 2^n u_{2^n} = ?$

~~(u_n) șir ↓ $\Rightarrow \frac{u_{n+1}}{u_n} > 1$~~

Criteriul lui Cauchy:

$$\sum x_n \sim \sum 2^n \cdot x_{2^n}$$

Crit raportului $L = \lim_{n\to\infty} \frac{u_{2^{n+1}}}{u_{2^n}} = ?$

Crit lui Raabe $R = \lim_{n\to\infty} n\left(\frac{u_{2^n}}{u_{2^{n+1}}} - 1\right)$

Știm că $\frac{u_{2^n}}{u_{2^{n+1}}} < 1 \Rightarrow$ ~~$\Rightarrow 0$~~

$$R = \lim_{n\to\infty} n\left(\frac{u_{2^n}}{u_{2^{n+1}}} - 1\right) = ?$$

$$\sum u_n \text{ e C} \Rightarrow \lim_{n\to\infty} 2^n u_{2^n} = 0$$

Exercițiul 3

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a) $f : (0, \frac{\pi}{2}) \to \mathbb{R}$

$$f(x) = \frac{\sin x \left[\ln \pi - \ln\left(x + \frac{\pi}{2}\right)\right]}{(2^x - 1)}$$

$$f\left(\frac{\pi}{2}\right) = \frac{\sin \frac{\pi}{2} \left[\ln \pi - \ln \pi\right]}{2^{\pi/2} - 1} = 0.$$

$$f(0) = \frac{0 \left[\ln \pi - \ln \frac{\pi}{2}\right]}{0} \Rightarrow \text{pb e în } 0.$$

Comparăm ~~integrala~~ cu funcția $g : (0, \frac{\pi}{2}] \to \mathbb{R}$

$$g(x) = \frac{1}{(x-0)^p} = \frac{1}{x^p}$$

$$L = \lim_{x \to 0} \frac{x^p \sin x \left[\ln \pi - \ln\left(x + \frac{\pi}{2}\right)\right]}{2^x - 1}$$

$$L = \lim_{x \to 0} \frac{x^p \cdot \sin x \cdot \ln \frac{\pi}{x + \frac{\pi}{2}}}{2^x - 1} \quad x^{-1} \cdot x^{-1}$$

$$L = \lim_{x \to 0} x^p \sin x \ln \frac{\pi}{x + \frac{\pi}{2}} (2^x - 1)^{-1} \quad p = -2 \Rightarrow$$

$$\Rightarrow L = \lim_{x \to 0} \overbrace{\frac{\sin x}{x}}^{\to 1} \ln \frac{\pi}{x + \frac{\pi}{2}} \overbrace{\left(\frac{2^x - 1}{x}\right)}^{\to \ln 2}{}^{-1}$$

$$L = \ln \frac{\pi}{0 + \frac{\pi}{2}} \cdot \ln 2 = \ln^2 2 \geq 0$$

$f(x) \geq 0.$

$p = -2 < 1$

$\Rightarrow \int f(x) = C.$

b) $\int_2^3 x^2 \sqrt{2x-3}\, d(\ln x)$

fie $f:[2,3] \to \mathbb{R}$ $f(x) = x^2\sqrt{2x-3}$

f e mărg pe $[2,3]$

fie $g: [2,3] \to \mathbb{R}$ $g(x) = \ln x$ $g'(x) = \frac{1}{x}$

g' e cont pe $[2,3]$. $\Rightarrow$

$\Rightarrow \int_2^3 x^2\sqrt{2x-3}\, d(\ln x) = \int_2^3 x^2\sqrt{2x-3}\cdot(\ln x)'\, dx$

$= \int_2^3 x\sqrt{2x-3}\, dx$

$u = 2x-3 \Rightarrow u+3 = 2x \Rightarrow x = \frac{u+3}{2}$

$du = 2\,dx$

$x = 3 \Rightarrow u = 2\cdot 3 - 3 = 3$

$x = 2 \Rightarrow u = 1$

$\Rightarrow \int_1^3 \frac{u+3}{2}\cdot\sqrt{u}\,\frac{du}{2} = \frac{1}{4}\left(\int_1^3 \frac{u\sqrt{u}}{1}du + \int_1^3 3\sqrt{u}\,du\right)$

$= \frac{1}{4}\left(\int_1^3 u^{\frac{3}{2}}du + 3\int_1^3 u^{\frac{1}{2}}du\right)$

$= \frac{1}{4}\,\frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1}\Big|_1^3 + 3\,\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Big|_1^3$

$= \frac{1}{\cancel{4}_{2}}\cdot\frac{\cancel{2}}{5}\,u^{\frac{5}{2}}\Big|_1^3 + \cancel{3}\cdot\frac{2}{\cancel{3}}\,u^{\frac{3}{2}}\Big|_1^3$

$= \frac{1}{10}\left(3^{\frac{5}{2}} - 1\right) + 2\left(3^{\frac{3}{2}} - 1\right)$

$$= \lim_{n\to\infty} \frac{\cancel{n^3}\left(-3 - \frac{3}{n} - \frac{3}{n^2}\right)}{\cancel{n^3}\left(1 + \frac{3}{n} + \frac{3}{n^2} + \frac{1}{n^3}\right)} = -3 < 1 \Rightarrow \text{D}.$$

$\Rightarrow$ m e ABS. CONV.

$\frac{n^3}{2}$

Exercițiul 4 Băciulescu Paul

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$$\sum_{n\geq 1}\left(\frac{n^3}{3^n+a^n}\right)x^n$$ În funcție de a, raza de c.

Calculăm $\lambda = \lim_{n\to\infty}\frac{|\cancel{x}_{n+1}|}{|\cancel{x}_n|}$, șirul $\cancel{x}_n = \frac{n^3}{3^n+a^n}$

$$\lambda = \lim_{n\to\infty}\frac{(n+1)^3}{3^{n+1}+a^{n+1}}\cdot\frac{3^n+a^n}{n^3}$$

$$\lambda = \lim_{n\to\infty}\frac{(n^3+3n^2+3n+1)(3^n+a^n)}{n^3(3^{n+1}+a^{n+1})}$$

$$\lambda = \lim_{n\to\infty}\frac{\cancel{n^3}\left(1+\frac{3}{n}+\frac{3}{n^2}+\frac{1}{n^3}\right)(3^n+a^n)}{\cancel{n^3}(3^{n+1}+a^{n+1})}$$

Distingem următoarele cazuri: I $a\in(-\infty,3]$

II $a\in(3,\infty)$

I $a\in(-\infty,3] \Rightarrow a\leq 3$

$$\lambda = \lim_{n\to\infty}\frac{\overbrace{\left(1+\frac{3}{n}+\frac{3}{n^2}+\frac{1}{n^3}\right)}^{\to 1}\cancel{3^{n+1}}\left(\frac{1}{3}+\overbrace{\left(\frac{a}{3}\right)^{n}}^{\to 0}\cdot\frac{1}{3}\right)}{\cancel{3^{n+1}}\left(1+\underbrace{\left(\frac{a}{3}\right)^{n+1}}_{\to 0}\right)}$$

$$a\leq 3 \Rightarrow \left(\frac{a}{3}\right)^{n+1}\to 0$$

$$\Rightarrow \lambda = \frac{1}{3}\in(0,\infty) \Rightarrow R = \frac{1}{\lambda} = 3$$

II $a \in (3,\infty) \Rightarrow a > 3 \Rightarrow \cancel{\left(\frac{3}{3}\right)^{n+1} \to 0}$ $\left(\frac{3}{a}\right)^{n+1} \to 0$.

$$\lambda = \lim_{n\to\infty} \frac{\underbrace{\left(1+\frac{3}{n}+\frac{3}{n^2}+\frac{1}{n^3}\right)}_{\to 1} \cancel{a^{n+1}} \left(\left(\frac{3}{a}\right)^{\overset{\to 0}{n}} \cdot \frac{1}{a} + \frac{1}{a}\right)}{\cancel{a^{n+1}}\left(1+\underbrace{\left(\frac{3}{a}\right)^{n+1}}_{\to 0}\right)}$$

$\lambda = \frac{1}{a} \in (0,\infty)$, $a \in (3,\infty) \Rightarrow R = a$.

Pt cazul $a=3 \Rightarrow R=3 \Rightarrow (-3,3) \subseteq \mathcal{C}$

verificăm pentru $x=-3 \Rightarrow \sum \frac{n^3 (-3)^n}{3^n+3^n}$

I $\sum \frac{(-1)^n \cdot n^3 \cdot 3^n}{3^n+3^n} = \sum \frac{(-1)^n n^3 \cdot \cancel{3^n}}{2\cancel{3^n}}$

$x=3$

$= \sum \frac{(-1)^n n^3}{2}$

II $\sum \frac{n^3}{2}$

$\lim_{n\to\infty} \frac{n^3}{2} = \infty \Rightarrow \sum \frac{n^3}{2}$ e D. $\Rightarrow$

I $\sum \frac{(-1)^n n^3}{2}$ Studiez AC $\sum \frac{n^3}{2}$

Crit rap $\lim_{n\to\infty} \frac{(n+1)^3}{\cancel{2}} \cdot \frac{\cancel{2}}{n^3} = \lim \frac{\cancel{n^3}\left(1+\frac{3}{n}+\frac{3}{n^2}+\frac{1}{n^3}\right)}{\cancel{n^3}}$

$= 1 \Rightarrow$ nu știm

Raabe $\lim_{n\to\infty} n\left(\frac{n^3}{\cancel{2}} \cdot \frac{\cancel{2}}{(n+1)^3} - 1\right)$

$\lim_{n\to\infty} n\left(\frac{\cancel{n^3} - \cancel{n^3} - 3n^2 - 3n - 1}{n^3+3n^2+3n+1}\right)$

$= \lim_{n\to\infty} \frac{-3n^3 - 3n^2 - n}{n^3+3n^2+3n+1}$