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The bullwhip effect in a 3-stage supply chain considering multiple retailers using a moving average method for demand forecasting

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ABSTRACT

The bullwhip effect is an undeniable phenomenon in supply chains that has a negative effect on their performance and efficiency. There are a variety of causes for the appearance of the bullwhip effect, one most important of which is the existence of uncertain demand. Due to the variety of causes as well as in order to recognize relevant factors, it is important to be able to quantify the bullwhip effect in a supply-chain environment. In this study, we aim to quantify the bullwhip effect in a 3-stage supply chain with multiple retailers. First, we quantify the bullwhip effect, order rate variance ratio (OVR) and inventory variance ratio (IV) in a pipeline supply chain, for which we develop a relation to calculate the bullwhip effect. Then we extend it to a supply chain with multiple retailers. We analyze the impact of service levels on the bullwhip effect for both defined supply chain situations. This analysis, which highlights the importance of the impact of service levels on the bullwhip effect, can be considered the main contribution of this paper. In addition, we survey the influence of the correlation coefficient on the bullwhip effect.

1. Introduction

Supply chains are typically composed of suppliers, manufacturers, distributers, retailers and customers [1]. In classical supply chains, members and facilities tend to fulfill customer expectations and demand [2]. Supply chain management (SCM) is defined as a set of strategies and decisions that improve supply chain performance [3]. One critical and significant issue is lack of accurate information about the amount of demand and setting the right order quantities, which is one of the most important decisions in SCM [4]. This lack of information increases deviation of orders from real demand. As orders move upstream in supply chains, this variation increases [5]. Therefore, a lot of fluctuation in order quantities is observed, and this causes many serious problems [6]. This phenomenon is called the bullwhip effect and has been defined by many researchers [7–10]. According to Dominguez et al., the bullwhip effect is increasing fluctuation in order and demand variance going upstream in supply chains [11]. The bullwhip effect has an adverse effect on supply chains, including high inventories and low service levels [7]. In addition, it can decrease the quality of products and increase manufacturing and transportation

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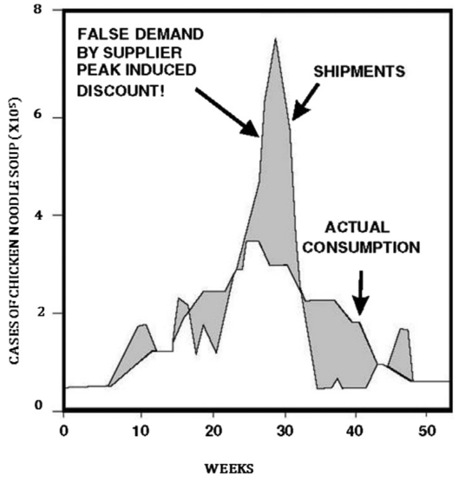

Fig. 1. Bullwhip effect as shown by Fisher.

costs [10]. Moreover, it has a negative effect on supply chain performance and receives increasing attention in supply chains [12]. Quantification of the bullwhip effect is required in order to analyze its effect and to study its causes.

In this study, we develop a new relation for quantifying the bullwhip effect in a 3-stage supply chain. We use the moving average method (MA) to estimate the expected value and standard deviation of retailer demand. In our model, the service level is taken into account, which differentiates our work from the mathematical relation developed by Chen et al. [13]. We consider a supply network consisting of a single supplier, a unique DC and multiple retailers. We consider the correlation between retailers' orders. We show the influence of service level on the bullwhip effect in the defined situation.

We initially review the literature related to our study in Section 2. Section 3 develops the required mathematical relations. We first study a pipeline supply chain considering a single retailer (Part 3.1). Then we extend our work to multiple retailers in the supply chain (Part 3.2). Finally, in Section 4, we present conclusions and future research suggestions.

2. Literature review

Amplification of the demand variance concept in industrial systems was introduced by Forrester (1958) [14]. He was a pioneer who studied the bullwhip effect and defined it as demand amplification. He believed that this is the system dynamics problem that is controllable through reducing delays [15]. Sterman (1989) enumerated four critical sources of demand amplification: demand signal processing, the rationing game, order batching and price variations. Demand amplification is not a new concept, and many researchers have been interested in working on it. This is shown in Fig. 1 by Fisher (1997) [16].

The words “bullwhip effect" were used for the first time by Lee et al. [17]. They defined this effect and studied four sources of this phenomenon. Lambrecht and Dejonckheere [18] evaluated a bullwhip effect explorer, which investigates different inventory policies and chooses the best in terms of the bullwhip effect. Many researchers have since studied this effect.

Various previous studies have centered on the bullwhip effect. O'Donnell et al. studied minimizing the bullwhip effect [19] using a genetic algorithm. They categorized studies before 2006 that tried to mitigate the bullwhip effect. Geary et al. [16] reviewed previous publications and prepared a historical review of the bullwhip effect in supply chains. Paik and Bagchi [20] categorized the main causes of the bullwhip effect and enumerated its six main causes. Canella and Ciancimino the effect of collaboration and order smoothing on the bullwhip effect and presented some concepts for calculating the performance of supply chains. Their study was the third that review prior studies and presented a good categorization of the studies [21]. Bhattacharya and Bandyopadhyay also studied the causes of the supply chains. They classified the causes into two categories: operational causes and behavioral causes [9]. Trapero et al. [22] studied the impact of information sharing and forecasting accuracy on the bullwhip effect. Some researchers have worked on pipeline 3-stage supply chains. Sucky [23] tried to eliminate lead-time in calculating the bullwhip effect and showed how risk pooling affects it. Schmidt the importance of aggregation of retailer demand and explained that estimating the mean and variance of retailer demand is not accurate because of time -dependent demand [24]. Nepal et al. [14] presented an analysis of the bullwhip effect in a 3-stage supply chain. They focused on the bullwhip effect during the life cycle. Akkermans and Voss used a system dynamics model to study how the bullwhip effect influences service; they identified three types of specifications that can potentially have a bad effect on service [25]. Cannella et al. investigated the effect of order and inventory smoothing on supply performance in a traditional 3-stage supply chain. They used OVR and IV metrics [26]. Cannella et al. studied the role of information technology in operational performance and improvement in the bullwhip effect; they used OVR and IV indices to calculate demand fluctuation [27]. Bruccoleri et al. [28] studied the effect of inaccuracy of inventory records on

Table 1
Studies of the bullwhip effect showing that most use order -up-to policy.

Study	Performance metrics	Model structures	Demand pattern	Type of warehouse
Chen et al. [13]	OVR	- Two echelon traditional - Multi echelon	Auto regressive	Distribution center
Disney and Towill [32]	BW	Two echelon VMI supply chain	Step	"
Dejonckheere et al. [33]	OVR	Traditional production inventory system	- Sinusoidal - Step - i.i.d	"
Chatfield et al. [34]	"	Two four-echelon supply chains	- i.i.d	"
Dejonckheere et al [35]	"	Two four-echelon supply chains	"	"
Disney et al. [36]	- OVR - IV	Traditional production inventory system	"	"
[37]	BW	"	Step	"
Disney et al [38]	- OVR -IV	"	- i.i.d - Auto Regressive - Moving Average	"
Kim et al. [39]	OVR	Two five-layer supply chains	i.i.d	"
Chen and Disney [40]	- OVR - IV	Traditional production inventory system	- Auto Regressive - Moving Average	"
Boute et al. [41]	OVR	"	i.i.d	"
Hosoda et al. [42]	"	Two-echelon Epos supply chain	Real life data set	"
Jakšič and Rusjan [43]	"	Two-echelon traditional supply chains	Sinusoidal	"
Kelepouris et al [44]	"	"	Real life data set	"
Bayraktar et al. [45]	BW	"	Online linear demand forecast with seasonal swings	"
Haughton [46]	BW	Two-echelon traditional supply chains	Multiplicative Seasonal	"
Agrawal et al. [47]	- OVR - IV	"	Auto Regressive	"
Sucky [23]	BW	Three-echelon traditional supply chains	Moving Average	"
Xie [48]	BW	Two-echelon traditional supply chains	Fuzzy forecasting	"
Coppini et al [49]	BW	"	Exponential Smoothing	"
Cannella and Ciancimino [21]	- OVR -IV - BW	Multi-echelon traditional supply chains	Smoothing	"
Cho and Lee [50]	BW	Two-echelon traditional supply chains	- Moving Average - Seasonal Auto Regressive	"
Li et al. [51]	BW	"	Moving Average	"
Syntetos et al. [52]	BW	Three-echelon traditional supply chains	Different methods	"
Cantor and Katok [53]	BW	Two-echelon traditional supply chains	Seasonal	"
Ciancimino et al [54]	BW	Three-echelon traditional supply chains	Exponential Smoothing	"
Nepal et al [14]	BW	"	"	"
[12]	BW	"	"	"
Chatfield and Pritchard [55]	BW	Multi-echelon traditional supply chains	Moving Average	"
Buchmeister et al. [56]	BW	"	Seasonal	"
Li et al. [57]	BW	Two-echelon traditional supply chains	Damped Trend forecasting	"
Costantino et al. [58]	- OVR -IV	Multi-echelon traditional supply chains	- Moving Average - Exponential Smoothing	"
Nagaraja et al [59]	BW	Two-echelon traditional supply chains	SARMA	"
Dominguez et al [11]	BW	Multi-echelon traditional supply chains	Moving Average	"

OVR and IV. Dominguez et al. [29] used two different strategies in a serial linked supply chain network to mitigate the bullwhip effect. Cannella studied order-up -to policies and illustrated how the performance of information exchange can be improved; he used OVR and IV [30]. Dominguez et al. [31] illustrated how the influence of returns in the bullwhip effect relies on supply chain configuration.

Table 1 summarizes many studies of the bullwhip effect and shows that most use order -up-to policies. This is because of using this policy in our study.

In this paper, we will show that the service level considerably affects the bullwhip effect. In addition, correlation between retailer demand is studied and its impact on the bullwhip effect is taken into account. We will develop a new relation to quantify the bullwhip effect in a pipeline 3-stage supply chain to accomplish these analyses. After that, we will extend it to a multi-retailer situation.

3. Model definitions

3.1. The bullwhip effect in a pipeline supply chain

A basic 3-stage supply chain involves a supplier or manufacturer, and a warehouse or distribution center; a retail store is assumed. We assume that a single product has to be transported from supplier to retailer [60] (see Fig. 2). Another assumption is the horizon, which is considered for t periods.

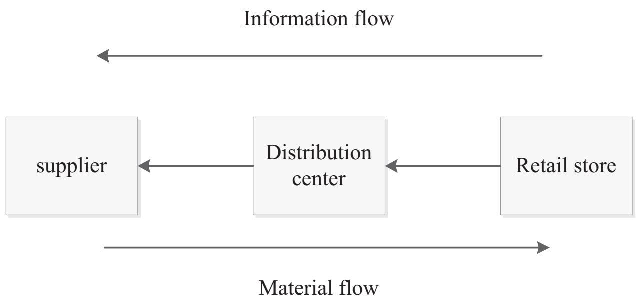

Fig. 2. A pipeline 3-stage supply chain.

Analysis of complicated supply chains in terms of the bullwhip effect needs to begin from examining a basic one and then extending the scenario based on the results. Therefore, we review a study by Chen et al. [13] that examined and quantified the bullwhip effect. We use this approach to quantify the bullwhip effect in a single retailer supply chain. First, we must specify the conditions under which we intend to quantify the effect.

3.1.1. Inventory policy

It is assumed that the inventory system is being managed and studied for every period. In addition, order q_t is sent at the beginning of every period. The size of the order must be sufficient to satisfy retailer demand D_t. Retailer demand is a stochastic parameter, and the distribution center has no information about that. It is assumed that the review is done at the beginning of every period t, and q_t is ordered by the supplier based on an order-up-to policy by the distribution chain as used in reviewed literature [13,23]. The size of the order that the distribution center sends to the supplier at the inception of period t is given by the following equation:

$$q_t = y_t - y_{t-1} + D_{t-1}, \tag{1}$$

where y_t and y_{t-1} are desired inventory levels at the end of periods t and $t-1$ respectively. The target of this inventory policy is to increase the actual inventory level to desired inventory level y_t [61]. In addition, D_{t-1} is retailer demand. It is placed at the end of period $t-1$ and is satisfied at period. If $q_t < 0$, we assume that this surplus inventory goes back to the supplier without any cost [17,62].

The point to notice is that, in this policy, it is assumed that the customer accepts no shortage at the end of the horizon or end period. Therefore, a shortage is not allowed at the end period. However, the order -up -to policy allows shortages in any other periods, and if the system face a shortage in a period, the shortage is backlogged [63].

There are two approaches to meeting retailer demand. The first is similar to one brought up [13], the autoregressive demand (AR1) model [64].

Another approach is for when we have no information about retailer demand. We assume that retailer demand is an identically distributed random variable that is stochastic, stationary, and independent over time [23]. We use the second approach and perform our calculation and analysis based on this assumption. We assume that the lead-time between the distribution center and retailer is a constant amount that is shown as L. We use a simple order-up-to inventory policy to compute target inventory level y_t with the following formula [65]:

$$y_t = \hat{D}_t^L + z\hat{\sigma}_t^L, \tag{2}$$

where $\hat{D}_t^L$ is the estimate of mean demand during lead-time, $\hat{\sigma}_t^L$ is the estimate of standard deviation demand during lead-time, and z is a constant chosen to the desired service level. Notice that z is a managerial factor that shows the amount of standard deviation demand that is held as safety stock [66]. If the distribution function of demand is a normal distribution, this form of order-up-to policy will be optimal [13].

3.1.2. Forecasting technique

For computing the mean and variance of demand during lead-time, we have to use a forecasting method to estimate these factors. This is because we do not have any information about retailer demand. We use a simple N-period moving average. N is the parameter of moving average that shows the number of periods used for estimating retailer demand. Achieving to the formulas for $\hat{D}_t^L$ and $\hat{\sigma}_t^L$, at first, $\hat{D}_t^L$ must be determined. We know that $\hat{D}_t^L$ is the summation of retailer demand from period t to period $t+L-1$. Therefore, we have:

$$D_t^L = D_t + D_{t+1} + \cdots + D_{t+L-1} = \sum_{i=0}^{L-1} D_{t+i}. \tag{3}$$

Because we do not determine demand in these periods, we have to use the moving average technique to estimate demand during the period. Therefore, we use following formulations:

$$\hat{D}_{t+k} = \hat{D}_t = \frac{1}{N}\sum_{i=1}^{N} D_{t-i} = \frac{1}{N}(D_{t-1} + D_{t-2} + \cdots + D_{t-N}). \tag{4}$$

This means that if we intend to estimate demand for any period in the future, we need to compute the mean of the N former periods. Then we can use it as the estimate for any following period.

$$\hat{D}_t^L = \hat{D}_t + \hat{D}_{t+1} + \cdots + \hat{D}_{t+L-1} \tag{5}$$

$$\hat{D}_t^L = L \times \hat{D}_t = \frac{L}{N}\sum_{i=1}^{N} D_{t-i}. \tag{6}$$

In addition, for calculating $\hat{\sigma}_t^L$, we use the moving average to estimate $\hat{\sigma}_t$, which is computed by the following equation [23]:

$$\hat{\sigma}_t^2 = \hat{\sigma}_{t+k}^2 = \frac{1}{N}\sum_{i=1}^{N} (D_{t-i} - \hat{D}_t)^2 \tag{7}$$

By using Eq. (3), σ_t^L is accomplished as the following state:

$$(\sigma_t^L)^2 = Var(D_t^L) = Var\left(\sum_{i=0}^{L-1} D_{t+i}\right) = \sum_{i=0}^{L-1} Var(D_{t+i}) + 2\sum\sum_{i<j} Cov(D_{t+i}, D_{t+j}).$$

There are two points: The first is that because of our former assumption, it is supposed that:

$$Cov(D_{t+i}, D_{t+j}) = 0.$$

The other is that due to lack of information about retailer demand, we have to estimate variance of retailer demand in every period and use $\hat{\sigma}_t^L$ instead of σ_t^L. Hence, we can find out $\hat{\sigma}_t^L$ by:

$$(\hat{\sigma}_t^L)^2 = L \times \hat{\sigma}_t^2 = \frac{L}{N}\sum_{i=1}^{N} (D_{t-i} - \hat{D}_t)^2$$

$$\hat{\sigma}_t^L = \sqrt{L} \times \hat{\sigma}_t = \sqrt{L} \times \sqrt{\frac{\sum_{i=1}^{N} (D_{t-i} - \hat{D}_t)^2}{N}}. \tag{8}$$

Notice that the estimated values of $\hat{D}_t^L$ and $\hat{\sigma}_t^L$ can be changed every period. This is because of getting new information; hence, the distribution center's order-up-to level can be updated every period [7].

Now we can estimate the amount of commodities that the distribution center orders for the supplier in order to increase inventory to the desired level. We can quantify q_t with the following calculations:

$$\begin{aligned} y_t &= \hat{D}_t^L + z\hat{\sigma}_t^L \\ q_t &= y_t - y_{t-1} + D_{t-1} \\ &= (\hat{D}_t^L - \hat{D}_{t-1}^L) + z(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L) + D_{t-1}. \end{aligned}$$

Using Eqs. (7) and (9), the order quantity can be calculated as follows:

$$q_t = (1 + \frac{L}{N})D_{t-1} + (-\frac{L}{N})D_{t-N-1} + z\sqrt{L}(\hat{\sigma}_t - \hat{\sigma}_{t-1}) \tag{9}$$

We consider a numerical example to illustrate how the distribution center determines order specifications. In this example, we use a 3-period moving average assuming a 99% service level with two periods of lead-time. The data set is generated from a normal distribution with a mean of 50 and a standard deviation of 15. The calculations were performed by MATLAB; the diagrams were generated by Microsoft Excel. Table 2 shows the conclusions.

To illustrate the effect of lead-time and parameter of MA (N) on the variance of q_t, we compute the variance of q_t with different lead-times and N. Then we visualize them in Fig. 3 as follows.

Fig. 3 illustrates the constant increase in variance of q_t with the increase of lead-time. This shows that lead-time is a crucial factor in variance of order quantity, and increases in lead-time have a negative impact on the performance of the supply chain. Fig. 3 shows that lead-time has a direct impact on the bullwhip effect. The point to notice is that this trend is expected for every N in MA and every data set.

Fig. 4 shows the decrease of variance of q_t by increasing the parameter of MA. This trend is similar for every data set and every lead-time. This means that the bullwhip effect has an inverse relationship with the N in MA.

Table 2
Determination of distribution center order quantities.

t	0	1	2	3	4	5	6	7	8	9	10
D_t	46	65	42	31	73	87	34	70	57	51	86
$\hat{D}_t$				51.0	46.0	48.7	63.7	64.7	63.7	53.7	59.3
$\hat{D}_t^L$				102.0	92.0	97.3	127.3	129.3	127.3	107.3	118.7
$\hat{\sigma}_t^2$				100.7	200.7	316.2	566.2	502.9	488.2	221.6	62.9
$(\hat{\sigma}_t^L)^2$				201.3	401.3	632.4	1132.4	1005.8	976.4	443.1	125.8
y_t				135.1	138.7	155.9	205.7	203.2	200.1	156.4	144.8
q_t				177.1	34.6	90.3	136.8	31.5	66.9	13.2	39.4
t		11	12	13	14	15	16	17	18	19	20
D_t		39	37	58	41	37	46	44	67	53	84
$\hat{D}_t$		58.7	54.0	44.7	45.3	45.3	41.3	42.3	52.3	54.7	58.7
$\hat{D}_t^L$		117.3	108.0	89.3	90.7	90.7	82.7	84.7	104.7	109.3	117.3
$\hat{\sigma}_t^2$		397.6	512.7	89.6	82.9	82.9	13.6	14.9	108.2	89.6	397.6
$(\hat{\sigma}_t^L)^2$		795.1	1025.3	179.1	165.8	165.8	27.1	29.8	216.4	179.1	795.1
y_t		183.0	182.6	120.5	120.7	120.7	94.8	97.4	138.9	140.5	183.0
q_t		42.3	36.6	−4.1	41.2	37.0	20.1	46.6	108.6	54.6	42.3

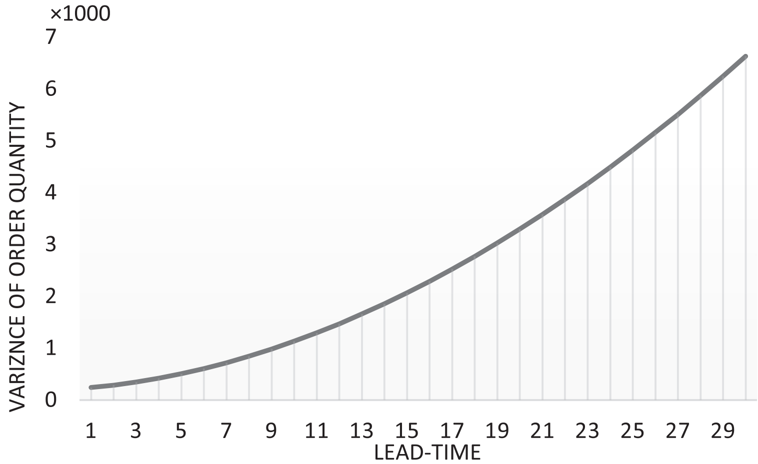

Fig. 3. Trend of variance of $\boldsymbol{q_t}$ by changing lead-time.

3.1.3. Quantifying the bullwhip effect

To quantify the bullwhip effect, we can use the following equation [13]:

$$BW = \frac{Var(q)}{Var(D)}, \tag{10}$$

where $Var(D)$ is variance of retailer demand, and $Var(q)$ shows variance of distribution center order quantity. Example 1 shows that the bullwhip effect in a supply chain can be demonstrated as Fig. 5, which indicates the quantities of retailer demand and the distribution center's order. In addition, we can see that fluctuation in distribution center order quantities is much greater than fluctuation in retailer demand.

However, Cannella and Ciancimino have given other relationships for the bullwhip effect [21]. These equations are as follows:

1- Order rate variance ratio:

$$ORV = \frac{\sigma_q^2/\mu_q}{\sigma_D^2/\mu_D}. \tag{11}$$

This equation is an extension of Eq. (10) and shows the instability of orders in the supply chain. In this equation, σ_q^2 and μ_q are used to show the corresponding variance and mean of distribution center order quantity. In addition, σ_D^2 and μ_D are the variance and mean of retailer demand, respectively.

1- Inventory variance ratio:

$$IV = \frac{\sigma_I^2/\mu_I}{\sigma_D^2/\mu_D} \tag{12}$$

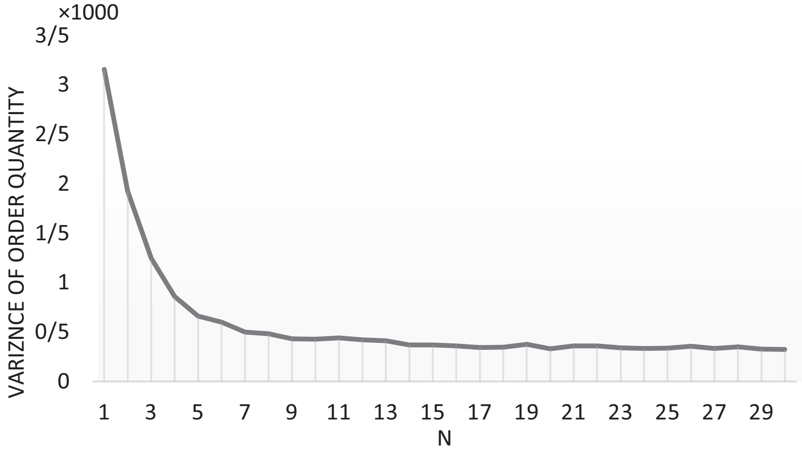

Fig. 4. Trend of variance of q_t by changing N in MA.

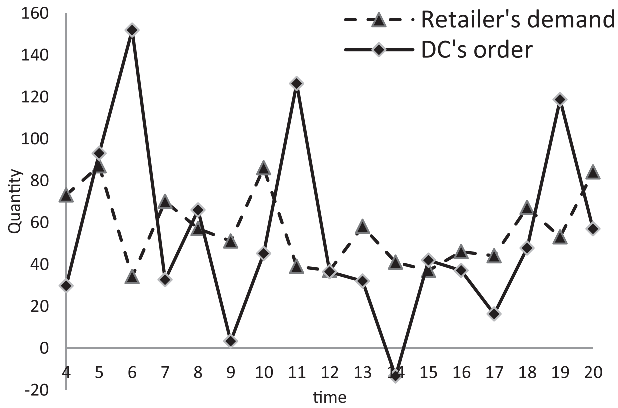

Fig. 5. Illustration of the bullwhip effect.

This relation was proposed by Disney and Towill to measure net stock or inventory instability [67]. It measures the fluctuation of distribution center inventory in the supply chain. Disney and Lambercht considered that increases in inventory fluctuation have an unfavorable effect on cost per period [67]. The notations σ_I^2 and μ_I respectively show the variance and mean of inventory held in the distribution center.

Now we quantify the bullwhip effect and develop a new relation to calculate it. For computing the bullwhip effect, we need to find the variance of distribution center order quantity. It is obtained by Lemma 1.

Lemma 1. *The variance of distribution center order quantity is given by the following equation*:

$$\tilde{V}ar(q_t) = \left(1+\frac{L}{N}\right)^2 Var(D_{t-1}) + \left(-\frac{L}{N}\right)^2 Var(D_{t-N-1}) + z^2 LVar(\hat{\sigma}_t) + z^2 LVar(\hat{\sigma}_{t-1}) \tag{13}$$

Proof. We use Eq. (9) to prove it. At first, we get variance from two sides of Eq. (9). Hence, the following calculations can be done:

$$\begin{aligned} q_t &= y_t - y_{t-1} + D_{t-1} \\ &= \left(1+\frac{L}{N}\right)D_{t-1} + \left(-\frac{L}{N}\right)D_{t-N-1} + z\sqrt{L}(\hat{\sigma}_t - \hat{\sigma}_{t-1}) \\ Var(q_t) &= Var\left(\left(1+\frac{L}{N}\right)D_{t-1} + \left(-\frac{L}{N}\right)D_{t-N-1} + z\sqrt{L}(\hat{\sigma}_t - \hat{\sigma}_{t-1})\right) \\ &= \left(1+\frac{L}{N}\right)^2 Var(D_{t-1}) + \left(-\frac{L}{N}\right)^2 Var(D_{t-N-1}) + z^2 LVar(\hat{\sigma}_t - \hat{\sigma}_{t-1}) + 2\left(1+\frac{L}{N}\right)\left(-\frac{L}{N}\right)Cov(D_{t-1}, D_{t-N-1}) + \\ &\quad 2z\sqrt{L}\left(1+\frac{L}{N}\right)Cov(D_{t-1}, (\hat{\sigma}_t - \hat{\sigma}_{t-1})) + 2z\sqrt{L}\left(-\frac{L}{N}\right)Cov(D_{t-N-1}, (\hat{\sigma}_t - \hat{\sigma}_{t-1})) \end{aligned}$$

Based on the preceding assumption, we can conclude that:

$$\mathrm{Cov}(\mathrm{D}_{t-1}, \mathrm{D}_{t-N-1}) = 0$$

In addition, Chen et al showed that [13]:

$$\mathrm{Cov}\left(\mathrm{D}_{\mathrm{t}-i}, \hat{\sigma}_{\mathrm{t}}\right) = 0 \quad \forall i = 0, 1, \ldots, N$$

Moreover, it is specified that:

$$Var(\hat{\sigma}_t - \hat{\sigma}_{t-1}) = Var(\hat{\sigma}_t) + Var(\hat{\sigma}_{t-1}) - 2Cov(\hat{\sigma}_t, \hat{\sigma}_{t-1})$$

We assume that covariance between standard deviation in period t and t + 1 is zero. This assumption is practically true. We use the previous data for retailer demand to estimate $\hat{\sigma}_{\mathrm{t}}^{\mathrm{L}}$, and we know that retailer demand is independent; therefore, this $\mathrm{Cov}(\hat{\sigma}_{\mathrm{t}}^{\mathrm{L}}, \hat{\sigma}_{\mathrm{t}-1}^{\mathrm{L}})$ is nearly zero. Accordingly, the result is the following assumption:

$$Cov(\hat{\sigma}_t, \hat{\sigma}_{t-1}) = 0$$

Hence:

$$Var(q_t) = \left(1 + \frac{L}{N}\right)^2 Var(D_{t-1}) + \left(-\frac{L}{N}\right)^2 Var(D_{t-N-1}) + z^2 LVar(\hat{\sigma}_t) + z^2 LVar(\hat{\sigma}_{t-1})$$

Lemma 2. *We assume that retailer demand has a normal distribution with unknown parameters μ and σ. Therefore, the variance of distribution center orders follows* Eq. (14) *and is given as follows*:

$$\tilde{V}ar(q) = \left(1 + \frac{2L}{N} + \frac{2L^2}{N^2} + 2z^2 L\left(\left(\frac{N-1}{N}\right) - \frac{2}{N}\left(\frac{\Gamma(N/2)}{\Gamma(N-1/2)}\right)^2\right)\right)\sigma^2 \tag{14}$$

where $Var(q)$ is the variance of distribution center order quantity, σ^2 is the variance of retailer demand, and $\Gamma(\alpha)$ is calculated by the following equation:

$$\Gamma(\alpha) = \int_{-\infty}^{y} y^{\alpha-1} e^{-y} dy$$

Proof. It is shown that if X is a sample of normal distribution with unknown parameters μ and σ, and S is the estimated standard deviation; S is determined by the following equation [68]:

$$S = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Thus, the expected value of S can be calculated as follows:

$$E(S) = \sqrt{\frac{2}{n-1}} \frac{\Gamma(n/2)}{\Gamma((n-1)/2)} \sigma. \tag{15}$$

In addition, we have:

$$E(S^2) = \sigma^2. \tag{16}$$

$E(x)$ is used as the expected value of x. We will prove Eqs. (15) and (16) in Appendix A.
By using Eq. (7), we can find that:

$$\hat{\sigma} = \sqrt{\frac{N-1}{N}} S \to E(\hat{\sigma}) = \sqrt{\frac{N-1}{N}} E(S)$$

$$E(\hat{\sigma}) = \sqrt{\frac{2}{N}} \frac{\Gamma(N/2)}{\Gamma((N-1)/2)} \sigma.$$

In addition, we can show that:

$$E(\hat{\sigma}^2) = \left(\frac{N-1}{N}\right) \sigma^2.$$

Hence, to calculate the variance of distribution center order quantity, we have to calculate the variance of $\hat{\sigma}$, which is obtained as follows:

$$Var(\hat{\sigma}) = E(\hat{\sigma}^2) - (E(\hat{\sigma}))^2 = \left(\left(\frac{N-1}{N}\right) - \frac{2}{N}\left(\frac{\Gamma(N/2)}{\Gamma((N-1)/2)}\right)^2\right)\sigma^2. \tag{17}$$

As we know, retailer demand in every period follows the same distribution function, therefore:

$$Var(D_{t-1}) = Var(D_{t-N-1}) = \sigma^2$$

By using the previous calculation and Eq. (13), the proof is completed.

Now we can quantify the bullwhip effect. As shown in Eq. (10), we use Lemmas 1 and 2 to quantify the bullwhip effect. As a result, we can perform the following calculations:

$$BW = \frac{Var(q)}{Var(D)} = \frac{1}{\sigma^2}\left(1 + \frac{2L}{N} + \frac{2L^2}{N^2} + 2z^2L\left(\left(\frac{N-1}{N}\right) - \frac{2}{N}\left(\frac{\Gamma(N/2)}{\Gamma((N-1)/2)}\right)^2\right)\right)\sigma^2$$

$$BW = 1 + \frac{2L}{N} + \frac{2L^2}{N^2} + 2z^2L\left(\left(\frac{N-1}{N}\right) - \frac{2}{N}\left(\frac{\Gamma(N/2)}{\Gamma((N-1)/2)}\right)^2\right). \tag{18}$$

To calculate OVR and IV, it is necessary to find more relations and parameters. For OVR, we have to find the expected value of order quantity. Disney and Towill show that for the stationary state in the long period, the mean of order quantity is equivalent to the mean of retailer demand [21]. Therefore, OVR is equal to BW. To find the IV, we need to calculate the expected value and the variance of inventory held in the distribution center in each period. They are calculated as follows:

$$E(I) = \int_0^q (q-x) f_{D_L}(x)dx \tag{19}$$

$$Var(I) = E(I^2) - (E(I))^2 = \int_0^q (q-x)^2 f_{D_L}(x)dx - (E(I))^2, \tag{20}$$

where $E(I)$ and $Var(I)$ are respectively the expected value and variance of inventory; q is the order quantity; and due to lack of information, we have to use the estimated order quantity instead of the correct quantity. Furthermore, $f_{D_L}(x)$ is the distribution function of retailer demand during lead-time.

Therefore, the following equation shows the inventory variance:

$$IV = \frac{\sigma_I^2/\mu_I}{\sigma_D^2/\mu_D}$$

$$IV = \frac{\hat{D}\left(\int_0^q (q-x)^2 f_{D_L}(x)dx - \left(\int_0^q (q-x) f_{D_L}(x)dx\right)^2\right)}{\hat{\sigma}^2 \int_0^q (q-x) f_{D_L}(x)dx}. \tag{21}$$

Evidently, because of lack of information, we have to use the estimated variance and expected value of retailer demand. Moreover, to find $f_{D_L}(x)$, we just state that this is a normal distribution, and the parameters are unknown. Therefore, we use the estimated amount for parameters in our calculations.

3.1.4. Computation results and simulation

In order to perform sensitivity analysis for the two concepts (BW and IV), we consider some examples below. In these examples, we use a normal distribution with a mean of 50 and a standard deviation of 15 to generate data for retailer demand. The calculation was performed with MATLAB (Table 2), and the diagrams were created with Microsoft Excel.

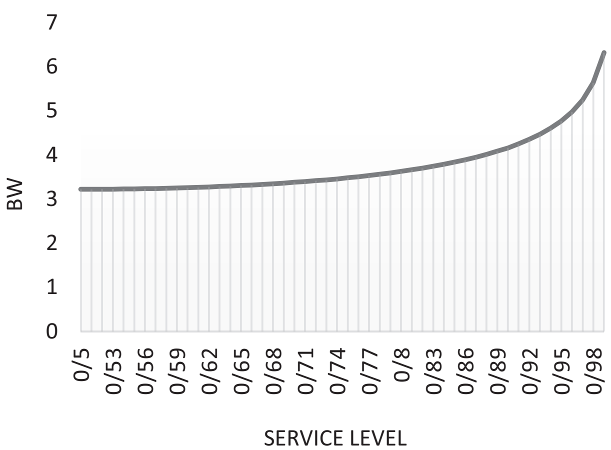

Fig. 6. The impact of service level on BW.

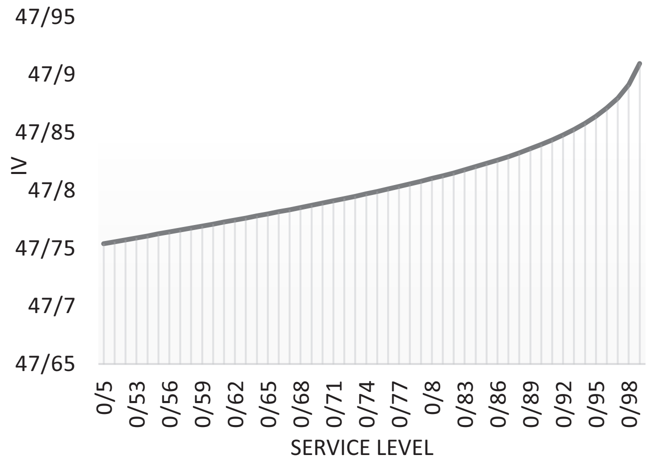

Fig. 7. The impact of service level on IV.

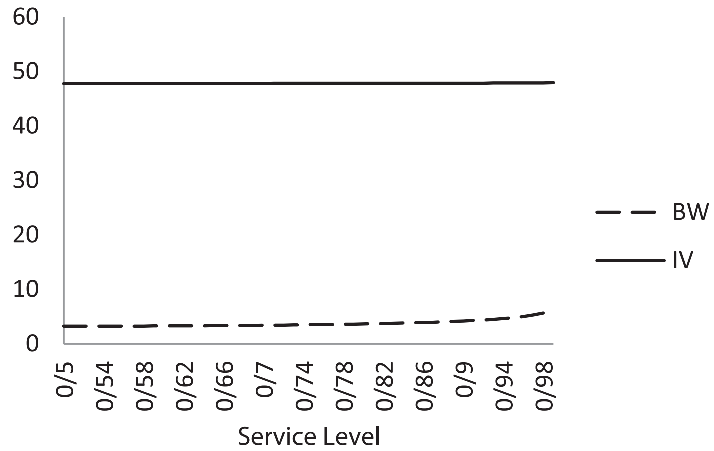

Fig. 8. Impact of service level on BW and IV.

First, we analyze the impact of service level on the bullwhip effect. This highlights the significant impact of service level (Fig. 6). This finding supports the need for the development of Eq. (18).

This figure illustrates the pattern of an increase in the bullwhip effect with respect as a result of an increase in the service level. For instance, considering two service level situations (70% and 88%), Fig. 6 shows that a 10% increase in the required service level at these points produces increases of nearly 13% and 60% in BW for the given points respectively. That is, a more significant impact on BW is expected when a higher service level is required. This result is very important from a practical point of view, since in the recent marketplace, firms usually aim for the highest range of service levels. One major reason is that in a higher service level situation, greater order size fluctuation is required in order to promptly cope with changes in demand. From this, we can conclude that at a higher service level, a more accurate measure of the bullwhip effect is also a clear need.

In this analysis, we presume 200 periods, while N in MA is considered to be five. The same is considered for Fig. 7 below.

Fig. 7 illustrates the impact of service level on IV. Two points should be noted. One is that service level has a positive impact on IV. The second is that the impact of service level on IV is not severe. In addition, the amount of IV is not changed very much by changing the service level.

Fig. 8 shows the effects of service level on BW and IV.

In Fig. 8, we can see that service level has a much larger influence on BW than on IV. Therefore, the impact of service level on the IV can be ignored.

Fig. 9 shows the influence of lead-time on BW and IV. Even though lead time has a positive impact on both BW and IV, this impact is more intense for BW. This makes sense, since lead-time is directly used in calculating the bullwhip effect.

Fig. 10 illustrates the effect of the parameter (N) in MA on BW and IV.

This figure shows that the number of periods in MA (N) has a negative impact on the bullwhip effect. This impact is similar for IV. However, it is much more sensitive for the bullwhip effect.

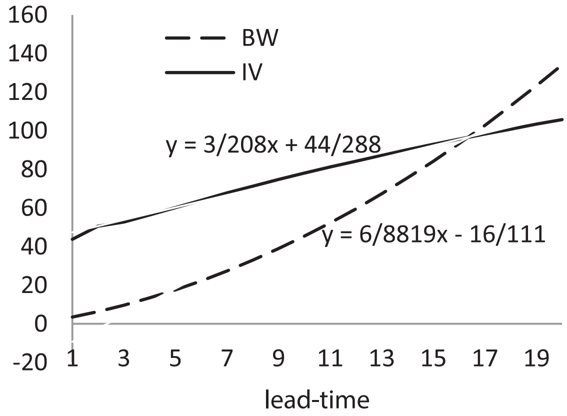

Fig. 9. Impact of lead-time on BW and IV.

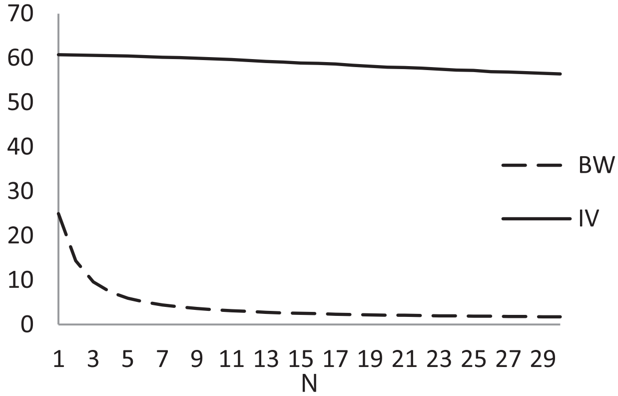

Fig. 10. Effect of number of periods in MA on BW and IV.

3.2. The bullwhip effect in supply chain with multiple retailers

In order to quantify the bullwhip effect in a supply chain with multiple retailers, there are some assumptions that we take into account. These assumptions are given as follows:

- The distribution function for all retailers is the same. This means that they follow normal distribution with identical parameters.
- Demand for every two retailers is correlated and the same, with the equivalent correlation coefficients ρ.

The first thing to take into account is the policy for order quantity. When there are multiple retailers in the supply chain, order policy is changed and transferred to the following equation [23]:

$$q_t = y_t - y_{t-1} + \sum_{j=1}^{m} D_t^j \tag{22}$$

This is an extension of Eq. (1), where D_t^j is demand of retailer j in period t. q_t, and y_t were defined previously.

In addition, the formulation of y_t does not change and is given in Eq. (2). However, the values for $\hat{D}_t^L$ or $E(D_t^L)$ and $\hat{\sigma}_t^L$ are different from those given before. $E(D_t^L)$ is the expected value of all retailer demand during lead-time and $\hat{\sigma}_t^L$ is the standard deviation of retailer demand. Thus, Eq. (23) can be proved [23]:

$$y_t = LE\left(\sum_{j=1}^{m} D_t^j\right) + z\sqrt{L}\sqrt{Var\left(\sum_{j=1}^{m} D_t^j\right)}. \tag{23}$$

The reason is that retailer demand equals the sum of demand of all retailers during lead-time. Eqs. (5), (6) and (7) help us find Eq. (23). To estimate the amount of D_t^j, we use the MA method as it is used in previous equations, and it results in the following equation, which is similar to Eq. (6).

$$E(D_t^j) = \frac{1}{N}\sum_{i=1}^{N} D_{t-i}^j \quad \forall j = 1, 2, ..., m$$
$$E(D_t) = \frac{1}{N}\sum_{j=1}^{m}\sum_{i=i}^{N} D_{t-i}^j. \tag{24}$$

Hence, the amount of distribution center order quantity is given as follows:

$$q_t = \left(1+\frac{L}{N}\right)\sum_{j=1}^{m} D_{t-1}^j + \left(-\frac{L}{N}\right)\sum_{j=1}^{m} D_{t-N-1}^j + z\sqrt{L}\left(\sqrt{Var\left(\sum_{j=1}^{m} D_t^j\right)} - \sqrt{Var\left(\sum_{j=1}^{m} D_{t-1}^j\right)}\right). \tag{25}$$

Calculation of the estimated variance of order quantity is performed in Lemmas 3 and 4.

Lemma 3. *Based on prior assumptions, the following equation is the outcome of the previous one:*

$$\sigma_D^2 = Var\left(\sum_{j=1}^{m} D_t^j\right) = m(1+(m-1)\rho)\sigma^2. \tag{26}$$

Proof. To find the variance of retailer demand, the following equations are given [68,69]:

$$Var\left(\sum_{j=1}^{m} D_t^j\right) = \sum_{j=1}^{m} Var(D_t^j) + 2\sum\sum_{i<j} Cov(D_t^i, D_t^j).$$

As we know, the variance of retailer demand is the same and the covariance of any two retailers are identical. Therefore, we can derive the following equations:

$$\sum_{j=1}^{m} Var(D_t^j) = m\sigma^2$$
$$2\sum\sum_{i<j} Cov(D_t^i, D_t^j) = 2\binom{m}{2}\rho\sqrt{Var(D_t^i)}\sqrt{Var(D_t^j)} = m(m-1)\rho\sigma^2.$$

In consequence, Eq. (26) is proved.

Lemma 4. *Based on prior assumptions, we can derive the following statement:*

$$\tilde{V}ar(q) = m(1+(m-1)\rho)\left(1+\frac{2L}{N}+\frac{2L^2}{N^2}+2z^2L\left(\left(\frac{N-1}{N}\right)-\frac{2}{N}\left(\frac{\Gamma(N/2)}{\Gamma(N-1/2)}\right)^2\right)\right)\sigma^2. \tag{27}$$

Proof. To prove Eq. (27), we have to get variance from two sides of Eq. (25). Based on the former arguments given in Lemma 1 and Lemma 2, the following statement can be shown:

$$Var(q_t) = \left(1+\frac{L}{N}\right)^2 Var\left(\sum_{j=1}^{m} D_{t-1}^j\right) + \left(-\frac{L}{N}\right)^2 Var\left(\sum_{j=1}^{m} D_{t-N-1}^j\right) + 2z^2 LVar\left(\sqrt{Var\left(\sum_{j=1}^{m} D_t^j\right)}\right).$$

Based on Eqs. (25) and (27), it is clear that:

In addition, based on Eq. (17), Var(σ) can be obtained, and therefore, Eq. (27) is easily obtained.

To calculate the bullwhip effect, we need to define it for a multi-retailer supply chain. This is the same as Eq. (10), which is as follows:

$$BW = \frac{Var(q)}{\sum_j Var(D_j)} \tag{28}$$

The only difference between Eqs. (10) and (28) is the calculation of retailer demand. This equation measures the proportion of variance of order quantity and sum of variance of retailer demand. To calculate the variance of retailer demand $\sum_j \mathrm{Var}(\mathrm{D_j})$ is used. We use $\sum_j \mathrm{Var}(\mathrm{D_j})$ instead of $\mathrm{Var}(\sum_j \mathrm{D_j})$. The reason is that we have to find the ratio of variance of order quantity to find the right variance of retailer demand. We know that $\mathrm{Var}(\sum_j \mathrm{D_j})$ is that variance in the distribution center's view. Thus, we use $\sum_j \mathrm{Var}(\mathrm{D_j})$ as the right amount of variance of retailer demand.

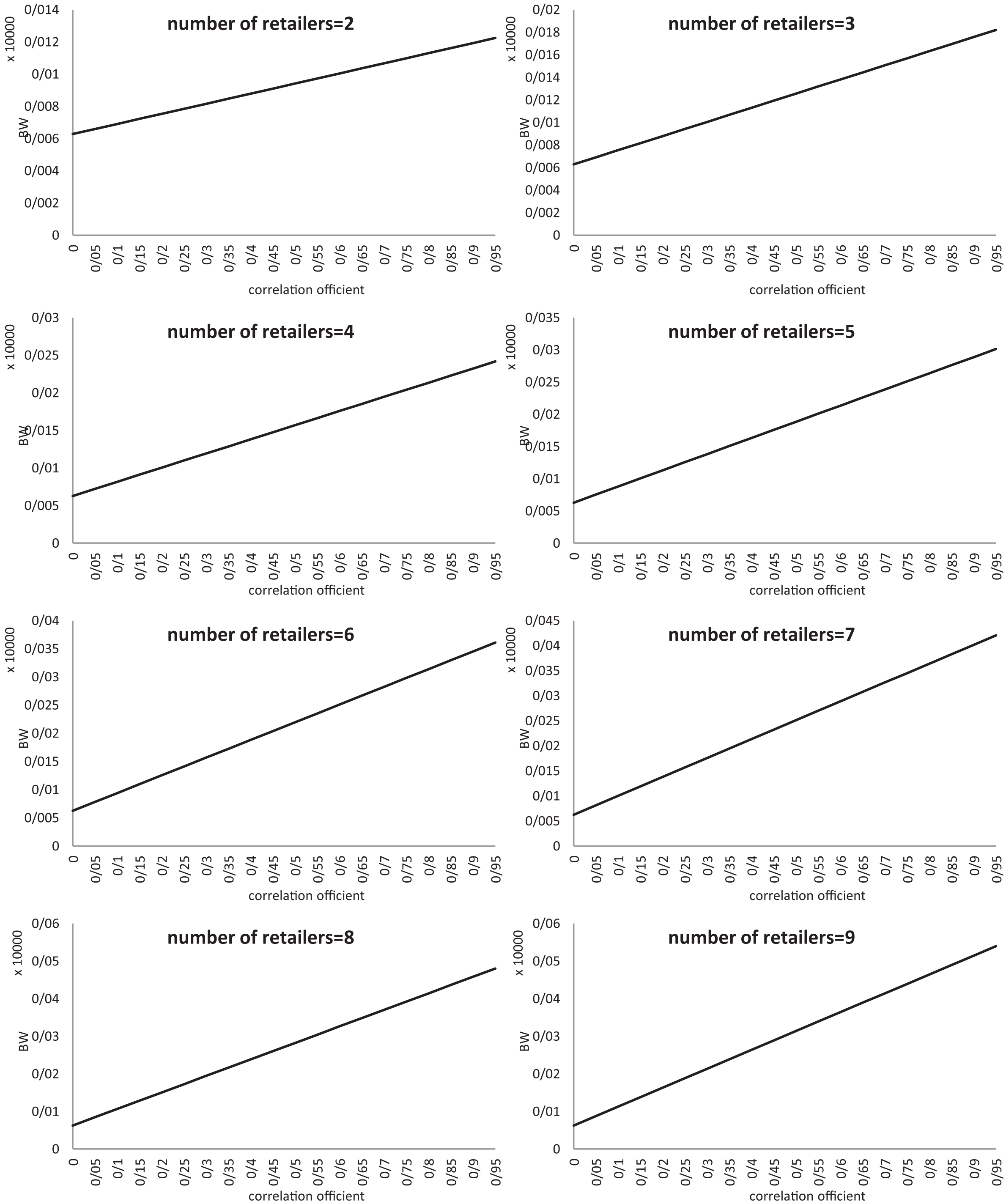

Fig. 11. Impact of correlation coefficient on BW in a supply chain considering different numbers of retailers.

Therefore, we can quantify the bullwhip effect by the following equation:

$$BW = (1 + (m-1)\rho)\left(1 + \frac{2L}{N} + \frac{2L^2}{N^2} + 2z^2L\left(\left(\frac{N-1}{N}\right) - \frac{2}{N}\left(\frac{\Gamma(N/2)}{\Gamma(N-1/2)}\right)^2\right)\right). \tag{29}$$

The equation that is used for calculating IV is the same as Eq. (18). The values for q, $E(D)$ and σ^2 are changed in calculation. To calculate these parameters, we use Eqs. (22) to (26).

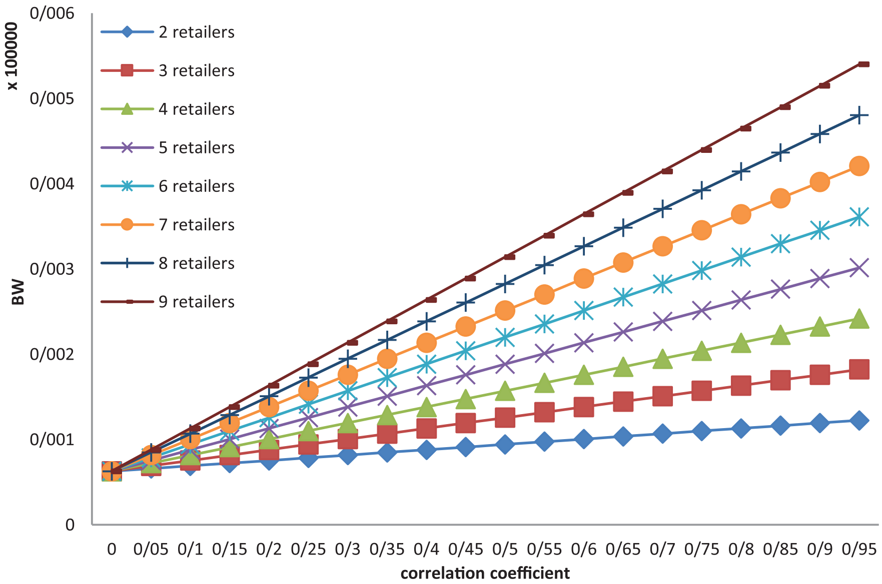

Fig. 12. Impact of number of retailers on BW.

A very important point to notice is that based on previous assumptions, orders for any two retailers have the same correlation; we have to assume that the correlation coefficient is nonnegative. The reason is that if the orders of the first and second retailer show a negative correlation, and the orders of the first and third retailers have a negative correlation, then the orders of the second and third retailers must have a positive correlation. Thus, if orders for all pairs of retailers have the same correlation, the correlation coefficient has to be nonnegative.

3.2.1. Computation results and simulation

In this section, an example is provided to show how the bullwhip effect is altered by changing the correlation coefficient. We assume a 3-stage supply chain considering 2–10 retailers and quantify the bullwhip effect that is transferred to the distribution center. Fig. 11 illustrates the impact of the correlation coefficient on the bullwhip effect. It shows the significant impact of correlation of retailer demand on the bullwhip effect in a supply chain. Changing the correlation coefficient considering different retailers shows the same behavior of the bullwhip effect. For all numbers of retailers, the bullwhip effect increases with an increasing correlation coefficient. This figure implies that the bullwhip effect strongly increases when the number of retailers in the supply network increases.

Fig. 12 shows BW of the supply chain with different retailers. In this figure, the increase in the bullwhip effect with the increasing the number of retailers is clearly illustrated. It is also obvious that the rate of increase is higher for supply chains with more retailers.

4. Sensitivity analysis and managerial insights

Sensitivity analysis and some managerial insights are discussed below. The focus is on BW, assuming a supply chain with a single retailer. According to Eq. (19), BW in a pipeline supply chain relates to three parameters: N, L and z. We show the paired impact of these parameters on the bullwhip effect. With one parameter (z) fixed, N and L are considered to be 3 and 5, respectively, while a service level of 99% is assumed. Mathematical software was used for this analysis.

We first show the paired impact of N and service level on the bullwhip effect, which is illustrated in Fig. 13.

Fig. 13 shows a direct relation between service level and the bullwhip effect. It also illustrates that N and BW have a reverse relationship.

Managerial insight 1: Service level is a strategic decision at the market level; a lower service level could result in loss of market share. However, if there is a preferred range for service level, finding the optimum point -of -service level in the desired range can be implemented by considering a multi-objective situation, including the bullwhip effect and the consequences of a lower service level.

Managerial insight 2: BW shows an increase in fluctuation in demand going upstream from downstream. This creates a lot of costs, including costs of inventory and backorders. In addition, the lack of smoothness is not suitable at all. Besides that, low service level leads to a decrease in demand and has a negative effect on brand as well as market in the long-term. Finding the optimum point -of -service level that has the most benefit for the supply chain can be implemented by considering a multi-objective problem, including the bullwhip effect and the cost of a low service level.

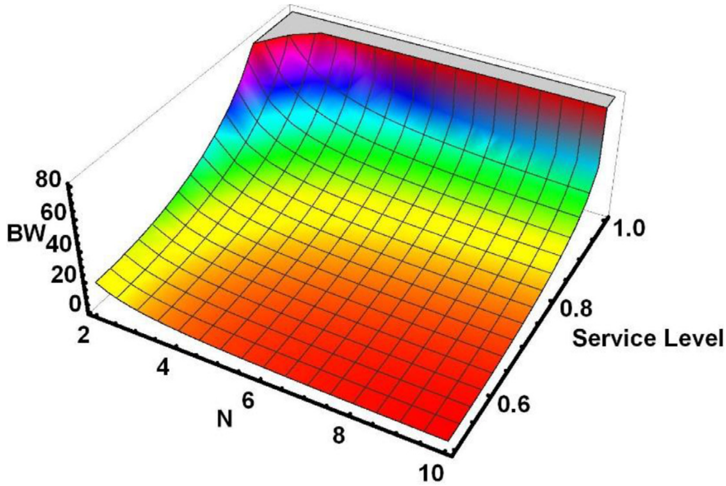

Fig. 13. Paired impact of *N* and service level on BW.

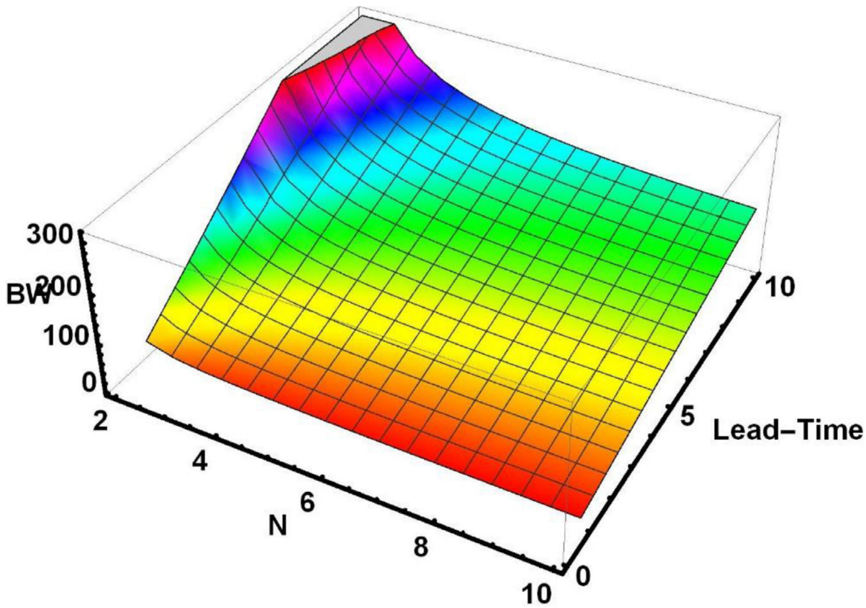

Fig. 14. Paired impact of *N* and lead-time on BW.

Managerial insight 3: N shows the periods that are considered for forecasting demand. A reverse relationship between BW and *N* can be explained as follows. The use of more previous periods for forecasting means considering more customers and leads to more precise forecasting. This leads to lower BW. However, this analysis can be used only for normal distribution of demand. For special demand like seasonal demand, using more *N* could cause imprecise forecasting that could result in more BW.

The following figure illustrated the paired impact of N and lead-time on the bullwhip effect.

Fig. 14 illustrates the direct relation between lead-time and BW. The intensity of this relation depends on the amount of *N*.

Managerial insight 4: As is clear in Fig. 14, the effect of large values for *N* and lead-time on BW is not very large. This shows that if forecasting of demand becomes precise, lead-time does not have a high impact on BW. Therefore, if the modeling of customer behavior is very difficult, it is better to use systems including vendor-managed inventory, just-in-time, agile or cross-docking in order to reduce lead-time. However, if forecasting is precise, expenditures for reducing the lead-time are not needed.

Fig. 15 shows the simultaneous impact of service level and lead-time on the bullwhip effect.

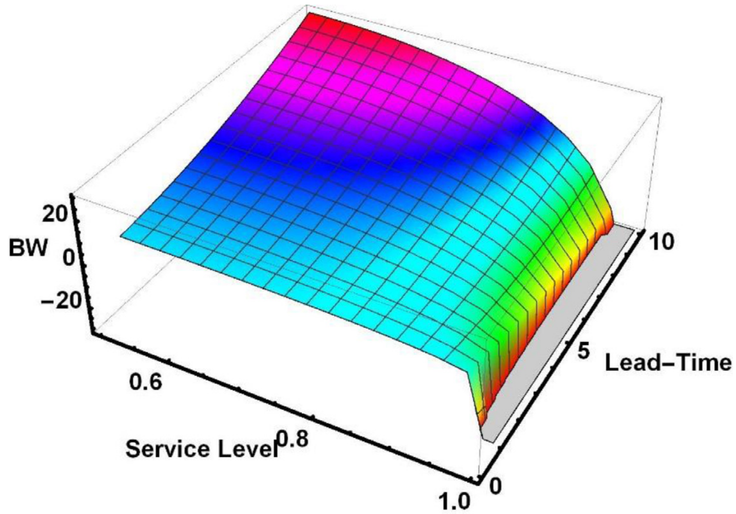

Fig. 15. pairwise impact of service level and lead-time on the BW.

Managerial insight 5: The above analysis indicates that the sensitivity of the bullwhip effect to the parameters increases when a higher service level is required. This finding supports more accurate measurement of the bullwhip effect, which is a main purpose of the current paper.

5. Conclusion

In this paper, we first quantified the bullwhip effect on order rate variance ratio (OVR) and inventory variance ratio (IV) in a pipeline 3-stage supply chain using the moving average technique to estimate mean and variance of retailer demand. A new relation was developed for calculating these values. We investigated the effects of service level, lead-time, and the parameter of the moving average method (N) on the bullwhip effect and the inventory variance ratio. The study identified that the relations that have previously been proposed for the bullwhip effect by using the moving average technique for demand estimation have ignored the impact of service level. Our proposed mathematical relation for the bullwhip effect takes into account the service level factor, and we have illustrated how significantly service level influences the bullwhip effect, which increases as the service level increases. It is also indicated that the bullwhip effect has a positive relation with the parameter of the moving average method as well as with lead-time. Second, we extended our proposed model to a supply chain considering multiple retailers. For this purpose, we assume identical demand distribution functions for retailers with equal correlation among retailer orders. Further, we assumed that $\mathrm{Cov}(\hat{\sigma}_t^L, \hat{\sigma}_{t-1}^L)$ is equal to zero; retailer demand follows a normal distribution function; and retailer demand is independent with regard to periods. However, for future research, any of these assumptions can be changed or relaxed according to the particular problem. In addition, we considered only one commodity in the supply chain. Therefore, for future research, multi -commodity supply chains could be taken into account. Further, multiple distribution center situations could also be considered as an extension of our work.

Appendix A

If X is a random variable with normal distribution with parameters μ and σ, and X_i is a sample of this distribution function, then it has been proven that [68]:

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1} = \Gamma\left(\alpha = \frac{n-1}{2}, \lambda = \frac{1}{2}\right),$$

where S^2 is estimated variance or sample variance with the following definition:

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}.$$

In addition, χ^2_n is a chi -square distribution and $\Gamma(\alpha, \lambda)$ is gamma distribution with the following function:

$$f_X(x) = \frac{\lambda e^{\lambda x} (\lambda x)^{\alpha-1}}{\Gamma(\alpha)}, x \geq 0,$$

where:

$$\Gamma(\alpha) = \int_{-\infty}^{y} y^{\alpha-1} e^{-y} dy.$$

Moreover, n is the number of the sample given from the normal distribution.
Therefore, we can find the expected value of S^2 by the following calculations [68]:

$$E(\frac{(n-1)S^2}{\sigma^2}) = \frac{(n-1)/2)}{1/2} = n-1$$

$$E(S^2) = \frac{\sigma^2}{n-1} E(\frac{(n-1)S^2}{\sigma^2}) = \frac{\sigma^2}{n-1} n-1 = \sigma^2.$$

To find the expected value of estimated standard deviation S we have

$$\frac{(n-1)S^2}{\sigma^2} \sim \Gamma(\alpha = \frac{n-1}{2}, \lambda = \frac{1}{2})$$

$$S^2 \sim \Gamma(\alpha = \frac{n-1}{2}, \lambda = \frac{n-1}{2\sigma^2})$$

$$Y = S^2 \to S = \sqrt{Y} \to E(S) = E(\sqrt{Y})$$

On the other hand, this proved that [68,69].
Therefore, it is given from above statements that:

$$E(Y^{\frac{1}{2}}) = \frac{\Gamma\left(\frac{n-1}{2} + \frac{1}{2}\right)}{\left(\frac{n-1}{2\sigma^2}\right)^{\frac{1}{2}} \Gamma\left(\frac{n-1}{2}\right)}.$$

Finally, it is given that:

$$E(S) = \sqrt{\frac{2}{n-1}} \frac{\Gamma(n/2)}{\Gamma((n-1)/2)} \sigma$$

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