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Petroleum demand forecasting for Taiwan using modified fuzzy-grey algorithms

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Abstract: *In this paper, we adopt the exponentially weighted moving average (EWMA) method to develop the residual modification EWMA grey forecasting model REGM(1,1) and combines it with fuzzy theory to derive the fuzzy REGM or the FREGM(1,1) model. The proposed model is used to forecast annual petroleum demand in Taiwan. The experimental results show that the mean absolute percentage errors, median absolute percentage error, and symmetric mean absolute percentage error of FREGM(1,1) model are higher by 23.71, 12.26, and 23.06% respectively, compared with those obtained using the traditional GM(1,1) model.*

Keywords: grey forecast, petroleum demand, EWMA, fuzzy theory

1. Introduction

Petroleum, the economic pillar of all industrialized countrixes, controls various aspects of economic development. Today, human life is almost inseparable from petroleum; power plants, transportation, plastic products, etc. are directly or indirectly linked with petroleum. The excessive use of petroleum has been linked to climate change and its associated detrimental effects (Guillet, 2010). Forecasting petroleum consumption is a vital part of energy policy of a country, especially for a developing country like Taiwan whose petroleum relies on import entirely. Today, issues concerning energy have become integral and important reference points for economic strategies and technology development for all countries. This has motivated many researchers to focus their work on petroleum consumption forecasting. For example, Furtado and Suslick (1993) investigated petroleum consumption in Brazil based upon logistic, learning, and translog models using the intensity of energy technique. Ji (2011) predicted petroleum consumption in China by comparing three sigmoidal growth models. Kumar and Jain (2010) applied three time series models to forecast energy consumption in India.

Although the first-order grey model with one variable [GM (1,1)] has been widely applied on energy consumption forecasting, such as Meng *et al.* (2011), Pao and Tsai (2011), and Niu *et al.* (2013), its predicting performance still could be improved. In this paper, we adopt the exponentially weighted moving average (EWMA) method to develop the residual modification EWMA grey forecasting model, namely REGM(1,1), and combine it with fuzzy theory to derive the fuzzy REGM or the FREGM(1,1) model. The proposed models are designed for the purpose of comparisons with traditional GM(1,1) models, and experimental results indicate that these designs significantly improve the forecasting accuracy of GM(1,1) models. The remainder of this paper is organized as follows. Section 2 reviews some related literature. Section 3 proposes our REGM(1,1) and FREGM (1,1) models. Section 4 describes evaluation criteria of models, and Section 5 presents some experimental results using Taiwan's petroleum demand. Finally, we draw some general conclusions. The next section provides a literature review of different forecast algorithms and grey theory.

2. Literature review

Traditional forecasting methods are widely applied in areas such as agriculture, industry, commerce, education, and transportation. Many researchers have invented different forecasting methods to enhance the forecasting precision in recent years. Typical models include the linear regression method (Chang and Tsai, 2008; Bianco *et al.*, 2009), non-linear dynamic model (Bradley and Jansen, 2004), time series method (Chu, 1998; Kayacan *et al.*, 2010), exponential smoothing method (Gardner, 1985; Tang and Yin, 2012), fuzzy time series method (Chen and Hsu, 2004; Qiu *et al.*, 2011), Markov chain method (Grimshaw and Alexander, 2011), and neural networks method (Crone *et al.*, 2011). Hsu (2011) noted that all the previously mentioned methods are quick, accurate, versatile, and easy to use. However, they have limitations. Although historical data collection is difficult, such methods require a large number of samples. Many scholars thus began using the residual grey forecasting model proposed by Deng (1982), which requires less data and smaller sample sizes, making forecasting faster and easier (Lin and Yang, 2003; Chen and Hsu, 2004; Chang *et al*., 2005; Tang and Yin, 2012).

In recent years, fuzzy time series have combined with traditional GM(1,1) forecasting models to improve forecasting accuracy. For example, Kuo and Chang (2003) adopted the grey theory and fuzzy theory to enhance the forecast of ship fire alarm systems. Wang (2004) predicted tourism demand using fuzzy time series and hybrid grey theory. Tsaur (2006) improved the accumulated generation operation (AGO) value in the grey theory to improve the forecast accuracy provided by the fuzzy theory. Zhao *et al*. (2007) combined the grey theory with the fuzzy theory to predict short-term mechanical failure and compare the results of the combination to those of the GM(1,1) model alone. Chung *et al*. (2010) integrated the grey theory with fuzzy control to predict the pH value of food.

To improve the accuracy of the forecast model, this study combines the EWMA method with fuzzy theory to propose a precise forecast model for predicting Taiwan's future petroleum energy requirements.

3. Methodology

Artificial intelligence has become very popular in recent years. It is a process of reasoning, learning, judging, thinking, and deciding about potential future issues/effects and is based on man-made simulations. Artificial intelligence forecasting methods include expert systems, genetic algorithms, neural networks, fuzzy theory, and grey theory. In cases where data are limited, the GM(1,1) model of the grey theory is commonly used for forecasting. We modify this model to develop a new forecasting model called the EWMA grey forecasting model or EGM(1,1). Combined with a residual modification, we propose a model called REGM(1,1) to improve the forecasting accuracy of the traditional GM(1,1) model. The amended fuzzy theory EGM(1,1) model, known as FEGM(1,1), and its combination with residual modification, known as FREGM(1,1), are also compared in this study. The accuracy of the forecast model is analyzed, and the results can provide a reliable reference index for decision-makers. These comparisons are executed in terms of the mean absolute percentage errors (MAPEs) of the various grey models (Figure 1).

3.1. The GM(1,1) model

The grey system theory was first proposed by Deng (1982). It was systematically modelled as the GM(1,1) model, using a small amount of incomplete information. The construction process of this model is as described in the succeeding texts.

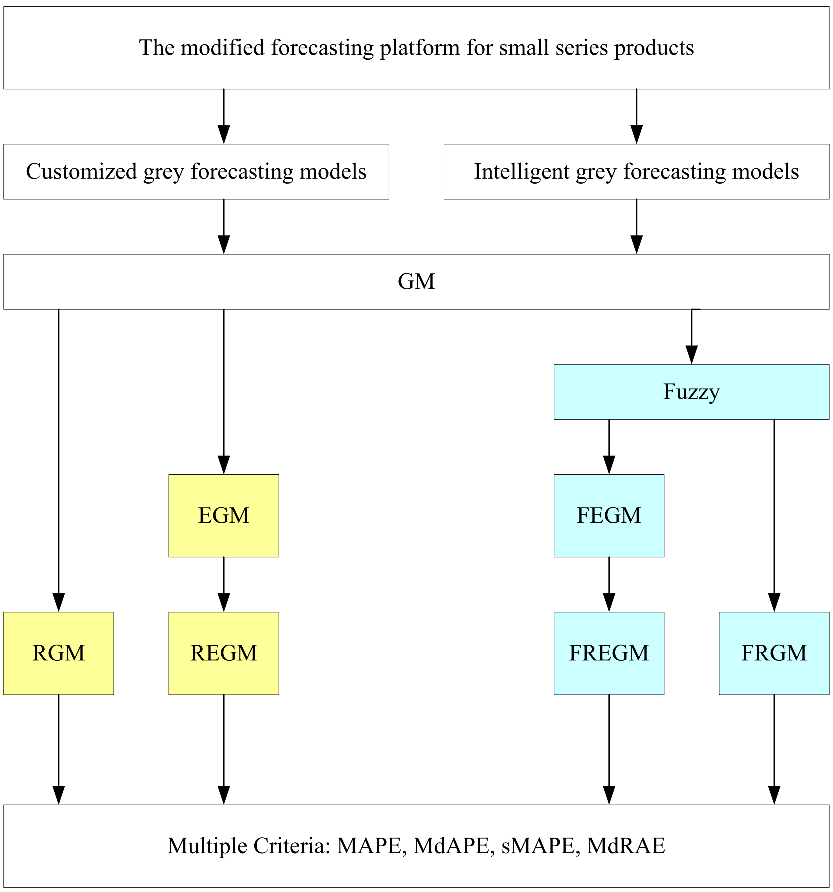

Figure 1: *The comparisons of grey model series.*

Step 1: Assume the original data sequence to be

$$x^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right\} \tag{1}$$

Step 2: A new series $x^{(1)}$ is generated by the AGO. Thus,

$$x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n)\right\} \tag{2}$$

where $x^{(1)}(1) = x^{(0)}(1)$, and $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 2, 3, \ldots, n$

Step 3: Calculate background values, $z^{(1)}$.

$$z^{(1)}(k) = (1-\alpha)x^{(1)}(k-1) + \alpha x^{(1)}(k), \quad k = 2, 3, \ldots, n \tag{3}$$

Step 4: Establish the grey differential equation

$$\frac{dx^{(1)}(k)}{dt} + ax^{(1)}(k) = b \tag{4}$$

where a is the developing coefficient and b is the grey input.

Step 5: Solve equation (4) using the least squares method. Then, the forecasting values can be obtained as

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-a(k-1)} + \frac{b}{a} \tag{5}$$

where $[a, b]^T = (B^T B)^{-1} B^T Y$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right]^T$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}$$

Step 6: The recovered data $\hat{x}^{(0)}(k)$ can be retrieved using the inverse AGO.

$$\hat{x}^{(0)}(k) = (1-e^{a})\left(x^{(0)}(1) - \frac{b}{a}\right)e^{-a(k-1)}, \quad k = 2, 3, \ldots, n \tag{6}$$

3.2. The EGM(1,1) model

When using the grey theory for forecasting, the background value will affect the accuracy of the forecast of GM(1,1). Generally, α is set as 0.5 in the GM(1,1) model. Hung *et al.* (2009) point out that different α values result in different model errors, and this property can be used to improve the forecast ability of the model. Therefore, this study utilizes the concept of the EWMA proposed by Roberts (1959) to modify the background value of GM (1,1). We call this model the EGM(1,1) model.

The EWMA is defined as

$$z_i = \lambda x_i + (1-\lambda)z_{i-1} \tag{7}$$

where $0 < \lambda \leq 1$. The initial value is used as the starting value of the EWMA. Parameter z_i is the weighted average of all past and current observations.

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \tag{8}$$

The weights $\lambda(1-\lambda)^j$ decrease geometrically with the age of the observation. That is, the most recent observation is given the greatest weight in reflecting data behaviour. Accordingly, $z^{(1)}(k-1)$ replaces $x^{(1)}(k-1)$ in equation (3) and modifies the background values as

$$z^{(1)}(k) = \hat{\alpha}x^{(1)}(k) + (1-\hat{\alpha})z^{(1)}(k-1), \quad k = 2, 3, \ldots, n \tag{9}$$

where the initial value of $z^{(1)}(1)$ is set to $x^{(1)}(1)$ in equation (2).

3.3. The REGM(1,1) model

Deng (1982) establishes the residual modification GM(1,1) model, known as RGM(1,1) model, and proves that it can effectively enhance the accuracy of GM(1,1). To improve the EGM(1,1) model, this study uses the residual errors of EGM(1,1) to develop the forecast values, and this model is called the REGM(1,1) model. The differences between the original series $x^{(0)}$ and the forecast values $\hat{x}^{(0)}_{EGM}$ from the EGM(1,1) model are defined as the residual series. Hence, the residual series $\varepsilon^{(0)}_{EGM}$ can be represented as

$$\varepsilon^{(0)}_{EGM} = \left(\varepsilon^{(0)}(2), \varepsilon^{(0)}(3), \varepsilon^{(0)}(4), \ldots, \varepsilon^{(0)}(n)\right) \tag{10}$$

where $\varepsilon^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}_{EGM}(k), \quad k = 2, 3, \ldots, n$. Transform the $\varepsilon^{(0)}_{EGM}$ to be the nonnegative residual series and represent as

$$\eta^{(0)}(k) = \varepsilon^{(0)}(k) - m, \quad k = 2, 3, \ldots, n \tag{11}$$

where $m = \min\{\varepsilon^{(0)}(2), \varepsilon^{(0)}(3), \varepsilon^{(0)}(4), \ldots, \varepsilon^{(0)}(n)\}$. Then, the steps in Section 3.1 are executed, and the residual forecast values are named $\hat{\varepsilon}^{(0)}_{EGM}(k+1) = \hat{\eta}^{(0)}(k+1) + m$. Hence, the forecast values of the REGM(1,1) model $\hat{x}^{(0)}_{REGM}(k)$ can be integrated by $\hat{\varepsilon}^{(0)}_{EGM}(k)$ and $\hat{x}^{(0)}_{EGM}$ as follows:

$$\hat{x}^{(0)}_{REGM}(k+1) = \hat{x}^{(0)}_{EGM}(k+1) + \hat{\varepsilon}^{(0)}_{EGM}(k+1), \quad k = 2, 3, \ldots, n \tag{12}$$

3.4. The adjusted fuzzy forecasting model

Zadeh (1965) proposes the fuzzy theory, which is essentially a mathematical description of linguistic fuzzy information and establishes a fuzzy time series. Song and Chissom (1993) derive the first-order time-invariant model and a time-variant model, both of which are fuzzy time series models, and use them to forecast enrolments at the University of Alabama. Hwang *et al.* (1998) define a new fuzzy relationship to forecast the enrolment rate at the University of Alabama. The experimental results show that the Hwang's model performs better than those of Song and Chissom. Moreover, Wang (2004) forecasts tourism demand using the grey theory and fuzzy time series. The empirical results show that the latter is suitable for tourism demand forecasting. Therefore, our study proposes a modified fuzzy time series model based on the fuzzy time series established by Hwang *et al.* (1998). Our modified model improves the rule that any positive number determines the range of the universe *U*. Using Hwang's procedures, a flow diagram is presented to calculate the forecast values of the adjusted fuzzy forecasting model (Figure 2).

3.5. The FRGM(1,1), FEGM(1,1), and FREGM(1,1) models

To improve the RGM(1,1), EGM(1,1), and REGM(1,1) models, we use the residual errors of the three models to forecast values using the adjusted fuzzy forecasting model in Section 3.4. The resulting models are known as the FRGM (1,1),FEGM(1,1), and FREGM(1,1) models, respectively. Figure 3 illustrates the Fuzzy-Grey models by Fuzzy controller.

4. Evaluation criteria

The forecast error measurement is an important criterion to evaluate the accuracy of the proposed forecasting models. Shcherbakov *et al.* (2013) demonstrated that it is impossible to choose only one measure for evaluating because of each error measure has the disadvantages leading to inaccurate evaluation of the forecasting results. This paper provides the common forecast error measures that are used in forecasting (Armstrong and Collopy, 1992; Fildes, 1992; Hyndman and Koehler, 2006; Makridakis and Hibon, 2009), namely the MAPE, median absolute percentage error (MdAPE), symmetric mean absolute percentage error (sMAPE), and median relative absolute error (MdRAE). Consider $x^{(0)}(k)$ to be the actual value at time k and $\hat{x}^{(0)}(k)$ to be the forecast value at time k, obtained from the use of the forecasting model. These forecast error measures are calculated as

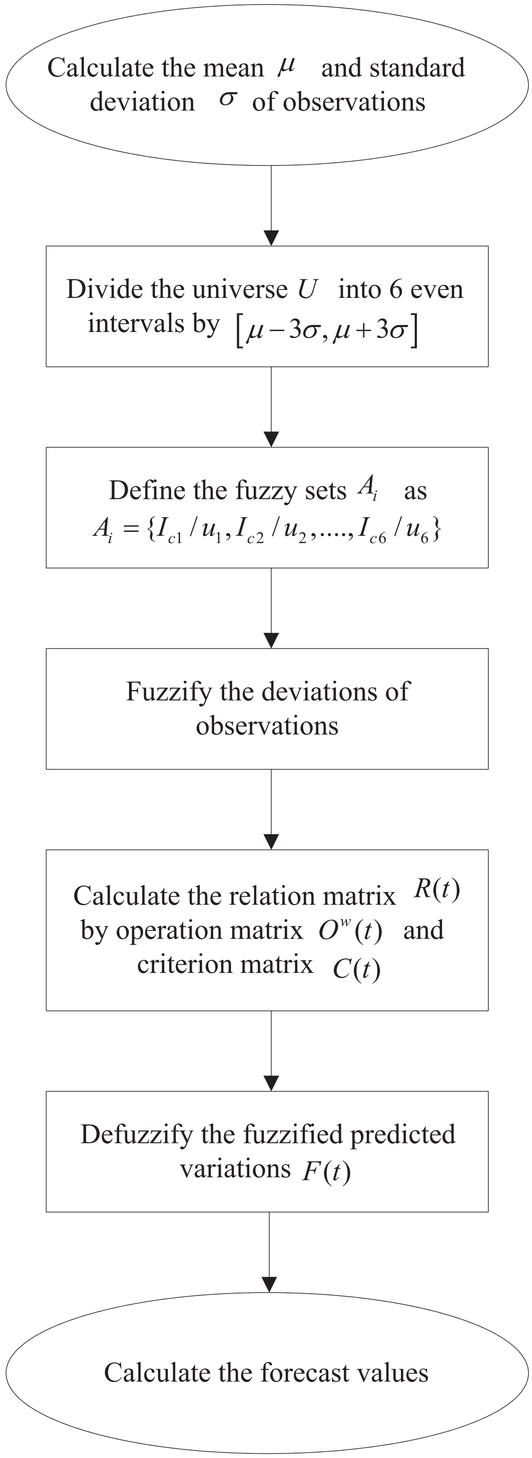

Figure 2: *A flow diagram to calculate the forecast values of the adjusted fuzzy forecasting model.*

$$MAPE = \frac{1}{n}\sum_{k=1}^{n} 100\cdot|p_k| \tag{13}$$

$$MdAPE = \underset{k=1,..,n}{median}(100\cdot|p_k|) \tag{14}$$

$$sMAPE = \frac{1}{n}\sum_{k=1}^{n} 200\cdot|s_k| \tag{15}$$

$$MdRAE = \underset{k=1,..,n}{median}|r_k| \tag{16}$$

where $p_k = \frac{|x^{(0)}(k)-\hat{x}^{(0)}(k)|}{x^{(0)}(k)}$, $s_k = \frac{|x^{(0)}(k)-\hat{x}^{(0)}(k)|}{x^{(0)}(k)+\hat{x}^{(0)}(k)}$, and $r_k = \frac{|x^{(0)}(k)-\hat{x}^{(0)}(k)|}{x^{(0)}(k)-\hat{x}^{*(0)}(k)}$. The $\hat{x}^{*(0)}(k)$ is the forecast value obtained using a reference model. In this paper, the GM(1,1) model is

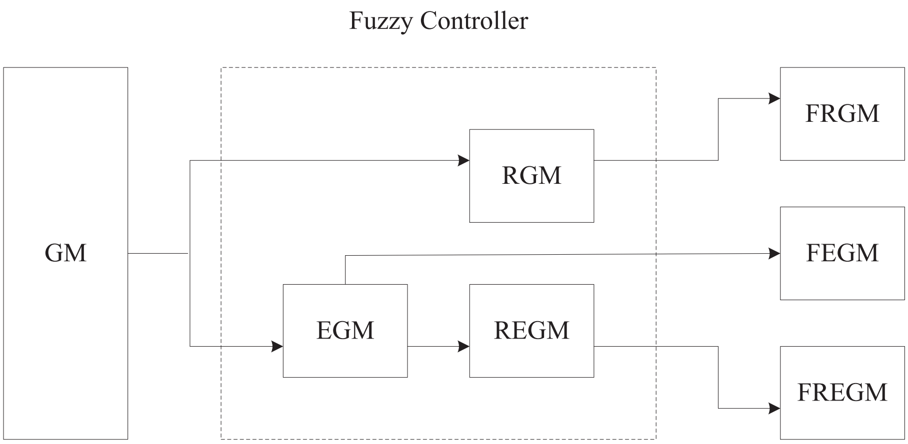

Figure 3: *Fuzzy-Grey models by Fuzzy controller.*

regarded as the reference model because the accuracy of proposed forecasting models is compared with it.

5. Empirical study

To forecast the petroleum demand in Taiwan, this study employs the petroleum demand data provided by the Bureau of Energy of the Ministry of Economic Affairs. As stated previously, it has been estimated that global petroleum reserves will last for a further 40 years or so. This study aims to provide timely and relevant forecast data as a reference for energy planning policy to Taiwan's government. Thus, this study focuses on improving the accuracy of the forecasting model.

5.1. *Empirical results of the customized grey forecasting models*

Using the available data on petroleum demand in Taiwan, the original data sequence can be written as $x^{(0)} = [37817.42, 38066.44, \ldots, 51481.92]$. The developing coefficient and grey input of the original GM(1,1) model are estimated by the least squares method using equations (2–4), such that $a = -0.020$ and $b = 39.242$, respectively. The forecast equation of GM(1,1) is as seen in the succeeding texts.

$$\hat{x}^{(0)}(k) = \left(1 - e^{-0.020}\right)\left(37817.42 + \frac{39.242}{0.020}\right)e^{0.020(k-1)},\quad \mathrm{k} = 2, 3, \ldots, n$$

To define the optimal value for α for the EGM(1,1) model, we need to minimize the objective value, namely the MAPE. Through trial and error, we find that the optimal value for α is 0.93. Simultaneously, the developing coefficient and grey input are $a = -0.023$ and $b = 38.756$, respectively. The EGM(1,1) model is as seen in the succeeding texts:

$$\hat{x}^{(0)}_{EGM}(k) = \left(1 - e^{-0.023}\right)\left(37817.42 + \frac{38.756}{0.023}\right)e^{0.023(k-1)},\quad \mathrm{k} = 2, 3, \ldots, n$$

Next, we forecast values using the RGM(1,1) and REGM (1,1) models. The differences between the original series $x^{(0)}$ and the forecast values $\hat{x}^{(0)}$ from the GM(1,1) model and the forecast values $\hat{x}^{(0)}_{EGM}$ from the EGM(1,1) model are defined as the residual series. We execute equations (2–4) to arrive at the forecast equations of RGM(1,1) and REGM(1,1) as seen in the succeeding texts.

$$\hat{x}^{(0)}_{RGM}(k) = \hat{x}^{(0)}(k) + \left(\varepsilon^{(0)}(2) - \frac{3.54}{0.0074}\right)(1 - e^{0.0074})e^{-0.0074(k-1)},\quad k = 3, 4, \ldots, n$$

$$\hat{x}^{(0)}_{REGM}(k) = \hat{x}^{(0)}_{EGM}(k) + \left(\varepsilon^{(0)}(2) - \frac{5.37}{0.0312}\right)(1 - e^{0.0312})e^{-0.0312(k-1)},\quad \mathrm{k} = 3, 4, \ldots, n$$

The actual values and forecast values of the GM(1,1), RGM (1,1), EGM(1,1), and REGM(1,1) models are presented in Table 1. The empirical results indicate that the REGM(1,1) model with $\alpha = 0.93$ has smallest MAPE, MdAPE, sMAPE, and MdRAE among these models. The MAPE, MdAPE, and sMAPE of REGM(1,1) model are higher by 5.67, 51.01, and 8.29% respectively, compared with those obtained using the traditional GM(1,1) model. Therefore, the REGM(1,1) model cannot only effectively reduce forecasting errors but it can also enhance the forecast accuracy of the GM(1,1) model.

5.2. *Empirical results of the intelligent fuzzy grey forecasting models*

To ameliorate the forecasting accuracies of the native EGM (1,1) and REGM(1,1) models, the fuzzy controller is incorporated with the customized grey forecasting models. Taiwan's petroleum demand data from 1995 to 2010 are used to investigate the forecasting accuracy of the modified fuzzy grey models. Following Hwang *et al.* (1998), we propose the following modifications.

Step 1: Determine the deviations of the forecast value of the GM(1,1) model.

Table 1: *Customized grey forecast values and errors of the petroleum demand in Taiwan from 1995 to 2010*

Year	$x^{(0)}(k)$	GM(1,1) $\hat{x}^{(0)}(k)$	RGM(1,1) $\hat{x}^{(0)}(k)$	EGM(1,1) $\alpha = 0.93$ $\hat{x}^{(0)}(k)$	REGM(1,1) $\alpha = 0.93$ $\hat{x}^{(0)}(k)$
1995	37817.42				
1996	38066.44	40416.81		40094.18	
1997	38765.64	41244.00	41537.45	41032.95	40788.69
1998	40377.97	42088.12	42355.84	41993.70	41904.50
1999	42855.42	42949.51	43191.70	42976.95	43038.05
2000	44260.09	43828.53	44045.37	43983.21	44190.00
2001	47214.77	44725.54	44917.22	45013.04	45361.02
2002	46900.68	45640.91	45807.62	46066.98	46551.83
2003	48365.80	46575.02	46716.93	47145.59	47763.10
2004	49990.30	47528.24	47645.54	48249.47	48995.54
2005	50008.77	48500.97	48593.85	49379.18	50249.88
2006	50565.32	49493.61	49562.24	50535.35	51526.85
2007	52944.62	50506.57	50551.13	51718.59	52827.16
2008	50011.89	51540.26	51560.92	52929.53	54151.59
2009	49375.49	52595.10	52592.05	54168.83	55500.88
2010	51481.92	53671.53	53644.94	55437.14	56875.80
MAPE		3.88	3.70	3.69	3.66
MdAPE		4.24	3.81	3.48	1.95
sMAPE		3.86	3.68	3.62	3.54
MdRAE			0.97	0.86	0.78

Step 2: Calculate the mean and standard deviation of the deviations ($\mu = 946.77$, $\sigma = 80.19$). We redefine the range of universe U such that $U = [\mu - 3\sigma, \mu + 3\sigma] = [706.21, 1187.32]$.

Step 3: Divide the universe U into six equal intervals. The six parameters $u_1, u_2, \ldots u_6$ are assigned to different intervals: $u_1 = [706.21, 786.39]$, $u_2 = [786.39, 866.58]$, $u_3 = [866.58, 946.77]$, $u_4 = [946.77, 1026.95]$, $u_5 = [1026.95, 1107.14]$, and $u_6 = [1107.14, 1187.32]$.

Step 4: Define the fuzzy sets on the universe U. Meanwhile, each fuzzy set is composed of the elements u and their corresponding memberships. Therefore, the six fuzzy sets $A_1, A_2, \ldots, A_6$ are expressed as follows:

$$A_1 = \{1/u_1, 0.5/u_2, 0/u_3, 0/u_4, 0/u_5, 0/u_6\}$$
$$A_2 = \{0.5/u_1, 1/u_2, 0.5/u_3, 0/u_4, 0/u_5, 0/u_6\}$$
$$A_3 = \{0/u_1, 0.5/u_2, 1/u_3, 0.5/u_4, 0/u_5, 0/u_6\}$$
$$A_4 = \{0/u_1, 0/u_2, 0.5/u_3, 1/u_4, 0.5/u_5, 0/u_6\}$$
$$A_5 = \{0/u_1, 0/u_2, 0/u_3, 0.5/u_4, 1/u_5, 0.5/u_6\}$$
$$A_6 = \{0/u_1, 0/u_2, 0/u_3, 0/u_4, 0.5/u_5, 1/u_6\}$$

Step 5: Fuzzify the deviations of the forecast value of the GM(1,1) model. For example, the deviation between the years 1996 and 1997 is 827.19. The degree for 1996 belongs to fuzzy set A_2.

Step 6: Make a fuzzy forecast of the deviations. Note that different window bases generate different forecast values. We consider different window bases to calculate the minimum error. Therefore, as an example, we take $w = 3$ and $t = 2000$ to construct F (2000). The related matrices are described in the succeeding texts.

$$O^3(2000) = \begin{bmatrix} fuzzy\ \ variation\ \ of\,1998 \\ fuzzy\ \ variation\ \ of\,1997 \\ fuzzy\ \ variation\ \ of\,1996 \end{bmatrix} = \begin{bmatrix} A_5 \\ A_4 \\ A_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0.5 & 1 & 0.5 \\ 0 & 0 & 0.5 & 1 & 0.5 & 0 \\ 0 & 0 & 0.5 & 1 & 0.5 & 0 \end{bmatrix}$$

$$C(2000) = [fuzzy\ variation\ of\ 1999] = [A_5] = [0 \quad 0 \quad 0 \quad 0.5 \quad 1 \quad 0.5]$$

$$\mathrm{R}(2000) = O^3(2000) \times C(2000)$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0.25 & 1 & 0.25 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}$$

$$F(2000) = [0 \quad 0 \quad 0 \quad 0.5 \quad 1 \quad 0.25]$$

Step 7: Defuzzify the predicted fuzzy variations from step 5. The membership in matrix $F(2000)$ has one maximum value at u_5. Therefore, the forecast value for 2000 will be the midpoint of the corresponding interval. Thus, the forecast value for 2000 is $(1026.95 + 1107.14)/2 = 1067.05$.

Step 8: Calculate the outputs. The forecast value is 53662.15 in 2000, namely the sum of the actual value in 1999 (52595.10) and the predicted variation in 2000 (1067.05).

The MAPE is measured to facilitate the evaluation of these models. Table 2 shows the MAPEs of the adjusted fuzzy forecasting model with different window bases ($w = 2$, 3, 4, and 5). According to the rule of thumb for MAPEs, the minimum MAPE is 3.77%, which corresponds to the adequate window base $w = 2$. Moreover, the MAPE increases as window base increases.

Next, we present an FRGM(1,1) model, an improvement over the RGM(1,1) model, by fuzzifying and defuzzifying the residuals.

Step 1: Calculate the residuals of the GM(1,1) model. The residual sequence $\varepsilon^{(0)}$ is the difference between the original sequence $x^{(0)}$ and the forecasted sequence $\hat{x}^{(0)}$ and can be obtained as seen in the succeeding texts.

$$\varepsilon^{(0)} = (-2350.37, -2478.36, \ldots, -2189.61)$$

Step 2: The residual forecast values $\hat{\varepsilon}^{(0)}$ can be obtained using steps 1–6 in Section 3.1. In this step, we take the $w = 5$ and $t = 2010$, for example. Thus, $F(2010) = [0 \quad 0.5 \quad 0.5 \quad 0 \quad 0 \quad 0]$.

The matrix F(2010) has two instances of the maximum value, one at u_2 and the other at u_3. Consequently, the residual forecast value for 2000 is the average or midpoint of the corresponding interval, u_2 and u_3, or $(-3079.43 - 1031.79)/2 = -2055.61$.

Step 3: The residual forecast value in 2010 (50539.49) is the sum of the GM(1,1) forecast value for 2009 (52595.10) and the residual forecast variation in 2010 (−2055.61).

Table 3 shows the MAPE values of the FRGM(1,1) model with different window bases ($w = 2$, 3, 4, and 5). According to the rule of thumb for MAPEs, the minimum MAPE is 3.31% corresponding to the adequate window base $w = 5$. Table 3 also indicates that the MAPE decreases as the window base increases.

The MAPEs of the FEGM(1,1) and FREGM(1,1) models with different window bases ($w = 2$, 3, 4, and 5) are presented in Tables 4 and 5. These tables indicate that the MAPE increases as the window base increases in the FEGM(1,1) model; however, for the FREGM (1,1) model, the MAPE decreases as the window base increases.

The intelligent grey forecast values and errors of the FGM(1,1), FRGM(1,1), FEGM(1,1), and FREGM(1,1) models are listed in Table 6. The empirical results show that the FREGM(1,1) model with $w = 5$ has smallest MAPE, sMAPE, and MdRAE among these models. The MAPE and sMAPE of FREGM(1,1) model are higher by 21.48 and 21.63% respectively, compared with those obtained using the FGM(1,1) model.

5.3. Empirical results of traditional forecasting methods

In this section, classical forecasting methods, namely the linear trend, the exponential smoothing, and the autoregressive integrated moving average (ARIMA), are investigated to compare the intelligent grey forecasting models. Taiwan's petroleum demand data from 1995 to 2010 are adopted to formulate a linear trend equation, the exponential smoothing with smoothing constant λ, and ARIMA(p, d, q) model, where p is the order of autoregression, d is the order of difference, and q is the order of moving average. The statistical results show that exponential smoothing method with $\lambda = 0.8$ and ARIMA (0,1,0) is a suitable model based on the minimal MAPE. Forecast values and errors of the petroleum demand by these traditional forecasting methods are listed in Table 7 and their corresponding forecasting equations as follows:

Linear trend equation $\quad \hat{x}^{(0)}(k) = 37671.43 + 1001.91 \cdot k, \quad k = 2, 3, \ldots, n$

Exponential smoothing $\quad \hat{x}^{(0)}(k) = 0.8 \cdot x^{(0)}(k-1) + 0.2 \cdot \hat{x}^{(0)}(k-1), \quad k = 2, 3, \ldots, n$

ARIMA $\quad \hat{x}^{(0)}(k) = 2380.97 + x^{(0)}(k-1), \quad k = 2, 3, \ldots, n$

From Table 7, it shows that the ARIMA(0,1,0) model has the best forecasting performance among these traditional forecasting methods. Simultaneously, these methods can also enhance the prediction accuracy of the GM(1,1) model.

Table 2: *MAPE values of FGM(1,1) model with different window bases*

Window base w	2	3	4	5
MAPE	3.77	4.03	3.89	4.04

Table 3: *MAPE values of FRGM(1,1) model with different window bases*

Window base w	2	3	4	5
MAPE	4.70	3.84	3.70	3.31

Table 4: *MAPE values of FEGM(1,1) model with different window bases*

Window base w	2	3	4	5
MAPE	3.65	3.93	3.81	3.98

Table 5: *MAPE values of FREGM(1,1) model with different window bases*

Window base w	2	3	4	5
MAPE	4.16	3.35	3.25	2.96

Table 6: *Intelligent grey forecast values and errors of the petroleum demand in Taiwan from 1995 to 2010*

Year	$x^{(0)}(k)$	FGM(1,1) $w=2$ $\hat{x}^{(0)}(k)$	FRGM(1,1) $w=5$ $\hat{x}^{(0)}(k)$	FEGM(1,1) $w=2$ $\hat{x}^{(0)}(k)$	FREGM(1,1) $w=5$ $\hat{x}^{(0)}(k)$
1995	37817.42				
1996	38066.44				
1997	38765.64				
1998	40377.97				
1999	42855.42				
2000	44260.09	43776.00		43886.42	
2001	47214.77	44695.11		44818.44	
2002	46900.68	45632.22	46765.20	45768.46	46531.51
2003	48365.80	46547.59	46656.75	46683.83	46508.23
2004	49990.30	47481.69	47590.86	47617.94	47586.84
2005	50008.77	48475.01	50592.08	48624.17	50845.16
2006	50565.32	49487.83	49516.81	49649.91	49820.43
2007	52944.62	50480.47	50509.45	50642.55	50976.60
2008	50011.89	51493.43	53570.05	51655.50	52159.84
2009	49375.49	52567.21	51532.28	52742.20	51216.35
2010	51481.92	53662.15	50539.49	53850.04	50301.21
MAPE		3.77	3.31	3.65	2.96
MdAPE		3.76	3.53	3.48	3.72
sMAPE		3.79	3.31	3.66	2.97
MdRAE		1.01	0.95	0.94	0.70

Table 7: *Comparison between intelligent grey and traditional forecasting methods for the petroleum demand in Taiwan from 1995 to 2010*

Year	$x^{(0)}(k)$	GM(1,1) $\hat{x}^{(0)}(k)$	FREGM(1,1) $\hat{x}^{(0)}(k)$	Linear trend equation $\hat{x}^{(0)}(k)$	Exponential smoothing $\lambda=0.8$ $\hat{x}^{(0)}(k)$	ARIMA(0,1,0) $x^{(0)}(k)$
1995	37817.42					
1996	38066.44	40416.81		39675.25	37817.42	40198.39
1997	38765.64	41244.00		40677.16	38016.64	40447.41
1998	40377.97	42088.12		41679.07	38615.84	41146.61
1999	42855.42	42949.51		42680.98	40025.54	42758.94
2000	44260.09	43828.53		43682.89	42289.44	45236.39
2001	47214.77	44725.54		44684.80	43865.96	46641.06
2002	46900.68	45640.91	46531.51	45686.70	46545.01	49595.74
2003	48365.80	46575.02	46508.23	46688.61	46829.55	49281.65
2004	49990.30	47528.24	47586.84	47690.52	48058.55	50746.77
2005	50008.77	48500.97	50845.16	48692.43	49603.95	52371.27
2006	50565.32	49493.61	49820.43	49694.34	49927.81	52389.74
2007	52944.62	50506.57	50976.60	50696.25	50437.82	52946.29
2008	50011.89	51540.26	52159.84	51698.16	52443.26	55325.59
2009	49375.49	52595.10	51216.35	52700.07	50498.16	52392.86
2010	51481.92	53671.53	50301.21	53701.98	49600.02	51756.46
MAPE		3.88	2.96	3.54	3.37	3.35
MdAPE		4.24	3.72	3.47	3.66	2.21
sMAPE		3.86	2.97	3.79	3.43	3.26
MdRAE			0.70	0.94	0.86	0.91

From the viewpoint of the intelligent grey forecasting model, the proposed FREGM(1,1) model with $w=5$ has smaller MAPE, sMAPE, and MdRAE than the ARIMA (0,1,0) model. The MAPE and sMAPE of FREGM(1,1) model are higher by 11.64 and 8.89% respectively, compared with the ARIMA(0,1,0) model. Consequently, the FREGM(1,1) model cannot only effectively reduce forecasting errors of traditional forecasting methods but it can also enhance the forecast accuracy of the GM(1,1) model.

5.4. *Results*

Based on preset data from 1995 to 2010, the MAPE of FREGM(1,1) model is higher by 23.71, 21.48, and 11.64% respectively than the corresponding values of the GM

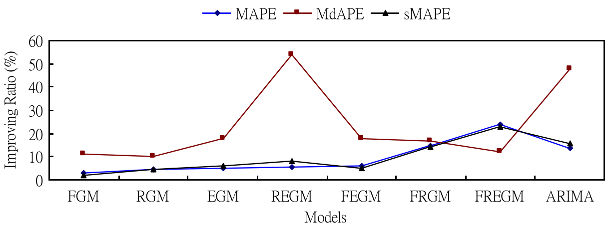

Figure 4: *The IR% comparisons of GM for petroleum demand of Taiwan.*

(1,1), FGM(1,1), and ARIMA(0,1,0) models; the sMAPE of FREGM(1,1) model is higher by 23.06, 21.63, and 8.89% respectively than the corresponding values of the GM (1,1), FGM(1,1), and ARIMA(0,1,0) models. Moreover, from the viewpoint of the MdRAE criterion, FREGM (1,1) model has the smallest MdRAE value, which indicates that the proposed model is robust among these models. These results show that the FREGM(1,1) model effectively reduces the error and enhances forecast accuracy. Figure 4 shows the improving ratios of various modified grey forecasting models compared with the traditional GM(1,1) model.

6. Conclusions

This study compared the accuracy of the grey theory and the fuzzy theory in forecasting petroleum demand for Taiwan. In recent years, many scholars have established different forecasting models to improve their forecasting abilities. However, there is still no appropriate model to adjust the background value of the grey forecasting model. Therefore, using the original GM(1,1) model combined with EWMA and the fuzzy theory, this study derived the EGM(1,1) and FEGM(1,1) models to enhance the accuracy of the grey forecasting model. It also compared the forecast results of the residual modification model REGM(1,1) and its fuzzy version, the FREGM(1,1) model.

The empirical results showed that the EGM(1,1) and FEGM(1,1) models can effectively improve accuracy of the traditional GM(1,1) model. Moreover, the accuracies of the EGM(1,1) and FEGM(1,1) models can be enhanced if residual modifications are adopted; the MAPE of the FREGM(1,1) model is higher by 23.71 and 21.48% compared with the MAPEs of the GM (1,1) and FGM (1,1) models respectively; the sMAPE of FREGM(1,1) model is higher by 23.06 and 21.63% compared with the sMAPE of the GM (1,1) and FGM(1,1) models respectively. Besides, The MAPE of FREGM(1,1) model is higher by 16.38, 12.17, and 11.64% respectively than the corresponding values of the linear trend method, exponential smoothing method, and ARIMA(0,1,0) models; the sMAPE of FREGM(1,1) model is higher by 21.63, 13.41, and 8.89% respectively than the corresponding values of the linear trend method, exponential smoothing method, and ARIMA(0,1,0) models. Therefore, Taiwan's annual petroleum demand will continue to grow in the future, and the FREGM(1,1) forecasting model can provide policy makers the most accurate information on petroleum demand planning. The further research of this approach will focus on the design, some new mechanisms for flexible cost optimizations, and control variable selections of industrial forecasting applications.

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