DSE 210: Probability and Statistics **Winter 2020**

Worksheet 1 — Sets and counting

1. (a) (0.5 points) Write down any set A of size 5.
   (b) (0.5 points) What is the formal notation for all sequences of three elements from A?
   (c) (0.5 points) How many such sequences are there, exactly?

   Solution:

   (a) A = {1, 2 , 3, 4, 5}
   (b) S = {(a, b, c) | $\forall$ a, b, c $\in$ A}
   (c) Sequence of 3 elements: $5^3 = 125$

2. (1.5 points) How many binary sequences of length 500 are there?

   Solution: 2^{500}

3. (1.5 points total, 0.5 points each) A and B are sets with $|A| = 3$ and $|B| = 4$.
   (a) What is the largest size $A \cup B$ could possibly have?
   (b) What is the smallest size $A \cup B$ could possibly have?
   (c) Repeat for $A \cap B$.

   Solution:

   (a) A and B are disjoint sets $\implies |A \cup B| = |A| + |B| = 7$
   (b) A is a subset of B $\implies |A \cup B| = |B| = 4$
   (c) Largest $|A \cap B|$ when A $\subset$ B $\implies |A \cap B| = |A| = 3$
   Smallest $|A \cap B|$ when A and B are disjoint $\implies |A \cap B| = 0$

4. (1.5 points) A donkey, an ox, a goat, and a tiger need to cross a river. They have a boat that can only hold one animal, so they need to go one at a time. How many different orderings are there?

   Solution:
   n = Number of animals
   Total possible outcomes = n! = 4! = 24

5. (1.5 points) How many sequences of 5 English characters are there?

   Solution: 26^5

6. ***Programming Question:*** (10 points) Write a Python code that calculates and returns the same answer as Question 5. You don't have to list the sequences.

   Solution:
   (Last Page)

7. (1.5 points) You have 10 good friends, and you want to choose 3 of them to accompany you on a trip. How many groups of three friends can you choose?

   Solution: Number of ways to pick k items from n items:
   $\binom{n}{k} = \dfrac{n!}{(n-k)!k!}$
   $\dfrac{10!}{7!3!} = 120$

8. (1.5 points) You have 10 different beer bottles, and you want to line 5 of them up on your mantelpiece. How many different arrangements can you make?

   Solution: Since this is a permutation problem, we can use the following formula:
   ${}^{n}P_k = \dfrac{n!}{(n-k)!} = \dfrac{10!}{5!} = 30240$

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Worksheet 2 — Probability spaces

9. (1.5 points total, 0.3 points each) Give a possible sample space Ω for each of the following experiments.

   (a) An election decides between two candidates A and B.

   (b) A two-sided coin is tossed.

   (c) A student is asked for the month and day-of-week on which her birthday falls.

   (d) A student is chosen at random from a class of ten students.

   (e) You choose the color of your new car's exterior (choices: red, black, silver, green) and interior (choices: black, beige).

   Solution:

   (a) $\Omega = \{$A, B$\}$

   (b) $\Omega = \{$H, T$\}$

   (c) Month = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
   Day-of-Week = $\{1, 2, 3, 4, 5, 6, 7\}$
   $\Omega = \{(1, 1), (1, 2), (1, 3), ... , (12, 5), (12, 6), (12, 7)\}$

   (d) $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

   (e) $\Omega = \{$(red, black), (red, beige), (black, black), (black, beige), (silver, black), (silver, beige), (green, black), (green, beige)$\}$

10. (1.5 points total, 0.5 points each) In each of the following situations, define the sample space Ω.

    (a) A fair coin is tossed 200 times in a row.

    (b) You count the number of people who enter a department store on a particular Sunday.

    (c) You open up *Hamlet* and pick a word at random.

    Solution:

    (a) $\Omega = \{\text{H, T}\}^{200}$

    (b) $\Omega = \{0, 1, 2, 3, ...\}$

    (c) $\Omega = \{$All words in Hamlet$\}$

11. (2 points total) Let A, B, and C be events defined on a particular sample space Ω. Write expressions for the following combinations of events:

    (a) (0.5 points) All three events occur.

    (b) (0.5 points) At least one of the events occurs.

    (c) (1 point) A and B occur, but not C.

Solution:

(a) $A \cap B \cap C$

(b) $A \cup B \cup C$

(c) $(A \cap B) \setminus$ C

12. (2 points total) Consider a sample space $\Omega = \{a, b, c\}$ with probabilities $\Pr(a) = 1/2$ and $\Pr(b) = 1/3$.

    (a) (0.5 points) What is $\Pr(c)$?

    (b) (0.5 points) How many distinct events can be defined on this space?

    (c) (1 point) Find the probabilities of each of these possible events.

    Solution:

    (a) Pr(c) = 1 - (Pr(a) + Pr(b)) = 1 - $\frac{1}{2}$ - $\frac{1}{3}$ = $\frac{1}{6}$

    (b) $2^3 = 8$

    (c) Pr({}) = 0, Pr({a}) = $\frac{1}{2}$, Pr({b}) = $\frac{1}{3}$, Pr({c}) = $\frac{1}{6}$, Pr({a, b}) = $\frac{5}{6}$, Pr({a, c}) = $\frac{2}{3}$, Pr({b, c}) = $\frac{1}{2}$, Pr({a, b, c}) = 1

13. (1.5 points total, 0.5 points each) A fair coin is tossed three times in succession. Describe in words each of the following events on sample space $\{H, T\}^3$.

    (a) $E_1 = \{HHH, HHT, HTH, HTT\}$

    (b) $E_2 = \{HHH, TTT\}$

    (c) $E_3 = \{HHT, HTH, THH\}$

    What are the probabilities of each of these events?

    Solution:

    (a) The first toss is heads (prob 1/2)

    (b) All tosses are identical (prob 1/4)

    (c) There are two heads (prob 3/8)

14. (1.5 points) Let A and B be events defined on a sample space Ω such that $\Pr(A \cap B) = 1/4$, $\Pr(A^c) = 1/3$, and $\Pr(B) = 1/2$. Here $A^c = \Omega \setminus A$ is the event that A *doesn't* happen. What is $\Pr(A \cup B)$?

    Solution:
    $\Pr(A \cup B)$ = Pr(A) + Pr(B) - $\Pr(A \cap B)$
    = (1 - 1/3) + 1/2 - 1/4 = 11/12

15. (2 points) A pair of dice are rolled. What is the probability that they show the same value?

    Solution:
    Sample space: $\Omega = \{(1, 1), (1, 2), (1, 3), \ldots , (6, 4), (6, 5), (6, 6)\}$
    Probabilities: For w $\in \Omega$, Pr(w) = 1/36
    Event of interest: F = $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$
    Pr(F) = 6/36 = 1/6

16. ***Programming Question:*** (10 points) Write a Python function that simulates rolling a pair of dice in Question 15. Simulate this experiment for $n \geq 1000$ trials and return the fraction that they show the same value. Is this fraction close to the probability you calculated before? (Hint: You can use random.choice([1, 2, 3, 4, 5, 6]) function to simulate a single die roll.)

    Solution:
    Last Page

17. (1.5 points) In Morse code, each letter is formed by a succession of dashes and dots. For instance, the letter S is represented by three dots and the letter O is represented by three dashes. Suppose a child types a sequence of 9 dots/dashes at random (each position is equally likely to be a dot or a dash). What is the probability that it spells out SOS?

    Solution:
    $|\Omega| = 2^9 = 512$
    Each sequence is equally likely with probability = 1/512
    Event of interest: F = {(., ., ., -, -, -, ., ., .)}
    Pr(F) = 1/512

18. (1.5 points) A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face (for instance, a six is three times as probable as a two). What is the probability of getting an even number in one throw?

    Solution:
    Faces of die: 1, 2, 3, 4, 5, 6 (Sums up to 21)
    Pr(1) = 1/21, Pr(2) = 2/21, Pr(3) = 3/21, Pr(4) = 4/21, Pr(5) = 5/21, Pr(6) = 6/21
    Pr(Getting an even number) = Pr(2) + Pr(4) + Pr(6) = 2/21 + 4/21 + 6/21 = 4/7

19. (Bonus: 4 points) A certain lottery has the following rules: you buy a ticket, choose 3 different numbers from 1 to 100, and write them on the ticket. The lottery has a box with 100 balls numbered 1 to 100. Three (different) balls are chosen. If any of the balls has one of the numbers you have chosen, you win. What is the probability of winning?

    Solution:
    The complement of this problem is: What is the probability that none of the numbers you pick match the winning numbers? You can compute this with: Pr(1st num is not winning) $\times$ Pr(2nd num is not winning) $\times$ Pr(3rd num is not winning), and then subtract from 1 to get the answer to the original problem:
    Pr(picking at least one winning number) = 1 - Pr(pick no winning numbers)
    1 - (97/100) $\times$ (96/99) $\times$ (95/98) = 0.088

20. (1.5 points) Five people of different heights are lined up against a wall in random order. What is the probability that they just happen to be in increasing order of height (left-to-right)?

    Solution:
    Number of ways for 5 people to line up, $|\Omega| = 5!$
    Number of ways for 5 people to line up in correct order, $|E| = 1$
    Pr(E) = 1/5! = 1/120

21. (1.5 points) Five people get on an elevator that stops at five floors. Assuming that each person has an equal probability of going to any one floor, find the probability that they all get off at different floors.

    Solution:
    Number of ways for 5 people to get off at 5 floors, $|\Omega| = 5^5$
    Number of ways for 5 people to all get off at different floors, $|E| = 5!$
    Then, Pr(E) = 5! / 5^5 = 24/625

22. (1.5 points) You are dealt five cards from a standard deck. What is the probability that the first four are aces and the fifth is a king?

    Solution:
    $\text{Pr(first 4 aces, 5th king)} = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times \frac{4}{48} = \frac{1}{3248700}$

23. (2 points) A barrel contains 90 good apples and 10 rotten apples. If ten of the apples are chosen at random, what is the probability that they are all good?

    Solution:
    Number of ways to choose 10 apples, $|\Omega| = \binom{100}{10}$
    Number of ways 10 apples to be good, $|E| = \binom{90}{10}$
    $\text{Pr(E)} = \frac{\binom{90}{10}}{\binom{100}{10}}$

24. (2 points) Four women check their hats at a concert, but when each woman returns after the performance, she gets a hat chosen randomly from those remaining. What is the probability that each woman gets her own hat back?

    Solution:
    Number of ways to pick hats, $|\Omega| = 4!$
    Number of ways that everyone gets their own hats, $|E| = 1$
    $\text{Pr(E)} = \frac{1}{24}$

25. (2 points) Assume that whenever a child is born, it is equally likely to be a girl or boy, independent of any earlier children. What is the probability that a randomly-chosen family with six children has exactly three girls and three boys?

    Solution:
    Using binomial theorem: Pr(3 boys and 3 girls)

$= \binom{n}{k} \times \text{p}^k \times \text{(1-p)}^{n-k}$
$= \binom{6}{3} \times 0.5^3 \times 0.5^3 = \dfrac{20}{64} = \dfrac{5}{16}$

26. (3 points total, 1 point each) Snow White asks three of the seven dwarfs, chosen at random, to accompany her on a trip.

    (a) What is the probability that Dopey is in this group?

    (b) What is the probability that both Dopey and Sneezy are in the group?

    (c) What is the probability that neither Dopey nor Sneezy are in the group?

    Solution:

    (a) Pr(Dopey is in this group) $= \dfrac{\binom{6}{2}}{\binom{7}{3}} = \dfrac{3}{7}$

    (b) Pr(Dopey and Sneezy are in this group) $= \dfrac{\binom{5}{1}}{\binom{7}{3}} = \dfrac{1}{7}$

    (c) Pr(Neither Dopey nor Sneezy are in this group) $= \dfrac{\binom{5}{3}}{\binom{7}{3}} = \dfrac{2}{7}$

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Worksheet 3 — Multiple events, conditioning, and independence

27. (3 points total, 0.6 points each) A coin is tossed three times. What is the probability that there are exactly two heads, given that:

   (a) the first outcome is a head?

   (b) the first outcome is a tail?

   (c) the first two outcomes are both heads?

   (d) the first two outcomes are both tails?

   (e) the first outcome is a head and the third outcome is a tail?

   Solution:
   the formula for conditional probability: Pr(E | F) = $\frac{Pr(E \cap F)}{Pr(F)}$

   (a) Pr(Two H | First is H) = $\frac{Pr(\{HHT, HTH\})}{Pr(\{H\})} = \frac{2/2^3}{1/2} = 1/2$

   (b) Pr(Two H | First is T) = $\frac{Pr(\{THH\})}{Pr(\{T\})} = \frac{1/2^3}{1/2} = 1/4$

   (c) Pr(Two H | First two are H) = $\frac{Pr(\{HHT\})}{Pr(\{HH\})} = \frac{1/2^3}{1/2^2} = 1/2$

   (d) Pr(Two H | First two are T) = $\frac{Pr(\{\})}{Pr(\{TT\})} = \frac{0}{1/2^2} = 0$

   (e) Pr(Two H | First is H, Third is T) = $\frac{Pr(\{HHT\})}{Pr(\{HHT, HTT\})} = \frac{1/2^3}{2/2^3} = 1/2$

28. (Bonus: 3 points total, 1.5 points each) A student must choose exactly two of the following three electives: art, French, or mathematics. The probability that he chooses art is 5/8, the probability he chooses French is 5/8, and the probability that he chooses both art and French is 1/4.

   (a) What is the probability that he chooses mathematics?

   (b) What is the probability that he chooses either art or French?

   Solution:
   Because each student must choose exactly two of three items, we can express the sample space as:
   P($A \cap F$) + P($A \cap M$) + P($F \cap M$) = 1

   (a) P(M) = 1 - P($A \cap F$) = 1 - 1/4 = 3/4

   (b) Art or French is a subset of all events in the sample space. Thus, the probability is 1.

29. (2 points) For a bill to come before the president of the United States, it must be passed by both the House of Representatives and the Senate. Assume that, of the bills presented to the two bodies, 60% pass the House, 80% pass the Senate, and 90% pass at least one of the two. Calculate the probability that the next bill presented to the two groups will come before the president.

    Solution:
    The general equation here is P($S \cap H$) = P(S) + P(H) - P($S \cup H$). All of these values are provided to us in the problem, giving a final answer of 80% + 60% - 90% = 50%

30. (Bonus: 4 points) In a fierce battle, not less than 70% of the soldiers lost one eye, not less than 75% lost one ear, not less than 80% lost one hand, and not less than 85% lost one leg. What is the minimal possible percentage of those who simultaneously lost one ear, one eye, one hand, and one leg?

    Solution:
    Since we are trying to lower-bound the intersection of all of the probabilities, we begin by assuming the lowest possible probabilities of all individual events (i.e. only 70% of soldiers lost an eye, etc.). Next, we can start lower-bounding the intersection of just the first two events:
    Pr(Lose Eye $\cup$ Lose Ear) $\leq$ 100%
    Pr(Lose Eye) + Pr(Lose Ear) - Pr(Lose Eye $\cap$ Lose Ear) $\leq$ 100%
    145% - Pr(Lose Eye $\cap$ Lose Ear) $\leq$ 100%
    Thus, the lower bound on Pr(Lose Eye $\cap$ Lose Ear) must be 45% ($\geq$ 45%). We can use this same technique recursively to lower-bound Pr((Lose Eye $\cap$ Lose Ear) $\cap$ Lose Hand) using the following equation:
    45% + 80% - Pr((Lose Eye $\cap$ Lose Ear) $\cap$ Lose Hand) $\leq$ 100%
    Applying the result of this lower bound and the same technique to the probability of losing a leg gives us the final answer of 10%.

31. (2 points) A card is drawn at random from a standard deck. What is the probability that:

    (a) it is a heart, given that it is red?
    (b) it is higher than a ten, given that it is a heart (interpret J, Q, K, A as having numeric value $11, 12, 13, 14$)?
    (c) it is a jack, given that it is higher than a 10?

    Solution:

    (a) P(Heart | Red) = $\dfrac{13/52}{1/2} = 1/2$
    (b) P(J, Q, K, A | Red) = $\dfrac{8/52}{1/2} = 4/13$
    (c) P(J | J, Q, K, A) = $\dfrac{4/52}{16/52} = 1/4$

32. (2 points) If $\Pr(B^c) = 1/4$ and $\Pr(A|B) = 1/2$, what is $\Pr(A \cap B)$?

    Solution:
    Re-arranging the formula for conditional probability gives us P(A $\cap$ B) = P(A | B) $\times$ P(B)
    = P(A | B) $\times$ (1 - P(B^c)) = (1/2) $\times$ (1 - 1/4) = 3/8

33. (4 points total, 1 point each) A die is rolled twice. What is the probability that the sum of the two rolls is > 7, given that:
    (a) the first roll is a 4?
    (b) the first roll is a 1?
    (c) the first roll is > 3?
    (d) the first roll is < 5?

    Solution:

    (a) $\text{P(Sum} > 7 \mid \text{First is 4)} = \dfrac{Pr(\{(4,4),(4,5),(4,6)\})}{Pr(\{4\})} = \dfrac{3/6^2}{1/6} = 1/2$

    (b) $\text{P(Sum} > 7 \mid \text{First is 1)} = \dfrac{Pr(\{\})}{Pr(\{1\})} = \dfrac{0}{1/6} = 0$

    (c) $\text{P(Sum} > 7 \mid \text{First is} > 3) = \dfrac{Pr(\{(4,4),(4,5),(4,6),(5,3),...,(5,6),(6,2),...,(6,6)\})}{Pr(\{4,5,6\})} = \dfrac{(3+4+5)/6^2}{3/6}$
    $= 2/3$

    (d) $\text{P(Sum} > 7 \mid \text{First is} < 5) = \dfrac{Pr(\{(2,6),(3,5),(3,6),(4,4),(4,5),(4,6)...,(6,6)\})}{Pr(\{1,2,3,4\})} = \dfrac{6/6^2}{4/6} = 1/4$

34. (4 points total, 2 points each) Two cards are drawn successively from a deck of 52 cards.
    (a) Find the probability that the second card is equal in rank to the first card. (Rank is defined according to the following ordering: $2, 3, \ldots, 10, J, Q, K, A$. The suit is irrelevant.)
    (b) Find the probability that the second card is higher in rank than the first card.

    Solution:

    (a) $\text{P(Second card is rank x} \mid \text{First card is rank x)} = \dfrac{\text{P(Second card is rank x} \cap \text{First card is rank x)}}{\text{P(First card is rank x)}}$
    $= \dfrac{4/52 \times 3/51}{1/13} = 3/51$
    Alternative: After we draw the first card, there will be exactly three cards left from the same rank. Thus, 3/51

    (b) P(ranks of both cards are different) = 1 - P(same rank) = 1 - 3/51 = 16/17
    P(Rank of the second is larger) = P(Rank of the first is larger)
    $\text{P(Rank of the second is larger)} = \dfrac{16/17}{2} = 8/17$

35. (4 points total, 2 each) A particular car manufacturer has three factories F_1, F_2, F_3 making 25%, 35%, and 40%, respectively, of its cars. Of their output, 5%, 4%, and 2%, respectively, are defective. A car is chosen at random from the manufacturer's supply.
    (a) What is the probability that the car is defective?
    (b) Given that it is defective, what is the probability that it came from factory F_1?

    Solution:
    Let D = car is defective, and F_i = car is from factory i.

(a) P(D) = P(D | F_1) P(F_1) + P(D | F_2)P(F_2) + P(D | F_3)P(F_3)
= (0.05)(0.25) + (0.04)(0.35) + (0.02)(0.40)

(b) P(F_1 | D) = $\frac{P(D|F_1)P(F_1)}{P(D)} = \frac{(0.05)(0.25)}{(0.0345)}$

36. (3 points) Suppose that there are equal numbers of men and women in the world, and that 5% of men are colorblind whereas only 1% of women are colorblind. A person is chosen at random and found to be colorblind. What is the probability that the person is male?

Solution:
Let M = male, F = female, and C = colorblind. Then,
P(M | C) = $\frac{P(C|M)P(M)}{P(C)} = \frac{P(C|M)P(M)}{P(C|M)P(M) + P(C|F)P(F)} = \frac{(0.05)(0.5)}{(0.05)(0.5) + (0.01)(0.5)}$

37. ***Programming Question:*** (10 points) Write a Python function that simulates the experiment above. Simulate this experiment for $n \geq 10000$ people and return the experimental probability P(Male | Colorblind). Check if the simulated answer matches the calculated probability. (Hint: For each person in the simulation, first simulate the gender then simulate the color blindness based on the gender.)

Solution:
Last Page

38. (Bonus: 2 points) A doctor assumes that his patients have one of the three diseases d_1, d_2, or d_3, each with probability 1/3. He carries out a test that will be positive with probability 0.8 if the patient has d_1, with probability 0.6 if the patient has d_2, and with probability 0.4 if the patient has d_3.

(a) What is the probability that the test will be positive?

(b) Suppose that the outcome of the test is positive. What probabilities should the doctor now assign to the three possible diseases?

Solution:
Let p = test is positive.

(a) P(p) = $P(p|d_1)P(d_1) + P(p|d_2)P(d_2) + P(p|d_3)P(d_3)$ = (0.8)(1/3) + (0.6)(1/3) + (0.4)(1/3)

(b) For each of the three diseases: $P(d_i|p) = \frac{P(p|d_i)P(d_i)}{P(p)}$

39. (1.5 points) One coin in a collection of 65 coins has two heads; the rest of the coins are fair. If a coin, chosen at random from the lot and then tossed, turns up heads six times in a row, what is the probability that it is the two-headed coin?

Solution:
Let A = two-headed coin and B = six heads in a row
P(A | B) = $\frac{P(B|A)P(A)}{P(B)} = \frac{(1)(1/65)}{(1)(1/65) + (64/65)(1/2)^6}$

40. (1.5 points) A scientist discovers a fossil fragment that he believes is either some kind of tiger (with probability 1/3) or mammoth (with probability 2/3). To shed further light on this question, he conducts a test which has the property that for tigers, it will come out positive with probability 5/6 whereas for

mammoths it will come out positive with probability just 1/3. Suppose the test comes out negative. What is the probability, given the outcome of the test, that the fossil comes from a tiger?

Solution:
Let T = tiger and N = test is negative.
P(T | N) = $\dfrac{P(N|T)P(T)}{P(N)} = \dfrac{(1/6)(1/3)}{(1/6)(1/3) + (2/3)(2/3)}$

41. (Bonus: 2 points) Sherlock Holmes finds paw prints at the scene of a murder, and thinks that they are either from a dog, with probability 3/4, or from a small bear, with probability 1/4. He then discovers some unusual scratches on a nearby tree. The probability that a dog would produce these scratches is 1/10, while the probability that a bear would is 3/5. What is the probability, given the presence of scratches, that the animal is a bear?

Solution:
Let B = bear and S = scratch.
P(B | S) = $\dfrac{P(S|B)P(B)}{P(S)} = \dfrac{(3/5)(1/4)}{(1/10)(3/4) + (3/5)(1/4)}$

42. (2 points total, 1 point each) A coin is tossed three times. Consider the following five events:

- A: Heads on the first toss
- B: Tails on the second toss
- C: Heads on the third toss
- D: All three outcomes the same
- E: Exactly one head

(a) Which of the following pairs of events are independent?

(1) A, B
(2) A, D
(3) A, E
(4) D, E

(b) Which of the following triples of events are independent?

(1) A, B, C
(2) A, B, D
(3) C, D, E

Solution:

(a) Item (1) is independent since the outcomes of each individual toss do not affect one another.
Item (2) is also independent, but it may be more difficult to see why. We compute P(A) = 1/2, and P(D) = $2/2^3$. We also compute P(A, D) = P((HHH)) = $1/2^3$. Thus, we see that P(A)P(D) = P(A, D) = 1/8.
Note that, events A and D are independent if and only if P(A)P(D) = P(A ∩ D)

(b) The equation for independence is also valid for multiple events. Thus, events A, B, C are independent if and only if P(A)P(B)P(C) = P(A ∩ B ∩ C). The only item that satisfies this inequality is Item (1).

43. (Bonus: 4 points total, 1 point each) You randomly shuffle a standard deck and deal two cards. Which of the following pairs of events are independent?

    (1) $A = \{\text{first card is a heart}\}, B = \{\text{second card is a heart}\}$

    (2) $A = \{\text{first card is a heart}\}, B = \{\text{first card is a 10}\}$

    (3) $A = \{\text{first card is a 10}\}, B = \{\text{second card is a 9}\}$

    (4) $A = \{\text{first card is a heart}\}, B = \{\text{second card is a 10}\}$

    Solution:
    Item (2) is independent as the drawing of the suit and face of a single card do not affect one another. We can verify this with a bit of math: P(A) = 1/4 and P(B) = 4/52. The chance of drawing the ten of hearts is simply P(A, B) = 1/52. Thus, P(A)P(B) = P(A, B) = 1/52.
    We also verify that item (4) is independent. Again, P(A) = 1/4 and P(B) = 4/52. Now, P(A, B) is the number of two-carded hands that contains a heart in the first card, and a ten in the second. The sample space is 52 × 51 since order matters. The final probability is therefore P(A,B) $= \dfrac{(12)(4) + (1)(3)}{(52)(51)} =$ 1/52. Again, P(A)P(B) = P(A, B).
    Note that, (12)(4)+(1)(3) is found by following logic: Number of hearts except 10 of hearts × Number of 10s + Number of 10 of hearts × Number of remaining 10s

44. (2 points total, 1 point each) A student applies to UCLA and UCSD. He estimates that he has a probability of 0.5 of being accepted at UCLA and a probability of 0.3 of being accepted at UCSD. He further estimates that the probability that he will be accepted by both is 0.2.

    (a) What is the probability that he is accepted at UCSD if he is accepted at UCLA?

    (b) Is the event "accepted at UCLA" independent of the event "accepted at UCSD"?

    Solution:
    Let S = accepted to UCSD and L = accepted to UCLA.

    (a) P(S|L) $= \dfrac{P(S, L)}{P(L)} = \dfrac{0.2}{0.5}$

    (b) No, because P(S|L) $\neq$ P(S).

Worksheet_Q&A

January 13, 2020

1 Worksheet 1

1.1 WS1: Problem 5

[9]:
```
# Solution
alpha = [chr(i) for i in range(97, 123)]
alpha_size = len(alpha)

ans = alpha_size**5
```

[ ]:
```
"""
Optional
"""
# Determine how many 3 letter english words are possible
# (The words need not have meaning)
# (i). With repetition of letters
# (ii). Without repetition of letters

# SOlution
alpha = [chr(i) for i in range(97, 123)]
alpha_size = len(alpha)

# (i)
ans = alpha_size*alpha_size*alpha_size

# (ii)
ans = list(itertools.permutations(alpha, 3))
```

2 Worksheet 2

2.1 WS2: Problem 16

2.1.1 A pair of dice are rolled. What is the probability that they show the same value?

[4]:
```
import random
```

```
def Simulation(n=100, match=0):
    for _ in range(n):
        output1 = random.choice([1, 2, 3, 4, 5, 6])
        output2 = random.choice([1, 2, 3, 4, 5, 6])
        if output1 == output2:
            match += 1
    print("Probability of match:\t", match/n)

Simulation(n=10000, match=0)
```

```
Probability of match:	0.1645
```

3 Worksheet 3

4 WS3: problem 37

Suppose that there are equal numbers of men and women in the world, and that 5% of men are colorblind whereas only 1% of women are colorblind. A person is chosen at random and found to be colorblind. What is the probability that the person is male? #### Simulate the above experiment and check if the simulated results match your answer.

4.0.1 Solution:

$$Pr(M) = Pr(F) = 0.5 \tag{1}$$
$$Pr(B|M) = 0.05 \tag{2}$$
$$Pr(B|F) = 0.01 \tag{3}$$
$$\tag{4}$$
$$\tag{5}$$
$$Pr(M|B) = \frac{Pr(B|M) \cdot Pr(M)}{Pr(B)} \tag{6}$$
$$Pr(M|B) = \frac{Pr(B|M) \cdot Pr(M)}{Pr(M) \cdot Pr(B|M) + Pr(F) \cdot Pr(B|F)} \tag{7}$$
$$Pr(M|B) = \frac{0.05 \cdot 0.5}{0.05 \cdot 0.5 + 0.01 \cdot 0.5} \tag{8}$$
$$Pr(M|B) = \frac{5}{6} = 0.833 \tag{9}$$

[7]:
```
import numpy as np

M_given_B = 0
blind = 0
for _ in range(1000):
    sex = np.random.choice(['M', 'F'])
    if sex == 'M':
        check = np.random.choice(['B', 'NB'], p = [0.05, 0.95])
```

```python
    else:
        check = np.random.choice(['B', 'NB'], p = [0.01, 0.99])
    if sex == 'M' and check == 'B':
        M_given_B += 1
    if check == 'B':
        blind += 1
print("Pr(M|B) = ",M_given_B/blind)
```

```
Pr(M|B) =  0.7142857142857143
```