Public debt and pensions in an overlapping generations model of a small open economy

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Contents

1 Introduction

- In Steigum (2010) we are looking at
  - Long term overlapping generations model.
  - Small open economy.
- Economic question that we want to analyze:
  - How do (1) public debt (e.g. due to temporary tax changes) and (2) the establishment of (or change in) a public old-age pension system *affect* the national wealth and the welfare of different generations?
- About the model
  - Extenstion of the overlapping generations models to a small open economy $\Rightarrow$ real interest rate is determined in the international capital market (exogenous).
  - Each generation is economically active in two periods.
    - First period: Working and accumulating wealth.
    - Second period: Not working and consuming all wealth.
  - For each period, two living overlapping generations.
  - Long run-growth model with production factors fully used $\Rightarrow$ Business cycles disregarded and money is neutral.

2 The model

2.1 Production

Solow part

Output Y_t is given by a standard production function:

$$Y_t = F(L_t, K_t) \text{ with } F_L > 0, F_{LL} < 0, F_K > 0, F_{KK} < 0 \text{ and } F_{LK} < 0 \tag{1}$$

Where L_t is the number of workers growing in each period at a rate of n. K_t is the capital stock.

Production function is homogenous of degree one. This implies that we can write

$$\frac{Y_t}{L_t} = F(1, \frac{K_t}{L_t}) \tag{2}$$

So if we define $y_t = \frac{Y_t}{L_t}$ and $k_t = \frac{K_t}{L_t}$, the production function on its intensive form can then be stated as

$$y_t = F(1, \frac{K_t}{L_t}) = f(k_t) \text{ with } f'(k_t) > 0 \text{ and } f''(k_t) < 0 \tag{3}$$

Marginal product of capital:

$$\frac{\partial Y_t}{\partial K_t} = \frac{\partial (f(k_t) L_t)}{\partial K_t} = f'(k_t) \tag{4}$$

Marginal product of labor:

$$\frac{\partial Y_t}{\partial L_t} = \frac{\partial(f(k_t)L_t)}{\partial L_t} = f(k_t) - f'(k_t)k_t \Rightarrow w_t = w(k_t) \tag{5}$$

Gross investment is formed according to

$$I_t = (K_{t+1} - K_t) + \delta K_t \tag{6}$$

In which δ is the depreciation rate of capital.

SOE part

Some simplifying assumptions made about the SOE

- Output can be traded internationally at prices determined by the world market (i.e. we have full purchasing power parity).
- Omit public consumption and public fixed investment. Real GDP is therefore

  $$Y_t = C_t + I_t + X_t \tag{7}$$

  Where C_t is the sum of consumption of the two overlapping generations and X_t is net export (measured in units of goods).
- Foreign real interest rate r^* is exogenous and constant over time
- Assume that $r > n$.
- Full interest parity domestic real interest rate $\Rightarrow r = r^*$.
- Perfect competition in the factor markets $\Rightarrow$
  - $f'(k) = r + \delta$ - capital intensity is constant
  - $w = w(k)$ - real wage level is constant
  - $I_t = (n + \delta)K_t$ - Gross investment is a constant share of domestic capital. [1]

2.2 National wealth

National wealth Ω_t^n is the sum of domestic capital stock K_t and net foreign assets K_t^*:

$$\Omega_t^n = K_t + K_t^* \tag{10}$$

[1] We find this expression using first that due to perfect capital markets

$$\frac{K_{t+1}}{L_{t+1}} = \frac{K_t}{L_t} \Rightarrow \frac{K_{t+1}}{K_t} = \frac{L_{t+1}}{L_t} \Rightarrow \frac{K_{t+1}}{K_t} - 1 = \frac{L_{t+1}}{L_t} - 1 \tag{8}$$

So we can write

$$\frac{\Delta K_{t+1}}{K_t} = \frac{\Delta L_{t+1}}{L_t} = n \tag{9}$$

Inserting for $\frac{\Delta K_{t+1}}{K_t}$ in the capital formation equation (6) then gives the gross investment equation stated above

The public sector can possess financial wealth or be indebted. Thus, national wealth Ω_t^n can also be expressed as the sum of private wealth Ω_t^p and government wealth Ω_t^g:

$$\Omega_t^n = \Omega_t^p + \Omega_t^g \tag{11}$$

We further assume that

- Domestic capital of the country is owned by private households.

2.3 Households

Households optimization problem under the given pension system

At the beginning of each period, a new generation is entering the economy. All generations live for two periods and have the same consumption preferences over time. We can describe this by the intertemporal utility function

$$U_t = U(c_{1,t}, c_{2,t+1}) \tag{12}$$

We assume that

- Inheritance is disregarded.
- Labor supply inelastic and each household supply one unit of labor in the first period of their life.
- The number of workers in generation t is L_t

The pension system is designed such that each household in the first period of life pays a proportional tax $w\tau_t$, where τ_t is the tax rate in period t. In the next period, the household receives $\theta_t w$ from the government, where θ_t is the pension rate received in period $t+1$ with full certainty based on the share of earlier wage income. Due to this, the intertemporal budget constrain of the consumers is defined as

$$c_{1,t} + \frac{c_{2,t+1}}{1+r} = (1-\tau_t)w + \frac{\theta_t w}{1+r} \equiv b_t \tag{13}$$

Maximizing (12) subject to (13) will generate the two demand functions[2]

$$c_{1,t} = c_1(r, \overset{+}{b_t}) \tag{14}$$

$$c_{2,t+1} = c_2(r, \overset{+}{b_t}) \tag{15}$$

Initial wealth for each generation is equal to zero, consequently in the first period of the life-cycle, the household accumulates wealth a_{t+1} that is equal to saving:

$$a_{t+1} = (1-\tau_t)w - c_1(r, b_t) \tag{16}$$

The effect on saving by an increasing in τ_t and θ_t is

$$\frac{\partial a_{t+1}}{\partial \tau_t} = -(1 - \frac{\partial c_1}{\partial b_t})w < 0 \tag{17}$$

[2] We assume here the two goods to be normal. Note also the functional forms of $c_1()$ and $c_2()$ are independent of time since all generations have the same preferences

$$\frac{\partial a_{t+1}}{\partial \theta_t} = -\frac{1}{1+r}\frac{\partial c_1}{\partial b_t} w < 0 \tag{18}$$

Since all wealth is consumed in the final period, total wealth of the private sector Ω^P_{t+1} at the end of period t is equal to the accumulated wealth of generation t:

$$\Omega^P_{t+1} = L_t a_{t+1} \tag{19}$$

Aggregate consumption and foreign current account

Aggregate private consumption in period t is equal to consumption for the pensioners (generation $t-1$) and the workers (generation t)

$$C_t = L_{t-1}c_{2,t} + L_t c_{1,t} \tag{20}$$

Substituting for the two demand functions from (14) and (15) give

$$C_t = L_{t-1}c_2\left(r, (1-\tau_{t-1})w + \frac{\theta_{t-1}w}{1+r}\right) + L_t c_1\left(r, (1-\tau_t)w + \frac{\theta_t w}{1+r}\right) \tag{21}$$

This shows the impact on total consumption of tax- and pension rates that affects the budgets of the two generations.

Net export is residually determined as

$$X_t = F(L_t, K_t) - (n+\delta)K_t - C_t \tag{22}$$

A change in fiscal policy that changes C_t has an impact on net exports with the opposite sign. Since we have that the increase in the net foreign assets ΔK^*_t is given by

$$\Delta K^*_{t+1} = X_t + rK^*_t \tag{23}$$

This also implies that changes in domestic consumption or saving also have an impact on net foreign assets.

2.4 The public sector

The change in net financial wealth of the public sector can be expressed as

$$\Omega^g_{t+1} - \Omega^g_t = r\Omega^g_t + \tau_t w L_t - \theta_{t-1} w L_{t-1} \tag{24}$$

Dividing this by the number of workers L_t

$$\frac{\Omega^g_{t+1}}{L_t} = \frac{\Omega^g_t}{L_t}(1+r) + \tau_t w - \theta_{t-1} w \frac{L_{t-1}}{L_t} \tag{25}$$

If we define $\omega^g_t = \frac{\Omega^g_t}{L_t}$, and using the relation $L_t = (1+n)L_{t-1}$, we can express this in terms of net government wealth per worker:

$$(1+n)\omega^g_{t+1} = (1+r)\omega^g_t + \left(\tau_t - \frac{\theta_{t-1}}{(1+n)}\right) w \tag{26}$$

National wealth per worker ω^n_{t+1} is owned by public and private wealth:

$$\omega^n_t = \omega^g_t + \omega^p_t \tag{27}$$

Which again is equal to the sum of domestic capital and net foreign wealth per worker

$$\omega^n_t = k_t + k^*_t \tag{28}$$

3 Steady state

Properties

In the steady state, population grows by n while capital intensity k and output per worker y remain constant. Further, for the SOE, the steady state also requires that the growth rate of net foreign assets is n. Since we have that

$$\Delta K^*_{t+1} = X_t + rK^*_t \tag{29}$$

Divide this by K^*_t

$$\frac{\Delta K^*_{t+1}}{K^*_t} = \frac{X_t}{K^*_t} + r \tag{30}$$

By setting the growth rate of net foreign asset equal to n, we find that net exports divided by foreign assets can be expressed as

$$\frac{X_t}{K^*_t} = n - r \tag{31}$$

Dividing this by L_t and denoting net export per worker by x, we obtain

$$x = (n - r)k^* \tag{32}$$

So in steady state net export per worker must have the opposite sign as net foreign wealth per worker

Realized steady-state growth path for fiscal policy

To realise a steady-state growth path, fiscal policy must be designed (i.e. select τ in relation to θ) so that ω^g will be constant over time.

The public budget condition (26) is in steady state then given as

$$(r - n)\omega^g = \left(\frac{\theta}{(1+n)} - \tau\right) w \tag{33}$$

So the higher the steady-state-value of ω^g, the lower can the tax rate τ be set in relation to θ. The reason is that a positive ω^g gives the government a permanent wealth income.

Note further that when ω^g, τ and θ are selected so that (33) is satisfied, this implies that b for each household is constant. Consequently, individual consumption, saving, private wealth and national wealth are also constant.

4 Government policies

4.1 Temporary tax cut that increases government debt

We want to analyse the effects of a temporary tax cut that increases government debt to a new permanent level.

- The situation
  - Start in steady state equilibrium in periode $t = 0$.

- τ_0 and θ_0 is set such that $\omega_0^g = 0$.
- The pension rate rate is held constant, $\theta = \theta_0$.
- Government Policy: $\tau_1 < \tau_0 < \tau_2 = \tau^{NSS}$ and $\omega_2^g = \omega^g < 0$ $\Rightarrow |\Delta\tau_2| > |\Delta\tau_1|$.

- Welfare of different generations
  - Since $b_1 > b_2 = b^{NSS}$ implies higher consumption in both periods of the life-cycle for generation 1, while lower consumption for all later generations.

- National wealth formation over time
  - Period 1
    For *generation one* the tax reduction is spent on both increased consumption and saving. Since the saving is less than the increase in public debt, national wealth goes down.
  - Period 2
    For *generation two* the tax increase is spent on both decreased consumption and saving. At the same time *generation one* has a higher consumption than in the old steady state. In total, since $|\Delta\tau_2| > |\Delta\tau_1|$ there is a further decrease in national wealth.
  - New steady state
    New Steady state (NSS) solution for national wealth per worker affected by the change in public wealth

    Start by differentiate equation (33) for changes in $d\omega^g$ induced by changes in $d\tau$

$$d\tau = -\frac{(r-n)}{w} d\omega^g \tag{34}$$

Inserting for (16) and (19) in (27)

$$\omega^n = \omega^g + \frac{1}{1+n}\Big((1-\tau)w - c_1(r, (1-\tau)w + \frac{\theta w}{1+r})\Big) \tag{35}$$

From (35) the effect of $d\omega^n$ on $d\tau^g$

$$d\omega^n = d\omega^g + \frac{1}{1+n}\Big(-wd\tau + \frac{\partial c_1}{\partial b} wd\tau\Big) \tag{36}$$

Inserting from (34) gives

$$d\omega^n = d\omega^g - \frac{1}{1+n}\Big(-w + \frac{\partial c_1}{\partial b} w\Big)\frac{(r-n)}{w} d\omega^g \tag{37}$$

$$\frac{d\omega^n}{d\omega^g} = 1 + \frac{(r-n)}{(1+n)}\Big(1 - \frac{\partial c_1}{\partial b}\Big) \tag{38}$$

National wealth in steady state will fall even more than the increase in government debt. This is due to that the tax increase that compensates for a decline in public wealth income leads to smaller life-time savings for households in the new steady state.

4.2 A funded public pension

We want to consider a fiscal policy that increases the tax and pension rates in parallel fashion leaving the budget constrain b unchanged. From (13) we obtain

$$d\tau_t = \frac{w}{1+r} d\theta_t \tag{39}$$

- National wealth formation over time
  - Period 1
    Households reduce private saving and government wealth increases (e.g. du to the establishment of a public pension fund). National wealth and consumption remains unchanged, so no real effects. In short, we get a Richardian equivalence result.

4.3 Pay-as-you-go system for public old-age pension

Many countries, including Norway, have a system where pensions are financed on a current basis from tax revenues and do not involve accumulation of public wealth. In its pure form, this is called a pay-as-you-go system (PAYG-system).

- The situation
  - Start in steady state equilibrium in periode $t = 0$.
  - $\tau_0 = \theta_0 = 0$.
  - Government policy: $\Delta\tau_1 > 0$ with $d\theta = (1+n)d\tau \Rightarrow d\omega_t^g = 0$.
- Welfare of different generations
  - Period 1
    Generation 0 gets an unexpected additional income of $w_1 L_1 \Delta\tau$ which is entirely spent on consumption.
  - Period 2 and onwards
    Generation 1 and onwards get a negative change in their life cycle budget constrain of $db = -wd\tau + \frac{w}{1+r}d\theta = -wd\tau + \frac{1+n}{1+r}wd\tau = -\frac{(r-n)}{1+r}wd\tau$. Which implies a reduction in consumption.
  - In total, generation 0 earn and generation 1 and onwards they loose from the introduction of PAYG-system.