Public debt and pensions in an overlapping generations model of a small open economy

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Contents

1 Introduction

- In Steigum (2010) we are looking at
  - Long term overlapping generations model.
  - Small open economy.
- Economic question that we want to analyze:
  - How do (1) public debt (e.g. due to temporary tax changes) and (2) the establishment of (or change in) a public old-age pension system *affect* the national wealth and the welfare of different generations?
- About the model
  - Extenstion of the overlapping generations models to a small open economy $\Rightarrow$ real interest rate is determined in the international capital market (exogenous).
  - Each generation is economically active in two periods.
    * First period: Working and accumulating wealth.
    * Second period: Not working and consuming all wealth.
  - For each period, two living overlapping generations.
  - Long run-growth model with production factors fully used $\Rightarrow$ Business cycles disregarded and money is neutral.

2 The model

2.1 Production

Solow part

Output Y_t is given by a standard production function:

$$Y_t = F(L_t, K_t) \text{ with } F_L > 0, F_{LL} < 0, F_K > 0, F_{KK} < 0 \text{ and } F_{LK} < 0 \tag{1}$$

Where L_t is the number of workers growing in each period at a rate of n. K_t is the capital stock.

Production function is homogenous of degree one. This implies that we can write

$$\frac{Y_t}{L_t} = F(1, \frac{K_t}{L_t}) \tag{2}$$

So if we define $y_t = \frac{Y_t}{L_t}$ and $k_t = \frac{K_t}{L_t}$, the production function on its intensive form can then be stated as

$$y_t = F(1, \frac{K_t}{L_t}) = f(k_t) \text{ with } f'(k_t) > 0 \text{ and } f''(k_t) < 0 \tag{3}$$

Marginal product of capital:

$$\frac{\partial Y_t}{\partial K_t} = \frac{\partial (f(k_t) L_t)}{\partial K_t} = f'(k_t) \tag{4}$$

Marginal product of labor:

$$\frac{\partial Y_t}{\partial L_t} = \frac{\partial (f(k_t)L_t)}{\partial L_t} = f(k_t) - f'(k_t)k_t \Rightarrow w_t = w(k_t) \tag{5}$$

Gross investment is formed according to

$$I_t = (K_{t+1} - K_t) + \delta K_t \tag{6}$$

In which δ is the depreciation rate of capital.

SOE part

Some simplifying assumptions made about the SOE

- Output can be traded internationally at prices determined by the world market (i.e. we have full purchasing power parity).
- Omit public consumption and public fixed investment. Real GDP is therefore
$$Y_t = C_t + I_t + X_t \tag{7}$$
Where C_t is the sum of consumption of the two overlapping generations and X_t is net export (measured in units of goods).
- Foreign real interest rate r^* is exogenous and constant over time
- Assume that $r > n$.
- Full interest parity domestic real interest rate $\Rightarrow r = r^*$.
- Perfect competition in the factor markets $\Rightarrow$
  - $f'(k) = r + \delta$ - capital intensity is constant
  - $w = w(k)$ - real wage level is constant
  - $I_t = (n + \delta)K_t$ - Gross investment is a constant share of domestic capital. [1]

2.2 National wealth

National wealth Ω_t^n is the sum of domestic capital stock K_t and net foreign assets K_t^*:

$$\Omega_t^n = K_t + K_t^* \tag{10}$$

[1]We find this expression using first that due to perfect capital markets

$$\frac{K_{t+1}}{L_{t+1}} = \frac{K_t}{L_t} \Rightarrow \frac{K_{t+1}}{K_t} = \frac{L_{t+1}}{L_t} \Rightarrow \frac{K_{t+1}}{K_t} - 1 = \frac{L_{t+1}}{L_t} - 1 \tag{8}$$

So we can write

$$\frac{\Delta K_{t+1}}{K_t} = \frac{\Delta L_{t+1}}{L_t} = n \tag{9}$$

Inserting for $\frac{\Delta K_{t+1}}{K_t}$ in the capital formation equation (6) then gives the gross investment equation stated above

The public sector can possess financial wealth or be indebted. Thus, national wealth Ω_t^n can also be expressed as the sum of private wealth Ω_t^p and government wealth Ω_t^g:

$$\Omega_t^n = \Omega_t^p + \Omega_t^g \tag{11}$$

We further assume that

- Domestic capital of the country is owned by private households.

2.3 Households

Households optimization problem under the given pension system

At the beginning of each period, a new generation is entering the economy. All generations live for two periods and have the same consumption preferences over time. We can describe this by the intertemporal utility function

$$U_t = U(c_{1,t}, c_{2,t+1}) \tag{12}$$

We assume that

- Inheritance is disregarded.
- Labor supply inelastic and each household supply one unit of labor in the first period of their life.
- The number of workers in generation t is L_t

The pension system is designed such that each household in the first period of life pays a proportional tax $w\tau_t$, where τ_t is the tax rate in period t. In the next period, the household receives $\theta_t w$ from the government, where θ_t is the pension rate received in period $t+1$ with full certainty based on the share of earlier wage income. Due to this, the intertemporal budget constrain of the consumers is defined as

$$c_{1,t} + \frac{c_{2,t+1}}{1+r} = (1-\tau_t)w + \frac{\theta_t w}{1+r} \equiv b_t \tag{13}$$

Maximizing (12) subject to (13) will generate the two demand functions[2]

$$c_{1,t} = c_1(r, \overset{+}{b_t}) \tag{14}$$

$$c_{2,t+1} = c_2(r, \overset{+}{b_t}) \tag{15}$$

Initial wealth for each generation is equal to zero, consequently in the first period of the life-cycle, the household accumulates wealth a_{t+1} that is equal to saving:

$$a_{t+1} = (1-\tau_t)w - c_1(r, b_t) \tag{16}$$

The effect on saving by an increasing in τ_t and θ_t is

$$\frac{\partial a_{t+1}}{\partial \tau_t} = -(1 - \frac{\partial c_1}{\partial b_t})w < 0 \tag{17}$$

[2]We assume here the two goods to be normal. Note also the functional forms of $c_1()$ and $c_2()$ are independent of time since all generations have the same preferences

$$\frac{\partial a_{t+1}}{\partial \theta_t} = -\frac{1}{1+r}\frac{\partial c_1}{\partial b_t} w < 0 \tag{18}$$

Since all wealth is consumed in the final period, total wealth of the private sector Ω^P_{t+1} at the end of period t is equal to the accumulated wealth of generation t:

$$\Omega^P_{t+1} = L_t a_{t+1} \tag{19}$$

Aggregate consumption and foreign current account

Aggregate private consumption in period t is equal to consumption for the pensioners (generation $t-1$) and the workers (generation t)

$$C_t = L_{t-1} c_{2,t} + L_t c_{1,t} \tag{20}$$

Substituting for the two demand functions from (14) and (15) give

$$C_t = L_{t-1} c_2 \left(r, (1-\tau_{t-1})w + \frac{\theta_{t-1} w}{1+r} \right) + L_t c_1 \left(r, (1-\tau_t)w + \frac{\theta_t w}{1+r} \right) \tag{21}$$

This shows the impact on total consumption of tax- and pension rates that affects the budgets of the two generations.

Net export is residually determined as

$$X_t = F(L_t, K_t) - (n+\delta)K_t - C_t \tag{22}$$

A change in fiscal policy that changes C_t has an impact on net exports with the opposite sign. Since we have that the increase in the net foreign assets ΔK^*_t is given by

$$\Delta K^*_{t+1} = X_t + r K^*_t \tag{23}$$

This also implies that changes in domestic consumption or saving also have an impact on net foreign assets.

2.4 The public sector

The change in net financial wealth of the public sector can be expressed as

$$\Omega^g_{t+1} - \Omega^g_t = r\Omega^g_t + \tau_t w L_t - \theta_{t-1} w L_{t-1} \tag{24}$$

Dividing this by the number of workers L_t

$$\frac{\Omega^g_{t+1}}{L_t} = \frac{\Omega^g_t}{L_t}(1+r) + \tau_t w - \theta_{t-1} w \frac{L_{t-1}}{L_t} \tag{25}$$

If we define $\omega^g_t = \frac{\Omega^g_t}{L_t}$, and using the relation $L_t = (1+n)L_{t-1}$, we can express this in terms of net government wealth per worker:

$$(1+n)\omega^g_{t+1} = (1+r)\omega^g_t + \left(\tau_t - \frac{\theta_{t-1}}{(1+n)} \right) w \tag{26}$$

National wealth per worker ω^n_{t+1} is owned by public and private wealth:

$$\omega^n_t = \omega^g_t + \omega^p_t \tag{27}$$

Which again is equal to the sum of domestic capital and net foreign wealth per worker

$$\omega^n_t = k_t + k^*_t \tag{28}$$

3 Steady state

Properties

In the steady state, population grows by n while capital intensity k and output per worker y remain constant. Further, for the SOE, the steady state also requires that the growth rate of net foreign assets is n. Since we have that

$$\Delta K^*_{t+1} = X_t + rK^*_t \tag{29}$$

Divide this by K^*_t

$$\frac{\Delta K^*_{t+1}}{K^*_t} = \frac{X_t}{K^*_t} + r \tag{30}$$

By setting the growth rate of net foreign asset equal to n, we find that net exports divided by foreign assets can be expressed as

$$\frac{X_t}{K^*_t} = n - r \tag{31}$$

Dividing this by L_t and denoting net export per worker by x, we obtain

$$x = (n - r)k^* \tag{32}$$

So in steady state net export per worker must have the opposite sign as net foreign wealth per worker

Realized steady-state growth path for fiscal policy

To realise a steady-state growth path, fiscal policy must be designed (i.e. select τ in relation to θ) so that ω^g will be constant over time.

The public budget condition (26) is in steady state then given as

$$(r - n)\omega^g = \left(\frac{\theta}{(1+n)} - \tau\right) w \tag{33}$$

So the higher the steady-state-value of ω^g, the lower can the tax rate τ be set in relation to θ. The reason is that a positive ω^g gives the government a permanent wealth income.

Note further that when ω^g, τ and θ are selected so that (33) is satisfied, this implies that b for each household is constant. Consequently, individual consumption, saving, private wealth and national wealth are also constant.

4 Government policies

4.1 Temporary tax cut that increases government debt

We want to analyse the effects of a temporary tax cut that increases government debt to a new permanent level.

- The situation
  - Start in steady state equilibrium in periode $t = 0$.

- τ_0 and θ_0 is set such that $\omega_0^g = 0$.
- The pension rate rate is held constant, $\theta = \theta_0$.
- Government Policy: $\tau_1 < \tau_0 < \tau_2 = \tau^{NSS}$ and $\omega_2^g = \omega^g < 0$ $\Rightarrow |\Delta\tau_2| > |\Delta\tau_1|$.

- Welfare of different generations
  - Since $b_1 > b_2 = b^{NSS}$ implies higher consumption in both periods of the life-cycle for generation 1, while lower consumption for all later generations.

- National wealth formation over time
  - Period 1
    For *generation one* the tax reduction is spent on both increased consumption and saving. Since the saving is less than the increase in public debt, national wealth goes down.
  - Period 2
    For *generation two* the tax increase is spent on both decreased consumption and saving. At the same time *generation one* has a higher consumption than in the old steady state. In total, since $|\Delta\tau_2| > |\Delta\tau_1|$ there is a further decrease in national wealth.
  - New steady state
    New Steady state (NSS) solution for national wealth per worker affected by the change in public wealth

    Start by differentiate equation (33) for changes in $d\omega^g$ induced by changes in $d\tau$

$$d\tau = -\frac{(r-n)}{w} d\omega^g \tag{34}$$

Inserting for (16) and (19) in (27)

$$\omega^n = \omega^g + \frac{1}{1+n}\Big((1-\tau)w - c_1(r, (1-\tau)w + \frac{\theta w}{1+r})\Big) \tag{35}$$

From (35) the effect of $d\omega^n$ on $d\tau^g$

$$d\omega^n = d\omega^g + \frac{1}{1+n}\Big(-wd\tau + \frac{\partial c_1}{\partial b} wd\tau\Big) \tag{36}$$

Inserting from (34) gives

$$d\omega^n = d\omega^g - \frac{1}{1+n}\Big(-w + \frac{\partial c_1}{\partial b} w\Big)\frac{(r-n)}{w} d\omega^g \tag{37}$$

$$\frac{d\omega^n}{d\omega^g} = 1 + \frac{(r-n)}{(1+n)}\Big(1 - \frac{\partial c_1}{\partial b}\Big) \tag{38}$$

National wealth in steady state will fall even more than the increase in government debt. This is due to that the tax increase that compensates for a decline in public wealth income leads to smaller life-time savings for households in the new steady state.

4.2 A funded public pension

We want to consider a fiscal policy that increases the tax and pension rates in parallel fashion leaving the budget constrain b unchanged. From (13) we obtain

$$d\tau_t = \frac{w}{1+r} d\theta_t \tag{39}$$

- National wealth formation over time
  - Period 1
    Households reduce private saving and government wealth increases (e.g. du to the establishment of a public pension fund). National wealth and consumption remains unchanged, so no real effects. In short, we get a Richardian equivalence result.

4.3 Pay-as-you-go system for public old-age pension

Many countries, including Norway, have a system where pensions are financed on a current basis from tax revenues and do not involve accumulation of public wealth. In its pure form, this is called a pay-as-you-go system (PAYG-system).

- The situation
  - Start in steady state equilibrium in periode $t = 0$.
  - $\tau_0 = \theta_0 = 0$.
  - Government policy: $\Delta\tau_1 > 0$ with $d\theta = (1+n)d\tau \Rightarrow d\omega_t^g = 0$.
- Welfare of different generations
  - Period 1
    Generation 0 gets an unexpected additional income of $w_1 L_1 \Delta\tau$ which is entirely spent on consumption.
  - Period 2 and onwards
    Generation 1 and onwards get a negative change in their life cycle budget constrain of $db = -wd\tau + \frac{w}{1+r}d\theta = -wd\tau + \frac{1+n}{1+r}wd\tau = -\frac{(r-n)}{1+r}wd\tau$. Which implies a reduction in consumption.
  - In total, generation 0 earn and generation 1 and onwards they loose from the introduction of PAYG-system.

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PUBLIC DEBT AND PENSIONS IN AN OVERLAPPING GENERATIONS MODEL OF A SMALL OPEN ECONOMY

By Erling Steigum

1. Introduction

In this note, a long-term overlapping generations model of a small open economy is used to analyse how public debt and the establishment of (or change in) a public old-age pension system affect national wealth and the welfare of different generations.

The overlapping generations model was originally formulated for closed economies, where the life-cycle savings of the households were invested in domestic capital. Here we study a small open economy where the real interest is determined on the international capital market and is included as an exogenous variable in the model. Analytically, this is a simpler model than the closed one, at the same time as we succeed in obtaining several of the qualitative conclusions from the closed model in regard to how fiscal policy affects national wealth and intergenerational welfare. In to-days world economy, many small countries (Norway in particular) accumulate sizeable stocks of net foreign assets.

The length of the time period is chosen so that each generation is economically active in two periods only. In the first period of an individual's adult life, she is working and accumulates wealth and in the second period, all wealth and wealth income is consumed. In each period, there are thus two living overlapping adult generations, where the youngest represents the labour force in the economy. The model is a long-run growth model where the production factors are fully used. Business cycles in employment and GDP are disregarded and money is neutral.

2. Production and income

The production- and real investment side in the model is the same as that in the Solow model. The country's real GDP in period t (Y_t) is generated by a standard production function $Y_t = F(L_t, K_t)$, where the number of workers (L_t) grows at the rate n per period. The number of workers is equal to the number of young households. K_t is the domestic capital stock at the beginning of period t. The share δ of K_t depreciates in period t. The production function exhibits constant economies of scale. Therefore, it can be expressed as $y_t = f(k_t), f'>0, f''<0$. Here, $y_t = Y_t / L_t$ is labour productivity and $k_t = K_t / L_t$ constitutes capital intensity. From the Solow model it is known that $f'(k_t)$ equals the marginal product of capital. Moreover, the marginal product of the labour (the real wage) can be expressed as $f(k_t) - k_t f'(k_t)$, i.e. as a function of k_t only. This function is written as $w_t = w(k_t)$, and is increasing in k_t. Output can be traded internationally at a price determined on the world market, i.e. there is full purchasing power parity. Net export is X_t (measured in units of goods). Real GDP is therefore

$$Y_t = C_t + I_t + X_t \tag{1}$$

In (1), C_t is the sum of consumption of the two overlapping generations, and $I_t = (K_{t+1} - K_t) + \delta K_t$ is gross capital formation. To simplify the presentation, we omit public consumption and public fixed investment. There is full interest rate parity. Therefore, the domestic and foreign real rate of interest are equal. We assume that the real interest rate (r) is exogenous and constant over time, and that $r > n$. There is perfect competition in factor markets. The first-order condition $f'(k_t) = r + \delta$ determines the optimal capital intensity (k^*) corresponding to the same rate of return from domestic and foreign investment. Since the real wage is a function of capital intensity, the real wage will also be constant over time: $w = w(k^*)$. As k^* is constant over time, the growth rate of the domestic capital stock K_t must by n, the same as the growth rate of the number of workers. Then, gross investment will equal $I_t = (n+\delta)K_t$, i.e. gross investment is a constant share of domestic capital.

Going back to equation (1), we will see that any policy-induced change in private consumption must be reflected in a corresponding change in net exports (X_t) with the opposite sign. For the real economy, this means that an increase in savings leads to an accumulation of foreign claims (K_t^*) through an export surplus that creates a surplus in the foreign current account, i.e. an increase in net foreign assets during period t (ΔK_t^*):

$$\Delta K_t^* = X_t + rK_t^* \tag{2}$$

This will, in turn, increase wealth income and national income (Y_t^n) in addition to net domestic product $(Y_t - \delta K_t)$:

$$Y_t^n = (Y_t - \delta K_t) + rK_t^* \tag{3}$$

3. National wealth

National wealth (Ω_t^n) is the sum of the domestic capital stock and net foreign assets:

$$\Omega_t^n = K_t + K_t^* \tag{4}$$

The public sector can possess financial wealth or be indebted. Thus, national wealth can also be expressed as the sum of private (Ω_t^p) and government wealth (Ω_t^g):

$$\Omega_t^n = \Omega_t^p + \Omega_t^g \tag{5}$$

where Ω_t^p is the wealth of the private sector and Ω_t^g is the net wealth of the public sector at the beginning of period t (a negative Ω_t^g represents net public debt). We assume that the domestic capital of the country is owned by private households. Who holds the foreign debt/the foreign claims is of no importance in this model since domestic and foreign assets are perfect substitutes. To be concrete, we can imagine that the foreign assets are held by the private sector and that the public sector only has claims or debt against domestic households.

4. Households

Let us now analyse household behaviour. A new generation is entering the economy at the beginning of each period. The generation that is young in period *t* will be called generation *t*. All consumers live for 2 periods and have the same consumption preferences over time. These are described by the intertemporal utility function

$$U_t = U(c_{1,t}, c_{2,t+1}) \tag{6}$$

where $c_{1,t}$ and $c_{2,t+1}$ are consumption for a representative member of generation *t* in period *t* and period *t* +1, respectively. Inheritance is disregarded.

Labour supply is completely inelastic and we do not allow for the possibility of choosing between leisure and private consumption. Each housenhold/worker supplies one unit of labour in the first period of her life. The number of workers in generation *t* is thus L_t. Each receives a wage income (measured in units of goods) amounting to *w* and pays a proportional tax $w\tau_t$, where τ_t is the tax rate in period *t*. In the next period, the individual receives a pension payment from the government of $\theta_t w$, where θ_t is pension received in period *t* + 1 as the share of earlier wage income. The pension payment is known with full certainty. We can now define the consumer's intertemporal budget constraint

$$c_{1,t} + \frac{c_{2,t+1}}{1+r} = b_t \tag{7}$$

where the budget b_t is:

$$b_t = (1-\tau_t)w + \frac{\theta_t w}{1+r} \tag{8}$$

As shown by (8), the present value of consumption cannot exceed after-tax wage income plus the discounted pension payment from the government. Maximising (7) subject to (8) generates the demand functions

$$c_{1,t} = c_1(r, b_t) \tag{9a}$$

$$c_{2,t+1} = c_2(r, b_t) \tag{9b}$$

Note that the actual functional forms $c_1(.,.)$ and $c_2(.,.)$ are independent of time since all generations have the same preferences. We assume the two goods to be normal, i.e. an increase in b_t leads to an increase in consumption in both periods of the life-cycle. Changes in the tax and pension rates (the latter anticipated one period ahead) affect consumption through the budget b_t. A reduced tax rate or an increased old-age pension rate increase consumption in both periods. In the first period of the life-cycle, the household accumulates wealth a_{t+1} that is equal to saving:

$$a_{t+1} = (1-\tau_t)w - c_1(r, b_t) \tag{10}$$

Since the initial wealth for each new generation equals zero when entering the economy (no transfer of wealth between generations in the form of inheritance), a_{t+1} are both savings (the increase in wealth) in period t and total wealth for each young individual at the end of period t. Note that (10) is not aggregate private saving because there is no deduction for the old generation's consumption of wealth. Aggregate private saving is thus smaller than a_{t+1}.

By substituting (8) into (10) and partially differentiating with regard to τ_t and θ_t, we find the effects on savings of increasing the tax rate or an anticipated increase in the pension rate:

$$\frac{\partial a_{t+1}}{\partial \tau_t} = -\left(1 - \frac{\partial c_1}{\partial b_1}\right) w < 0 \tag{11a}$$

$$\frac{\partial a_{t+1}}{\partial \theta_t} = \frac{-1}{1+r} \cdot \frac{\partial c_1}{\partial b_t} w < 0 \tag{11b}$$

The economic interpretation of (11) is straightforward. An increase in taxes in the first period of the life-cycle reduces disposable income in period 1 and leads to a desire for lower consumption in both periods. There is thus a decrease in the savings in period 1. An anticipated increase in the pension rate increases disposable income in period 2. Now the household demands more consumption in both periods. In order to finance the increase in consumption in period 1, there is a decrease in saving.

In period 2 of the life-cycle, the wealth is consumed, among other things through the sales of assets to the next generation. Thus, the older generation has no wealth at the end of the period. The total wealth of the private sector (Ω_{t+1}^{p}) at the end of period t is thus equal to the accumulated wealth of generation t:

$$\Omega_{t+1}^{p} = L_t a_{t+1} \tag{12}$$

Aggregate private consumption in period t equals total consumption for generation t – 1 (the pensioners) and generation t (those active in the labour market):

$$C_t = L_{t-1}c_{2,t} + L_t c_{1,t} \tag{13}$$

Substituting the demand functions in (9) into (13) gives the following macro consumption function:

$$C_t = L_{t-1}c_2\left(r, (1-\tau_{t-1})w + \frac{\theta_{t-1}w}{1+r}\right) + L_t c_1\left(r, (1-\tau_t)w + \frac{\theta_t w}{1+r}\right) \tag{14}$$

This shows the impact on total consumption of tax- and pension rates that affect the budgets of the two generations. Moreover, note that the interest rate also affects macroeconomic consumption, although in general, we cannot establish its sign. This is due to the fact that the substitution and income effects have the opposite sign for the young generation. The old generation will, however, consume more, the higher is the interest rate, everything else equal.

Net exports (X_t) is residually determined as

$$X_t = F(L_t, K_t) - (n+\delta)K_t - C_t \tag{15}$$

cf (1). A change in the fiscal policy that changes C_t thus automatically has an impact on net exports with the opposite sign. This also means that changes in domestic savings have an impact on the current account and net foreign assets.

5. The public sector

The budget surplus of the public sector can be written as

$$\Omega_{t+1}^{g} - \Omega_t^{g} = r\Omega_t^{g} + \tau_t w L_t - \theta_{t-1} w L_{t-1} \tag{16}$$

Equation (16) expresses the growth in net financial wealth of the public sector, i.e. interest income and tax income minus pension payments. Dividing (16) by the number of workers L_t, and using the relation $L_t = (1+n)L_{t-1}$, the government budget constraint can be expressed in terms of net government wealth per worker:

$$(1+n)\omega_{t+1}^{g} = (1+r)\omega_t^{g} + \left(\tau_t - \frac{\theta_{t-1}}{1+n}\right)w \tag{17}$$

Here, $\omega_t^g = \Omega_t^g / L_t$ equals public wealth per worker. By determining τ_t and θ_t, the authorities can influence the development of public wealth over time..

National wealth per worker is denoted as ω_t^n. This is the sum of public and private wealth:

$$\omega_t^n = \omega_t^g + \omega_t^p \tag{18a}$$

where $\omega_t^p = \Omega_t^p / L_t = a_t/(1+n)$ is private wealth per worker at the beginning of period t, see (12). National wealth per worker is equal the sum of domestic capital and net foreign wealth per worker (k_t^*):

$$\omega_t^g + \omega_t^p = k_t + k_t^*. \tag{18b}$$

6. Steady state

In this model, the steady state growth rate is clearly n, the growth rate of the young population. For a given real rate of interest, capital intensity (k) and output per worker (y) will be constant. In an open economy, steady-state also requires that the growth rate of net foreign wealth is n. Let us take a closer look at the implication for net exports, using the definition of the current account as net exports foreign wealth income:

$$\Delta K_t^* = X_t + rK_t^*$$

Dividing this equation by K_t^* , we see that the left hand side will be equal to the growth rate of net foreign assets. Setting the latter growth rate to n, net exports divided by net foreign assets can be expressed as

$$\frac{X_t}{K_t^*} = n - r \tag{19a}$$

Dividing the numerator and the denominator on the left hand side of (19a) by L_t, and denoting net export per worker by x, we obtain the following expression for the steady-state level of net export per worker:

$$x = (n - r)k^* \tag{19b}$$

It follows that net export per worker will have the opposite sign as net foreign wealth per worker in steady state. From previous analysis, we also know that the domestic capital stock and gross investment grow at the same rate as the number of workers. From (1), it thus follows that also total consumption must grow at the steady state growth rate.

To realise a steady-state growth path, fiscal policy must be designed so that public wealth per worker increases at the rate n. Then ω^g will be constant over time. Observe that there will be different combinations of constant tax- and social benefit rates that correspond to a constant level of public wealth per worker. By setting $\omega_{t+1}^g = \omega_t^g$ for all t, the public budget condition (17) is expressed as

$$(r - n)\omega^g = \left(\frac{\theta}{1+n} - \tau\right)w \tag{20}$$

From this equation, we can see that the higher is the steady-state value of ω^g, the lower can the tax rate τ be set in relation to the pension rate θ. The interpretation is that a high level of public wealth gives the government a corresponding wealth income that allows for lower taxes or higher pension payments in steady state.

Choosing ω^g, τ and θ which satisfy (20), we note that the budget (b) for each household must be constant, see (8). Therefore, individual consumption and saving must also be constant. Also private wealth per worker (a) must be constant when the tax rate and the pension rate do not change over time. Hence ω^p and national wealth per worker are also constant.

7. Real economic effects of increased government debt and redistribution of the tax burdens among generations

Let us now analyse the effects of a temporary tax cut that increases the government debt permanently. We start in a steady-state equilibrium in period $t = 0$, and we assume that public wealth ω_0^g is zero initially. The initial tax and pension rates are consistent with zero government wealth. The pension rate is held constant. Assume now that the tax rate τ_1 is reduced by $\Delta\tau_1$ from period zero to period 1. Thus, the current value of net tax payments for generation 1 is reduced. This reduces public wealth by $\Delta\omega_1^g$, and thus involves net government debt. If the tax rate is increased to its former level in period 2, the government's debt will still increase because it has to pay interest on its debt. The government must therefore increase the tax rate to a higher level in the following periods in order to stabilize government debt per worker at the new permanent level. We assume that the tax rate increases exactly so much from period 1 to period 2 ($\Delta\tau_2$) that ω^g (which is now negative) is thereafter constant over time.

The tax rate must increase more in period 2 than it decreased in period 1 since the interest income to the public sector has been lower than in the initial steady state equilibrium. For generation 1 to get a tax cut, the tax burden of all later generations must be higher. In other words, for the budget b to increase for one generation, all generations born later will get a lower budget if the increase in public debt per worker is permanent. Clearly, this intergenerational redistribution will permit the low-tax generation to increase its consumption (and welfare) in both periods of the life-cycle. All later generations get lower consumption and welfare compared to the initial steady state.

Let us now outline how this policy affects private wealth formation over time. In the first period, some of the tax reduction goes to an increase in consumption and some to savings (for consumption in period 2), i.e. there is an increase in private wealth, but this increase is less than the increase in public debt. There is thus a decline in national wealth in period 1.

In period 2, there is an increase in the tax rate for the new young generation. Therefore, it consumes and saves less than previous generation. At the same time, the old generation has a higher consumption than in the old steady state due to the tax cut in period 1. Thus, there is a further decrease in national wealth.

After the decline in national wealth per worker in the course of two periods, national wealth per worker is constant over time. Therefore, a new steady state with lower public wealth and national wealth will be reached after two periods, the same time length as the life-cycle of an adult.

Let us see how the steady-state solution for national wealth is affected by the change in public wealth. First, we use (20) in order to find out how much the tax rate must increase for each unit decrease in ω^g. Differentiation gives (θ constant).

$$d\tau = -\frac{(r-n)}{w} d\omega^g \tag{21}$$

We now use (18) (after inserting from (10)) to see how the steady-state level of national wealth depends on government debt:

$$\omega^n = \omega^g + \frac{1}{1+n}\left[(1-\tau)w - c_t\left(r, (1-\tau)w + \frac{\theta w}{1+r}\right)\right] \tag{22}$$

We now differentiate ω^n with respect to ω^g with consideration being given to (21), which says that a change in government wealth will also change the steady-state tax rate. This gives

$$\frac{d\omega^n}{d\omega^g} = 1 + \left(\frac{r-n}{1+n}\right)\left(1 - \frac{\partial c_1}{\partial b}\right) \tag{23}$$

Since $r > n$ and the marginal propensity to save, ($1 - \partial c_1 / \partial b$), is positive, national wealth in steady state will fall even more than the increase in government debt. This is due to the fact that the necessary increase in the tax rate to compensate for a decline in public wealth income leads to smaller life-time savings for households in the new steady state. In addition to the decrease in public wealth, private wealth will also fall due to the long-run tax increase.

The tax postponement policy that we have analyzed clearly redistributes welfare among different generations. The generation that received a tax cut gains at the expense of all later generations. In a real economic sense, this takes place by the first generation incurring a larger foreign debt to the country (alternatively less foreign claims) which requires a larger export surplus in the future (see (19b)). There will thus be less room for consumption in the long run.

8. A funded public pension

Let us now consider a fiscal policy that increases the tax and pension rates in a parallel fashion, leaving the budgets (b) of each generation unchanged. This means that each generation can expect a bigger pension from the government when old, but on the other hand, they must pay more taxes as young. It follows from (11) that households will reduce private saving. Going back to (8), we see that if the tax and pension rates are increased such that

$$d\tau_t = \frac{d\theta_t}{1+r},$$

the budget b_t will not change for any generation. We begin in a steady-state equilibrium in period zero. In the first period, the government collects more taxes, and since there are no extra pension payments in this period, government wealth increases. This extra wealth can be interpreted as a public pension fund, and it is equal to the decline in the private saving of the young generation. Since the budgets of households don't change, no generation will want to change her consumption in any period, cf (9). But then the export surplus and foreign wealth are not affected by this policy either. Even if there is a tax increase for the individuals in the first period, and less private saving, there is such an increase in the pension payments that the original consumption plan can still be realised and is still optimal. Thus, the change in policy has no real economic effect. This result is often called the *The Ricardian Equivalence Result.* Ricardian equivalence holds in this case since a tax reduction or a tax increase that changes public wealth (alternatively net public debt) means that the same individuals get adjusted future taxes or social security in such a way that the present value of the total tax increase equals zero. Then, it will be optimal to change private savings so that the sum of public and private savings does not change. In the case with an increased tax in period 1 of the life cycle, private savings are reduced so that total savings are not affected. In economic terms, private households know that the government is building up a pension fund for them. Therefore, private saving is reduced correspondingly.

The Ricardian equivalence result builds on several conditions that do not necessarily completely hold in practice. For example, it disregards the fact that taxes can affect labour supply. Moreover, it is assumed that the capital market is perfect and that the consumers and the public sector can borrow or lend at the same interest rate. It is also assumed that the individuals have correct expectations and foresee that policy changes do not affect the present value of net tax over the life-cycle.

In the policy experiment analyzed in section 7 above, Ricardian equivalence breaks down because the budget constraints of the households in the tax-cut generation do not deduct the present value of taxes that future generations have to pay. The former generation therefore perceives the tax cut as a gain that permits higher consumption over the life-cycle.

9. Implementing a pay-as-you-go system for public old-age pension

Many countries, for example Norway, have a system for public old-age pensions where pensions are financed on a current basis from tax revenues and do not involve accumulation of public wealth in special pension funds. In its pure form, this type of public pension system is called a pay-as-you-go-system (PAYG-system). It means that there is an income transfer from tax payers who are active on the labour market to the old generation.

The PAYG-system will weaken the life-cycle motive for saving since the individuals know that they will get payments from the social security system in period 2 (see (11)). An increase in τ and θ will thus reduce the accumulation of private wealth just as in the funded system discussed in section 8.

Using the overlapping generations model, we shall now analyse the effects of introducing a PAYG-system for public old-age pensions. For simplicity, we assume that at the outset, the economy is in a steady-state with zero public wealth and that there are no taxes and transfers. In period 1, the tax rate is set ($\Delta\tau > 0$) at a permanently higher level, while the tax incomes are being paid out to the older people (generation 0). A difference from the funded system is that the tax revenues are transferred to the old generation at once, without any accumulation of government wealth. From the public budget restriction, (17), we see that if

$$\Delta\theta = (1+n)\Delta\tau \tag{24}$$

this policy will not lead to any change in public wealth, i.e. $\Delta\omega_1^g = 0$. Private savings will fall, however. This leads to the economy getting into a new steady state after one period (from period 2 and onwards). In period 1, generation 0 gets an unexpected additional income of $w_1 L_1 \Delta\tau,$ which is entirely spent on consumption. But what happens to consumption in generations 1, 2, 3 ….? They have all received a higher current value of net tax since the change in their life-cycle budget can be expressed as (cf (24))

$$\Delta b = -w\Delta\tau + \frac{(1+n)w}{1+r}\Delta\tau = -\frac{(r-n)w}{1+r}\Delta\tau < 0 \tag{25}$$

All generations from number 1 and onwards thus lose from the introduction of the PAYG-system since the present value of the pension payments less tax is negative. This applies as long as $r > n$. If $r \leq n$, the present value of participating in the redistribution system will be positive or zero, and the introduction of this system will constitute a Pareto improvement if $r < n$.

Intuitively, these results can be explained by participation in the redistribution system giving a rate of return amounting to n. This represents *the growth in the tax base on which transfers to pensioners is based*. The more quickly the tax base grows, the larger are the returns to participating in the PAYG-system. When the real interest rate is larger than n, all consumers will, however, gain from saving their wealth for old age themselves rather than participating in the PAYG- system.

If $r > n$, all generations except generation 0 will thus lose from the introduction of the PAYG-system, while generation 0 will gain from this.

10. Concluding remarks

In this note we have looked at long-run intergenerational effects of fiscal policy in a small open economy. Such effects arise if households have a planning horizon for saving and consumption that is equal to their expected lifetime. Then households would mainly care about government taxes and transfers within their own lifetimes and not more distant effects of current fiscal policy.

We first looked at the welfare implications for different generations (both those living now and unborn generations) of cutting taxes temporarily in order to raise the public debt permanently. This policy reduces national wealth, net foreign assets and future consumption. We then looked at a policy that increased taxes and pension benefits in such a way that the government accumulates a permanent public pension fund. This policy shifted saving from the private to the public sector, but did not redistribute consumption and welfare among different generations.

Finally, we analysed the effects of starting a pay-as-you-go system of public pensions, i.e. a pension system that does not involve accumulation of a public pension fund. We showed that the first old generation benefited from such a system, while all later generations lost, assuming that the real rate of interest is larger than the growth rate of the tax base. National wealth and net foreign assets would decline in the long run.

In this note, we have disregarded long-term productivity and real wage growth. It can be shown that if exogenous labour saving technical progress leads to an increase in the tax base at the "natural" (steady state) growth rate $g > n$, our results will hold if $r > g$.