A COMPENSATION-BASED POWER FLOW METHOD FOR WEAKLY MESHED DISTRIBUTION AND TRANSMISSION NETWORKS

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ABSTRACT

This paper describes a new power flow method for solving weakly meshed distribution and transmission networks, using a multi-port compensation technique and basic formulations of Kirchhoff's laws. This method has excellent convergence characteristics and is very robust. A computer program implementing this power flow solution scheme was developed and successfully applied to several practical distribution networks with radial and weakly meshed structure. This program was also successfully used for solving radial and weakly meshed transmission networks. The method can be applied to the solution of both the three-phase (unbalanced) and single-phase (balanced) representation of the network. In this paper, however, only the single phase representation is treated in detail.

I. INTRODUCTION

Recently at the Pacific Gas and Electric Company we developed a distribution network optimization software package. This development work called for a power flow solution algorithm with the following general characteristics:

1. Capable of solving radial and weakly meshed distributation networks with up to several thousand line sections (branches) and nodes (buses).

2. Robust and efficient.

The efficiency of such a power flow algorithm is of utmost importance as each optimization study requires numerous power flow runs. Furthermore, the extension of the application of this power flow method to three phase networks with distributed loads was also envisaged.

The Newton Raphson and the fast decoupled power flow solution techniques and a host of their derivatives have efficiently solved "well-behaved" power systems for more than two decades. Researchers, however, have been aware of the shortcomings of these solution algorithms when they are "generically" implemented and applied to ill-conditioned and/or poorly initialized power systems [1,2,3]. Hence, commerical power flow packages always modify these algorithms for enhanced robustness. The nature of modifications and the degree of improvement obtained varies for different packages. The Gauss-Seidel power flow technique, another classicial power flow method, although very robust, has shown to be extremely inefficient in solving large power systems.

Distribution networks, due to their wide ranging resistence and reactance values and radial structure, fall into the category of ill-conditioned power systems for the generic Newton-Raphson and fast decoupled power flow algorithms. Our experience with a basic Newton-Raphson power flow program for solving distribution networks was mostly unsuccessful as it diverged for the majority of the networks studied. Later we successfully used the Newton-Raphson based Western System Coordinating Council (WSCC) power flow program. This program, which is commonly used by the WSCC member utilities, includes several enhancements for increasing its convergence capabilities. Although the robustness of the program was acceptable, the computation time was excessive. In addition, the extension of the Newton-Raphson algorithm to the solution of the three phase networks, not even considering distributed load, would result in substantial deterioration of the numerical efficiency of the solution algorithm [4].

Efficient power flow algorithms for solving single and three phase radial distribution networks [5,6,7] have been extensively used by distribution engineers. These algorithms are not, however, designed to solve meshed networks.

In this paper, we propose a new method for the solution of weakly meshed networks. In this method, we first break the interconnected grid at a number of points (breakpoints) in order to convert it into one radial network. Each breakpoint will open one simple loop. The radial network is solved efficiently by the direct application of Kirchhoff's voltage and current laws (KVL and KCL). We then account for the flows at the breakpoints by injecting currents at their two end nodes. The breakpoint currents are calculated using the multi-port compensation method [8,9]. In presence of constant P,Q loads, the network is nonlinear causing the compensation process to become iterative. The solution of the radial network with the additional breakpoint current injections completes the solution of the weakly meshed network.

Our studies have shown that, typically, only a few iterations were required for the solution of distribution networks using the proposed power flow solution technique. For the weakly meshed transmission networks the number of iterations was higher, due to the additional nonlinearities introduced by generator buses (PV nodes). In all the cases studied the proposed power flow technique was significantly more efficient than the Newton-Raphson power flow algorithm while converging to the same solution.

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The numerical efficiency of the proposed compensation-based power flow method, however, diminishes as the number of breakpoints required to convert the meshed network to a radial configuration increases. This restricts the practical application of the method to weakly meshed networks.

In this paper we emphasize the application of the compensation-based power flow method to the distribution networks and provide several practical examples. A comparison with the Newton-Raphson based WSCC power flow program is also presented. We then discuss the application of the algorithm to weakly meshed transmission networks where again comparison with the WSCC power flow program is provided. A brief discussion of the extension of this method to three phase networks as well as the treatment of distributed loads and other practical considerations concludes the paper.

II. SOLUTION OF A RADIAL DISTRIBUTION NETWORK

In our algorithm, regardless of its original topology, the distribution network is first converted to a radial network. Hence, an efficient algorithm for the solution of radial networks is crucial to the viability of the overall solution method.

The solution method used for radial distribution networks is based on the direct application of the KVL and KCL. Similar techniques are also described in [5,6,7]. For our implementation, we developed a branch oriented approach using an efficient branch numbering scheme to enhance the numerical performance of the solution method. We first describe this branch numbering scheme.

Branch Numbering

In contrast to all classical power flow techniques which use nodal solution methods for the network, our algorithm is branch-oriented. Figure 1 shows a typical radial distribution network with n nodes, b(=n-1) branches and a single voltage source at the root node. In this tree structure, the node of a branch L closest to the root note is denoted by L1 and the other node by L2. We number the branches in layers away from the root node as shown in Figure 2. The numbering of branches in one layer starts only after all the branches in the previous layer have been numbered. This numbering scheme is very simple and straightforward, and has been implemented in our power flow program.

Solution Method

Given the voltage at the root node and assuming a flat profile for the initial voltages at all other nodes, the iterative solution algorithm consists of three steps:

1. Nodal current calculation: At iteration k, the nodal current injection, $I_i^{(k)}$, at network node i is calculated as,

$$I_i^{(k)} = (S_i/V_i^{(k-1)})^* - Y_i V_i^{(k-1)} \qquad i=1,2,\ldots,n \qquad (1)$$

where $V_i^{(k-1)}$ is the voltage at node i calculated during the $(k-1)^{th}$ iteration and S_i is the specified power injection at node i. Y_i is the sum of all the shunt elements at the node i.

2. Backward sweep: At iteration k, starting from the branches in the last layer and moving towards the branches connected to the root node the current in branch L, J_L, is calculated as:

$$J_L^{(k)} = -I_{L2}^{(k)} + \Sigma(\text{currents in branches emanating from node L2}) \qquad L=b,b-1,\ldots,1 \qquad (2)$$

where $I_{L2}^{(k)}$ is the current injection at node L2. This is the direct application of the KCL.

3. Forward sweep: Nodal voltages are updated in a forward sweep starting from branches in the first layer toward those in the last. For each branch, L, the voltage at node L2 is calculated using the updated voltage at node L1 and the branch current calculated in the preceding backward sweep:

$$V_{L2}^{(k)} = V_{L1}^{(k)} - Z_L J_L^{(k)} \qquad L=1,2,\ldots,b \qquad (3)$$

where Z_L is the series impedance of branch L. This is the direct application of the KVL.

Steps 1, 2 and 3 are repeated until convergence is achieved.

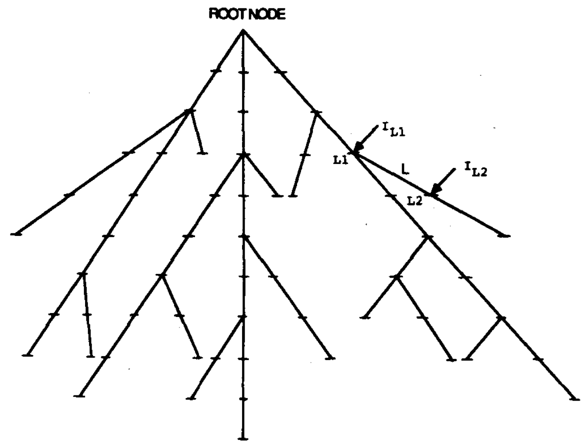

Fig.1 A typical radial distribution network

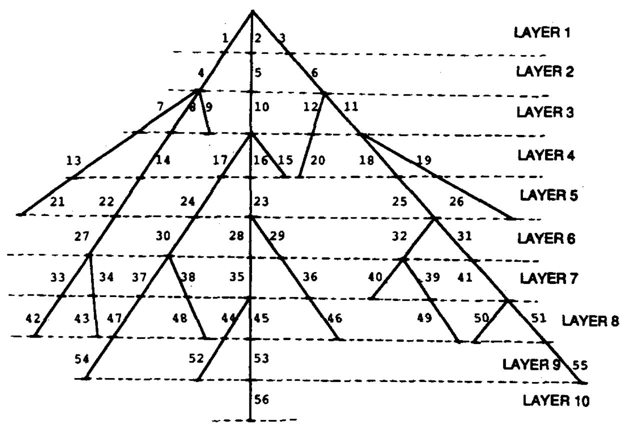

Fig.2 Branch numbering of the radial distribution network

Convergence Criterion

We used the maximum real and reactive power mismatches at the network nodes as our convergence criterion. As described in the solution algorithm, the nodal current injections, at iteration k, are calculated using the scheduled nodal power injections and node voltages from the previous iteration (equation (1)). The node voltages at the same iteration are then calculated using these nodal current injections (equations (2) and (3)). Hence, the power injection for node i at kth iteration, $S_i^{(k)}$, is calculated as:

$$S_i^{(k)} = V_i^{(k)}(I_i^{(k)})^* - Y_i|V_i^{(k)}|^2 \qquad (4')$$

The real and reactive power mismatches at bus i are then calculated as:

$$\begin{aligned} \Delta P_i^{(k)} &= \mathrm{Re}[S_i^{(k)} - S_i] \\ \Delta Q_i^{(k)} &= \mathrm{Im}[S_i^{(k)} - S_i] \end{aligned} \qquad i=1,2,\ldots,n \quad (4'')$$

Table 1 shows the values of the maximum real and reactive power mismatches at the various iterations of this radial network power flow solution algorithm for three practical distribution networks. An excellent rate of convergence can be observed for all three networks studied. This convergence behavior can be briefly explained as follows. The error that is incurred in estimating initial node voltages is propagated first to nodal and then to branch currents via equations (1) and (2). In the process of updating node voltages using equation (3), the error in branch currents is multiplied by the small line impedance, Z_L ($|Z_L|<<1$), and thereby rapidly attenuated.

DISTRIBUTION NETWORK CONFIGURATION	ITERATION NUMBER	MAXIMUM REAL POWER MISMATCH (KW)	MAXIMUM REACTIVE POWER MISMATCH (KVAR)
244 NODES	1	6.134	13.092
	2	0.301	0.567
R/X RANGE	3	0.008	0.024
	4	0.000	0.001
0.245-5.065	5	0.000	0.000
	1	5.994	5.597
544 NODES	2	1.691	1.132
	3	0.402	0.377
R/X RANGE	4	0.126	0.085
	5	0.031	0.029
0.409-5.063	6	0.010	0.007
	7	0.003	0.002
	8	0.000	0.000
1411 NODES	1	4.891	4.573
	2	0.719	0.222
R/X RANGE	3	0.088	0.037
	4	0.011	0.003
0.000-5.480	5	0.001	0.000
	6	0.000	0.000

Table 1 Convergence characteristics of the radial network solution algorithm

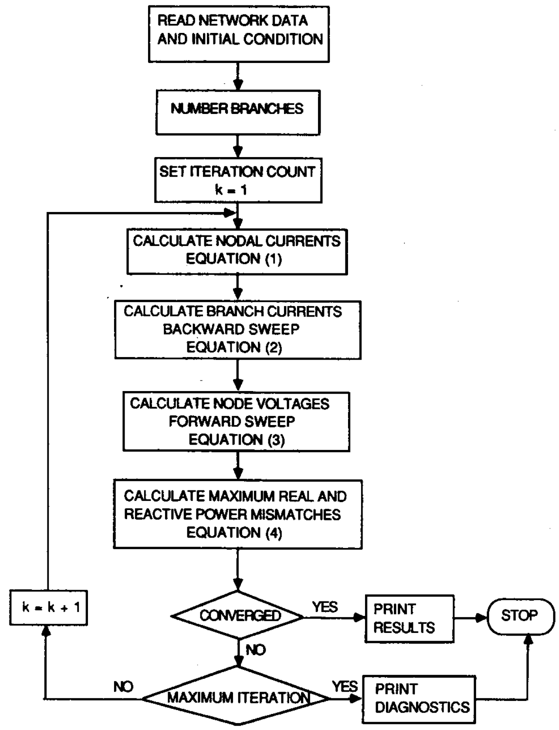

Fig.3 Power flow solution algorithm for the radial networks

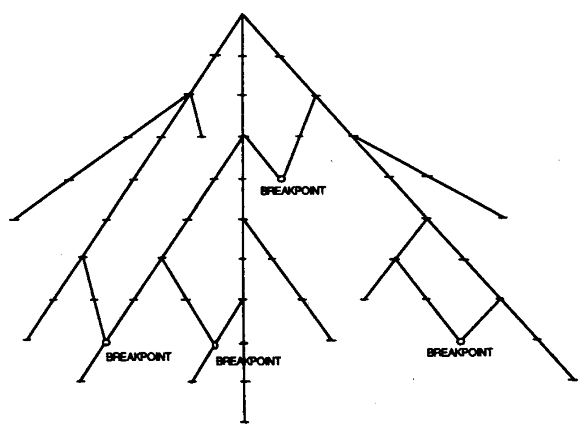

Fig.4 A weakly meshed distribution network

Figure 3 shows the flow chart of the overall power flow solution method for radial networks.

III. SOLUTION OF WEAKLY MESHED DISTRIBUTION NETWORKS

Figure 4 shows an example of a weakly meshed distribution network containing four simple loops. The radial network solution algorithm can not be directly applied to this network. Nevertheless, by selecting four breakpoints, this network can be converted to a radial configuration. The branch currents interrupted by the creation of every breakpoint can be replaced by current injections at its two end nodes, without affecting the network operating condition. This resulting radial network can now be solved by the radial network solution technique described earlier.

In applying the radial network solution algorithm, the current at breakpoint j, J_j, must be injected with opposite polarity at the two end nodes of the breakpoint. At iteration k:

$$I_{j1}^{(k)} = -\hat{J}_j^{(k)} \qquad j = 1, 2, \ldots, p \qquad (5)$$
$$I_{j2}^{(k)} = \hat{J}_j^{(k)}$$

where j1 and j2 correspond to the two end nodes of the breakpoint j, and $I_{ji}^{(k)}$ and $I_{j2}^{(k)}$ become the nodal current injections at these nodes, $\hat{J}_j^{(k)}$ is the breakpoint current and p the total number of breakpoints. In the presence of nodal currents at the breakpoint nodes, due to shunt elements and/or loads, $\hat{J}_j^{(k)}$ and $-\hat{J}_j^{(k)}$ must be added to these nodal currents. This process is schematically shown in Figure 5. Once $I_{j1}^{(k)}$ and $I_{j2}^{(k)}$ are updated, steps 2 and 3 of the radial network solution algorithm can be directly applied.

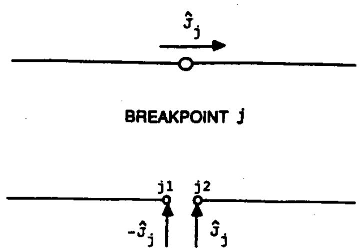

Fig.5 Breakpoint representation using nodal current injections

Calculation of Breakpoint Currents Using Compensation Method

Breakpoint currents are calculated using the multi-port compensation method [8]. Figure 6 illustrates the concept used in this approach. In this figure the radial network resulting from the opening of the breakpoints is shown as a multi-port circuit with breakpoint nodes forming the ports of the circuit. The calculation of breakpoint currents requires that the multi-port equivalent circuit for the radial network as seen from the ports of the breakpoints be established.

For a linear network, this multi-port equivalent circuit can be the Thevenin equivalent circuit of the radial network seen from the open ports created by the breakpoints. In this circuit the Thevenin voltage $\underline{\hat{V}}$ is the (px1) vector of open circuit breakpoint voltages, obtained from the power flow solution of the radial network ,$[\hat{Z}]$ the (pxp) non-sparse matrix of the breakpoint impedances (coefficients relating breakpoint currents and voltages) and $\underline{\hat{J}}$ is the (px1) vector of the desired breakpoint currents (Figure 7):

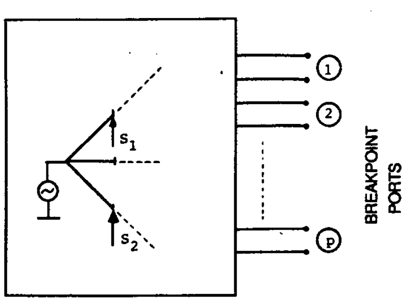

Fig.6 Multi-port equivalent of the network as seen from the breakpoint ports

$$\underline{\hat{V}} = [\hat{Z}]\ \underline{\hat{J}} \qquad (6)$$

In the presence of constant power loads the distribution network is, however, nonlinear and equation (6) cannot be directly used. Instead, as we shall explain, we calculate breakpoint currents iterively using the Thevenin equivalent circuit.

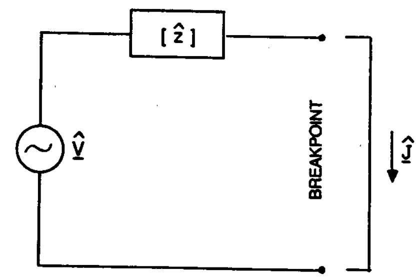

Fig.7 Thevenin equivalent circuit of the network as seen from the breakpoint ports

Calculation of Breakpoint Impedance Matrix

The breakpoint impedance matrix (Thevenin equivalent impedance) can be determined using the following method:

Equation (6) can be written as,

$$\begin{bmatrix} \hat{V}_1 \\ \vdots \\ \hat{V}_j \\ \vdots \\ \hat{V}_p \end{bmatrix} = \begin{bmatrix} \hat{Z}_{11} & \cdots & \hat{Z}_{1j} & \cdots & \hat{Z}_{1p} \\ \vdots & & \vdots & & \vdots \\ \hat{Z}_{j1} & \cdots & \hat{Z}_{jj} & \cdots & \hat{Z}_{jp} \\ \vdots & & \vdots & & \vdots \\ \hat{Z}_{p1} & \cdots & \hat{Z}_{pj} & \cdots & \hat{Z}_{pp} \end{bmatrix} \begin{bmatrix} \hat{J}_1 \\ \vdots \\ \hat{J}_j \\ \vdots \\ \hat{J}_p \end{bmatrix} \qquad (7)$$

According to equation (7), column j of the breakpoint impedance matrix will be equal to vector of breakpoint voltages for $\hat{J}_j$=1 p.u. and $\hat{J}_i$=0, i=1,2,...,p and i $\neq$ j. This corresponds to the application of 1 p.u. current at the breakpoint j with all loads and the source at the root node removed, which is, in turn, equivalent to the injection of 1 p.u. currents with opposite polarity at the two end nodes of the breakpoint j (equation (5)). In the absence of loads, the accurate solution of the power flow for the radial network can be achieved in one iteration. Each of the breakpoint voltages can be determined by subtracting the voltages at the two end nodes of the breakpoint. This process must be repeated for all breakpoints until all the columns of the breakpoint impedance matrix are calculated.

Iterative Compensation Process

The iterative compensation process for calculating the breakpoint currents, using the Thevenin equivalent circuit of Figure 7, is described in the following:

1. Calculate the Thevenin equivalent impedance (breakpoint impedance matrix $[\hat{Z}]$ of the radial network) maintaining it constant throughout the compensation process.

2. Calculate the Thevenin equivalent voltage (breakpoint voltage vector $\hat{V}$) of the radial network using the radial network solution algorithm (Figure 3) including the breakpoint currents calculated from the previous iteration of the compensation process. The initial values of the breakpoint currents are zero.

3. Calculate the incremental change in the breakpoint currents using the Thevenin equivalent circuit. At iteration m of the compensation process:

$$\underline{\Delta\hat{J}}^{(m)} = [\hat{Z}]^{-1}\,\underline{\hat{V}}^{(m)} \tag{8'}$$

4. Update the breakpoint currents. At iteration m:

$$\underline{\hat{J}}^{(m)} = \underline{\hat{J}}^{(m-1)} + \underline{\Delta\hat{J}}^{(m)} \tag{8''}$$

5. Repeat steps 2, 3 and 4 until convergence is reached (the maximum breakpoint voltage calculated at step 2 is within prescribed limits).

This fixed tangent solution method is schematically depicted in Figure 8 for a network having a single breakpoint. Computationally there is no need for the inversion of the breakpoint impedance matrix $[\hat{Z}]$. Complex matrix $[\hat{Z}]$ is factorized once in the beginning of the iterations and the forward and backward substitution is then used to calculate $\underline{\Delta\hat{J}}^{(m)}$ in equation (8'). Our test cases on practical distribution networks showed that the number of iterations required for the calculation of the breakpoint currents was less than 5 in most cases.

Figure 9 shows the flow chart of the overall power flow solution scheme.

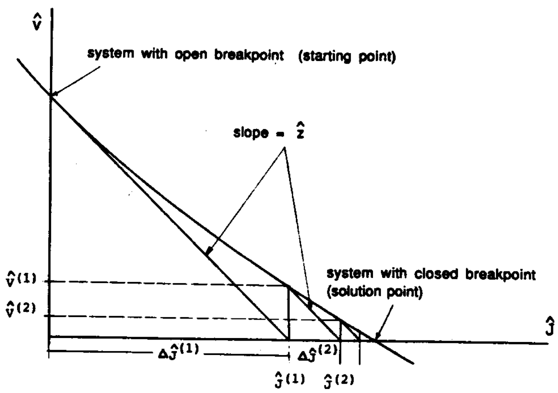

Fig.8 Graphic representation of the iterative compensation process

Selection of Breakpoints

Breakpoints are selected in order to convert meshed network into a radial configuration. In addition to this function breakpoints should be selected in such a way as to ensure the convergence of the overall solution algorithm. The latter requirement for the selection of breakpoints usually is satisfied by concentrating on the parts of the meshed network where the power flows are low. On the other hand, the power flows are the end product of the solution method and are not known at the time of breakpoint selection.

In weakly meshed distribution networks breakpoint selection does not affect the convergence performance of the solution method in any noticeable manner. Hence, we select them for the main purpose of opening the network loops. Under these circumstances the algorithm for identifying the breakpoints is very simple and becomes part of the branch numbering scheme described below:

1. Examine all branches and select those connected to the root node for the first branch layer

2. Store the node number of the far node of the branches in the branch layer just formed. For all these nodes raise a flag indicating that they have already been used

3. Examine all the remaining branches and select those connected to any of far nodes of the branches in the previous layer and place them in a new branch layer

4. If the end node of a branch numbered in step 3 has been used before (flag identification of step 2) a loop has been formed and a breakpoint must be created at this node

5. Repeat steps 2-4 until all branches are processed.

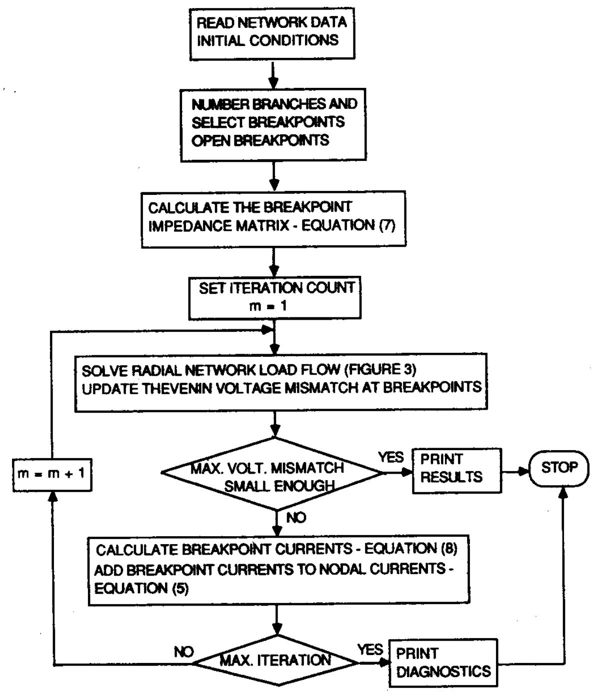

Fig.9 Compensation-based power flow method for the weakly meshed networks

IV. RESULTS FOR THE DISTRIBUTION NETWORKS

We developed the program WNETPF (Weakly meshed NETwork Power Flow) based on the proposed power flow solution algorithm. This program successfully solved several practical distribution networks with radial and meshed structures. Table 2 shows the performance results of this program alongside those of the Newton-Raphson based WSCC power flow program. We also used two other power flow programs using the generic Newton-Raphson and the Gauss-Seidel solution algorithms at this application stage. The generic Newton-Raphson solution algorithm only converged for the smallest network. The Gauss-Seidel power flow method converged in all cases while requiring in excess of 20,000 iterations.

For the cases reported in Table 2, the proposed algorithm converged in less than 14 iterations in 0.21 to 2.1 CPU seconds on a mainframe computer. Each iteration of this program corresponds to one iteration in the radial network solution algorithm prior to, during, and after the calculation of breakpoint currents (variable k in Figure 3). The number of outer iterations for calculating the breakpoint currents in the compensation process (variable m in Figure 9) was less than 6 in all three cases. A flat start was used in all cases with the tolerance for the real and reactive power mismatches set to 0.05 kW and 0.05 kvar.

Table 2 indicates that the proposed power flow program is significantly more efficient than the Newton-Raphson power flow method when studying radial and weakly meshed distribution networks. This conclusion is particularly crucial for: (a) on-line applications; (b) multiple power flow studies; (c) micro- and mini-computer applications.

The WNETPF program does not require double precision variables and uses only one two dimensional array (breakpoint impedance matrix), hence, avoids taxing computer resources.

DISTRIBUTION NETWORK CONFIGURATION	WSCC POWER FLOW		PROPOSED METHOD		MAX. MISMATCH	
	ITERATIONS	CPU TIME (SECONDS)	ITERATIONS	CPU TIME (SECONDS)	REAL POWER (KW)	REACTIVE POWER (KVAR)
244 NODES						
NO LOOPS	5	0.97	3	0.21	0.008	0.024
5 LOOPS	5	0.97	9	0.25	0.003	0.008
544 NODES						
NO LOOPS	5	1.99	5	0.48	0.031	0.029
9 LOOPS	5	1.96	14	0.65	0.001	0.000
1411 NODES						
NO LOOPS	5	5.31	4	1.47	0.011	0.003
36 LOOPS	5	5.27	10	2.10	0.000	0.000

Table 2 Performance results for distribution networks (IBM 3090-200 mainframe computer) - The total cpu time in seconds includes the time for initial processing of the network data and the iterative solution algorithm.

V. APPLICATION TO WEAKLY MESHED TRANSMISSION NETWORKS

In a weakly meshed transmission network, the swing bus is assigned as the root node. Then the branches are numbered and breakpoints selected in exactly the same manner used for the weakly meshed distribution networks. As a result a radial transmission network is created.

The solution algorithm for a radial transmission network is identical to that of a radial distribution network except for the processing of the generator (PV) nodes. For the PV node i having a specified power injection P_i^s and voltage magnitude $|V_i^s|$, we start the iterations of the radial network solution algorithm by assuming $V_i^{(o)} = |V_i^s| \angle 0$ and $Q_i^{(o)} = 0$. Steps 1 and 2 of the radial network solution method are then performed in the same manner as before. Step 3, however, must be modified.

At iteration k, the voltage magnitude at the generator node i, calculated using equation (3) must modified as,

$$V_{i\,new}^{(k)} = V_i^{(k)}\,|V_i^s|/|V_i^{(k)}| \tag{9}$$

$V_{i\,new}^{(k)}$ is then used for calculating the voltages at the end nodes of the branches in the next layer. The reactive power at the generator node i is then updated using the secant method as described in the Appendix.

Inclusion of generator nodes using this approach does not noticeably deteriorate the convergence properties of the radial network power flow solution algorithm. The efficiency of the solution method for the weakly meshed networks will, however, be affected by the introduction of the generator nodes. The nonlinearity in the transmission network caused by the additional generator nodes is more pronounced than that of the distribution networks having constant power loads alone. This results in an increased number of iterations for calculating the breakpoint currents using equation (8).

Table 3 shows the performance results of applying the proposed algorithm and the Newton-Raphson based WSCC power flow program to one radial and three weakly meshed transmission networks. These networks were synthesized from a practical 500kV transmission network with a high degree of series compensation. The table shows that for the networks with low number of breakpoints, the proposed load flow techniques is more efficient than the Newton-Raphson method. However, as the number of breakpoints increased, the proposed method required significantly higher number of iterations while the Newton-Raphson algorithm converged with the same number of iterations.

TRANSMISSION NETWORK CONFIGURATION	WSCC POWER FLOW		PROPOSED METHOD	
	ITERATIONS	CPU TIME (SECONDS)	ITERATIONS	CPU TIME (SECONDS)
42 BUS, 500KV NO PV NODES				
NO LOOPS	10	0.28	20	0.12
1 LOOP	10	0.28	21	0.12
3 LOOPS	10	0.30	32	0.13
5 LOOPS	10	0.28	61	0.15
42 BUS, 500 KV 4 PV NODES				
NO LOOPS	5	0.31	38	0.13
1 LOOP	5	0.31	100	0.18
3 LOOPS	6	0.35	116	0.18
5 LOOPS	5	0.31	120	0.19

Table 3 Performance results for transmission networks (IBM 3090-200 mainframe computer)

VI. PRACTICAL CONSIDERATIONS

Detailed representation of a distribution network requires [10]:

1. The three phase representation of the network to account for the actual load unbalances.
2. The distributed load representation along the distribution lines.
3. The representation of load tap changers, voltage regulators, boosters, etc.

Requirement 1 can be directly incorporated in the proposed power flow method by replacing voltage and current scalars in equations (1) thru (10), by (3X1) vectors of voltages and currents of the three phases. Under these circumstances, in equations (1), (3), (4) and (7), the admittances and impedances should be represented by (3X3) matrices.

Distributed loads along distribution lines can be approximated by lumped loads at system nodes for power flow calculations purposes. This, however, may require the addition of pseudo-nodes along some of the lines where part of the line load is lumped. The proposed power flow method is also capable of directly including distributed loads. This can be achieved by the modification of equations(1) and (3).

Distribution network equipment (regulators, boosters, etc.) can be modeled in the proposed algorithm without any restriction.

This method is directly applicable to distribution planning studies, where single phase representation of the network with lumped nodal loads are considered to be adequate. In addition we have successfully used this power flow method for our optimal network reconfiguration studies.

In the case of weakly meshed transmission networks, a complete system representation requires the inclusion of tap changers and phase shifters. The proposed solution algorithm is capable of including these components directly. This requires that the KVL and KCL be written for the mathematical model of the tap changers and phase shifters.

VII. CONCLUSION

This paper presents a new, compensation-based power flow method, for the solution of weakly meshed distribution and transmission networks. This technique is simple, straightforward, computationally efficient and numerically robust. Extensive study of the performance of this compensation-based power flow scheme shows that it is significantly more efficient than the Newton-Raphson power flow technique when used for solving radial and weakly meshed distribution and transmission networks.

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APPENDIX

At the k^{th} iteration, the reactive power injection required to maintain the voltage at the generator bus i, can be calculated using the secant method:

$$Q_i^c = \frac{Q_i^{(k-1)} - Q_i^{(k-2)}}{|V_i^{(k-1)}| - |V_i^{(k-2)}|}\left[\,|V_i^s| - |V_i^{(k-1)}|\,\right] + Q_i^{(k-1)}$$

where $|V_i^{(k-1)}|$ and $|V_i^{(k-2)}|$ are the voltage magnitudes at the node i calculated in the previous two iterations (equation (3) in step 3). The actual reactive power injection is determined as:

$$Q_i^{(k)} = Q_i^c \quad \text{if} \quad Q_i^{min} \le Q_i^c \le Q_i^{max}$$

$$Q_i^{(k)} = Q_i^{max} \quad \text{if} \quad Q_i^c > Q_i^{max}$$

$$Q_i^{(k)} = Q_i^{min} \quad \text{if} \quad Q_i^c < Q_i^{min}$$

where Q_i^{min} and Q_i^{max} are the respective minimum and maximum reactive power limits for the generator node i.

Discussion

R. P. Broadwater and **A. Chandrasekaran** (Tennessee Technological University, Cookeville, TN): The authors are to be complimented for addressing an area that has received little attention, distribution power flow analysis.

Radial Distribution System Analysis

1) A suggested improvement to the radial power flow is to sum load powers and power losses in the reverse trace (i.e., moving from the ending buses to the source bus) instead of summing load currents. This suggestion has been tested on a four-line section system as illustrated in Fig. D.1. For a nominally loaded case, both methods converged in four iterations. However, for a very heavily loaded case, the method of summing the

currents in the reverse trace diverged, whereas the method of summing the powers converged.

A brief and heuristic explanation of this phenomenon is as follows. Initially, when the currents are summed in the reverse trace, each current will contain an error proportional to the initially guessed voltage. If the initially guessed voltages are maintained constant and a succession of power flow problems are solved in which the loads are continually increased, the errors that are proportional to the initially guessed voltages will grow. For a sufficiently heavily loaded system, the initially guessed voltages fall outside the region of convergence, and the algorithm will diverge.

When the powers are summed in the reverse trace, the errors that exist when the source bus is reached involve only the power losses, and not the load powers. The power losses are always a small fraction of the load powers. Hence, using the power sum leads to good convergence for even heavily loaded systems.

2) Even though it is mentioned that the multiphase unbalanced systems are easily handled, the convergence characteristics claimed for the single-phase system may get severely impaired, since the error involved in current summation may become excessive.

3) The details of the distribution networks given in Table 1 do not include the loading levels of the systems. It would be instructive to know whether the systems are nominally loaded or lightly loaded. Further, the number of nodes may not be a direct indication of the size of the system since distributed loads can be modeled using any number of node points.

Weakly Meshed Transmission Systems

4) The impedance matrix of (7) of the paper appears to be the loop impedance matrix of the system with the breakpoint currents chosen as the loop currents. At the exact solution, the breakpoint voltage vector must go to zero. There appears to be a paradox here since for the constant impedance matrix assumed the solution for the currents is either trivial or infinite.

5) The handling of PV buses explained in section V does not mention whether the alternating current directions in the traces would affect convergence if the P value is higher than "downstream" loads.

General

6) The CPU time given in the table is said to include the initial processing time also. This may show the WSCC Power Flow Program in a bad light. Exact CPU time required for the iterations alone should be a better index.

7) In Fig. 2, the numbering of the distribution network is laid out on a grid in a very orderly fashion. With this scheme, it appears that choosing buses at large load centers may lead to conflicts. For instance, in Layer 2

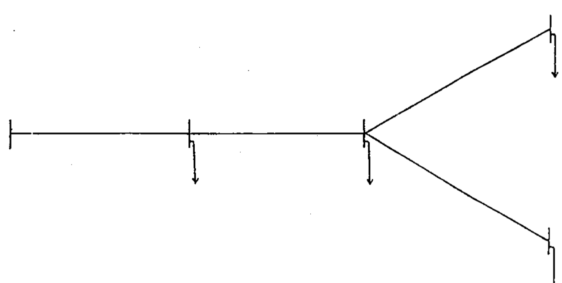

Fig. D.1. Four-line section test system.

suppose that a large load exists at the center of branch 4, but there is little or no load in the centers of branches 5 and 6. With the author's proposed scheme, it appears that additional buses would be inserted in branches 5 and 6 that would not be necessary for accurate load modeling.

8) It appears that if a component is inserted or added somewhere in the network, a complete renumbering of the network is required. Is this true?

9) Even though the WSCC Power Flow Program is shown to converge more slowly for the examples chosen in comparison to the proposed method, it is a moot point whether a few extra seconds of CPU time alone would be sufficient to choose any alternative method. The Newton's method has been so finely honed during the last two decades that negative experiences must be very few to be deemed almost nonexistent. Hence, the virtues of the proposed method must be highlighted from a different viewpoint.

K. Aoki, K. Nara, and **T. Sato** (Hiroshima University, Higashihiroshima, Japan): The authors have written an interesting paper on a load flow problem of radial power system. Although many algorithms have been developed for a load flow problem, the discussers believe that it is effective to develop an algorithm which utilizes special features of the problem; the radial power system is one of such special features. In load flow problem of a radial power system, the number of variables or computational burden can be reduced extremely by taking voltages of the ends (or tips) of the branches as independent variables, in comparison with taking all node voltages as variables. The method developed by the authors also utilizes such features. The discussers have a question that it might be fast to make a reduced variable problem as above and solve it directly by the Newton–Raphson method. Could the authors comment on whether they have ever tried such a reduced variable Newton–Raphson method, and if tried, how was the comparison of computation time between the two methods?

N. Vempati, B. K. Chen, and **R. R. Shoults** (University of Texas at Arlington, Arlington, TX): The authors have presented an algorithm for the load flow solution of weakly meshed distribution and transmission networks. The simplicity of the algorithm makes it a very interesting paper to read. However, the practical limitations of the algorithm prevent its general usage for networks with multiple loops. It would be inadvisable to have two programs, one to deal with strongly meshed networks and yet another for weakly meshed ones.

The results were based on tests performed on radial and weakly meshed networks using the positive sequence representation. The authors attempt to extrapolate the results to three-phase distribution and transmission networks without a report of such an analysis. Until such results are presented, the efficacy of such an algorithm is still in doubt.

The authors make an erroneous observation that power flow algorithms for meshed distribution systems have not been developed. One of the algorithms referred to [6] has the ability to analyze three-phase networks, irrespective of the complexity of the meshing. However, this algorithm was designed for distribution systems and therefore has the limitation of only one swing bus and no other voltage-controlled (P-V) bus. Subsequently this algorithm was modified to accept numerous P-V buses, thereby enabling its usage in the analysis of three-phase transmission networks [A]. We feel that the algorithm [6] based on an implicit Z-bus (i.e., bifactored Y-bus) formulation is superior to the one proposed by the authors.

How do the authors propose to model the open-wye/open-delta transformers in the distribution system analysis? Our simulations have shown that the injection currents due to the model are so large that the currents due to the loads are negligible. This affects the convergence characteristics of the algorithm directly. Any insight into the modeling and simulation of these transformers would be welcome.

References

[A] B. K. Chen, "Transmission System Unbalance Analysis," Ph.D. Diss., The University of Texas at Arlington, December 1986.

M. E. Baran and **F. F. Wu** (University of California, Berkeley, CA): This paper points out the reasons why a special power flow for distribution systems is needed and provides a computationally attractive method. It is a valuable contribution.

There are a few points on which we would appreciate the authors' clarification.

1) How does the convergence of the method for radial networks depend on the system parameters, in particular, the line resistances?

2) The application of the compensation method for weakly meshed networks is a very clever idea. The method uses an approximate of the linearized V-J function at the breakpoint. We wonder if the convergence of the method is always monotonic as implied in Fig. 8 or it exhibits oscillatory behavior.

3) In calculating the breakpoint impedance matrix, the authors have observed that the corresponding power flow solutions can be achieved in one iteration. Is this also true in cases where the shunts in the system are significant and cannot be neglected?

Dr. Dromey: The authors have investigated an aspect of load flow analysis that has received little attention in the literature to date. The wide range of R/X ratios in a distribution system can lead to difficulty with convergence and this problem is aggravated by the presence of adjacent long and short branches. In particular, the Newton–Raphson and decoupled algorithms can, in some circumstances, fail to converge for larger or ill-conditioned systems. The method presented in the paper produces an optimal ordering for solution and is very efficient, particularly where the number of loops is limited and the system is essentially radial in nature.

A number of questions arise in connection with the results. The paper infers that constant power loads are assumed. What is the effect on convergence for loads that have a mixture of p/v and q/v characteristics? No mention is made of the use of convergence factors. Was this studied and are there indications of optimal factors that can be considered generic for the radial solution and for breakpoint injection currents?

The authors mention that the efficiency of the algorithm deteriorates with the increase in the number of loops. In a large city with low voltage downtown networks, the number of loops required to interconnect a large number of radial sections can be in the hundreds. There is the additional problem of a significant number of very short cable sections used to balance the flows between sections. What degree of deterioration can be expected in the efficiency of solution for such a network where the coupling can be quite strong?

There is a significant advantage in being able to solve the networks described above on a desktop microcomputer which will be restricted in the memory available for processing. Would the authors like to suggest a sensible method of partitioning such a network to achieve solutions in an acceptable time? How would the compensation method be modified to account for the partitioning?

D. Shirmohammadi, H. W. Hong, A. Semlyen, and G. X. Luo: We would like to thank the discussers for their interest in our paper and for their questions and comments. Many of these constitute contributions to the topic of the paper. We will give our answers to each discusser separately.

Messrs. Broadwater and Chandrasekaran:

(1) We believe that the discussers' suggestion of adding up powers rather than currents in the backward sweep is interesting and their explanation quite plausible. We suspect however that the improved convergence is obtained with more computations. Moreover, we have not experienced any problems in dealing with heavily loaded networks and low node voltages. In fact, in the 544 node network example of the paper, due to heavy loading and lack of VAR support the voltages of some of the nodes are as low as 0.75 per unit. This network was efficiently solved by the proposed algorithm.

(2) The process of the attenuation of the errors described in the section of the paper on "Convergence Criterion" is completely general and would apply to multi-phase unbalanced as well as single-phase networks.

(3) All three examples included in the paper represent peak load conditions on the feeders. These were 4 MW, 3 MW, and 8 MW loads on the 244 node, 544 node, and 1411 node networks, respectively.

(4) We would like to clarify the discussers' question related to equations (6) and (7): the voltages there are internal Thevenin voltages while the breakpoint voltages are zero since the breakpoint ports are shortcircuited. We would also like to point out that the actual relationship between the break point voltages and currents is established through equation (8).

(5) We have not experienced any problem in the overall convergence of the proposed method in solving a variety of networks that included P,V buses. We agree, however, that there may be more efficient ways of handling P,V buses in order to minimize the impact on the convergence characteristics of the method.

(6) In contrast to the discussers assumption, the inclusion of initial processing time puts in more favorable light the WSCC Power Flow Program. For example, the time required for each iteration of the proposed algorithm is around 0.005 seconds for the 244 node example of the paper. At the same time, every iterator of the Newton-Raphson based WSCC Power Flow Program took 0.08 seconds for the same network (16 times more). Similar results were obtained for the 544 node and 1411 node networks.

(7) In our distribution network model we have assumed that loads are concentrated at network nodes. As a result, the introduction of a load at the center of branch 4 means the addition of a new node at this location. This will also add a new branch to the network. If there are no loads at the end node of branch 5, this branch and branch 10 can be combined and represented with a single branch. Branch 6 must exist, because two separate branches emanate from its end node.

(8) There is no need for the renumbering of the entire network as a result of the insertion or deletion of network components. Only branches in the layers below the inserted or deleted component must be renumbered.

(9) We do not agree with the discussers view on the merits of the proposed algorithm. These are well documented throughout the paper. Furthermore, we would like to point out that many major advances in the development of power system analysis techniques have resulted from methodologies that exploit the special structure of the power system. An example is the Fast Decoupled Power Flow which provides improved efficiency of "a few CPU-seconds" in the solution of the transmission network by exploiting the low R/X ratios prevalent in these networks. Yet, the impact of the Fast Decoupled Power Flow in the field of the transmission network analysis has been very significant.

Messrs. Aoki, Nara, and Sato:

As pointed out by the discussers, special features of particular load flow problems can be exploited to produce more efficient solution methods. In the approach of the paper, the transversal elements have been lumped with the loads (see our answers to Messrs. Vempati, Chen, and Shoults): this has made the factorization and the subsequent algebraic operations with the L and U matrices of a nodal approach equivalent to using tree-branch voltages and currents. These matrix calculations are implicit in the method and did not have to be performed explicitly. This is the clue of the computational efficiency of branch-oriented calculations in a radial network. However, at each node we have had to enforce the power (P, Q) constraints. Therefore, all bus voltages are used as variables. Because of this only little improvement of efficiency can be derived from the radial network structure in a Newton-type load flow.

Messrs. Vempati, Chen, and Shoults:

It is true that the solution method of the paper becomes less efficient as the number of loops increases. However, the overwhelming majority of all distribution networks and also the transmission systems of many countries are weakly meshed. Therefore, a special program, if it is more efficient than a general one, is certainly of interest. Very often advances in any field of knowledge are based on the special structure of a particular problem. One could cite innumerable examples where special programs are developed and used for particular situations.

We have not yet attempted the application of our method to multi-phase networks. However, as we explained in our response to Messrs. Broadwater and Chandrasekaran we do not foresee any deterioration in the convergence characteristics of this method when applied to multi-phase networks.

With regard to our statement about the lack of power flow algorithms for meshed distribution network, we note that we made two erroneous remarks. First, as the discussers rightly argue, the Z-bus solution algorithm of Ref. [6] is capable of solving meshed distribution networks. Second, we stated that Ref. [6] proposes a popular algorithm for distribution network analysis. Based on further investigation, however, we have found that this solution method is by no means popular among distribution engineers.

The discussers raise the problem of the relative computational efficiency of the method of Ref.[6] compared to the method of the paper. It is easy to show that a nodal approach (for example the one of Ref.[6]) requires more computation than the branch-oriented approach of the paper. We will show that the two differ computationally by two facts:

(a) The computations in the branch-oriented method are equivalent to the forward and backward substitutions of the nodal method but the factorization of a matrix is not needed.

(b) The forward and backward substitutions in a nodal approach are replaced in the branch-oriented method by additions and subtractions and no multiplications and divisions are needed.

Consider, for example, the simple radial network of Fig.D.1 of the discussion by Messrs. Broadwater and Chandrasekaran. Connect the root node to ground via a voltage source with V=0 but do not connect impedance branches to ground from the other nodes. Number the nodes and branches moving outward from the root. We will then have branch and bus voltages and currents. We can relate these to each other by the incidence matrices A_V and A_I,

$$V_{bus} = A_V V_{br} \quad \text{and} \quad I_{br} = A_I I_{bus} \tag{a}$$

It can be seen that the incidence matrices are square, lower triangular, and consist of elements ±1 only. Their inverses could also have been formed directly by inspection of the network graph. This reflects the fact that equations (a) represent directly the two Kirchhoff's laws. They correspond to the forward and backward sweeps used in the paper.

Let Y_{br} (diagonal) and Y_{bus} be the admittance matrices of the network. We have

$$Y_{br} = A_I \, Y_{bus} \, A_V \tag{b}$$

Clearly, the two incidence matrices are the transpose of each other.

Let us now solve the nodal problem

$$Y_{bus} \, V_{bus} = I_{bus} \tag{c}$$

Factorization of Y_{bus} yields

$$Y_{bus} = L \, D \, L^T \tag{d}$$

Comparing (d) with (b) we obtain

$$L^{-1} = A_I, \quad L^{-T} = A_V, \quad \text{and} \quad D = Y_{br} \tag{e}$$

so that the solution of (c) becomes

$$V_{bus} = L^{-T} D^{-1} L^{-1} I_{bus} = A_V \, (Z_{br} \, (A_I \, I_{bus})) = A_V \, (Z_{br} \, I_{br}) = A_V \, V_{br} \tag{f}$$

Equation (f) shows that the branch-oriented solution reproduces the matrix operations of the nodal approach without the need of preliminary formulation and factorization of a bus admittance matrix. To achieve this, it was essential to replace all shunt branches connected to buses by corresponding injections.

We note that reference [6] claims that the efficiency of its algorithm is comparable to that of the Newton-Raphson Power Flow while we have shown a substantial improvement over the Newton-Raphson Power Flow method using our algorithm.

We appreciate the information provided by the discussers on problems related to transformer modeling. We have not investigated this topic.

Messrs. Baran and Wu:

(1) The convergence of the method for a radial network is linear and dependent essentially on the line impedances $|Z|$ and the apparent powers $|S|$ of the loads. This can be seen from the following simplified convergence analysis, for a single line of impedance $Z = R + jX$ and load $S = P + jQ$. For this, eqn.(1) becomes $I = S^*/V^*$ or, in incremental form,

$$\Delta I = -\frac{S^*}{V^{*2}} \Delta V^* \tag{g}$$

The resultant change in voltage is, taking (g) into account with $V \approx 1$,

$$\Delta V^{new} = -Z \Delta I \approx Z S^* \, \Delta V^* \tag{h}$$

This equation shows that the convergence rate of $|\Delta V|$ (not $\Delta |V|$!) is given by $|ZS| = |Z| \, |S|$. It depends only indirectly on line resistances.

(2) Fig. 8 of the paper is used only to depict the basic mechanism of the fixed tangent solution algorithm used for calculating breakpoint current injections according to equation (8). Nevertheless, our experience with all distribution networks studied indicates a monotone and rapid convergence in the calculation of breakpoint currents.

(3) After a considerable amount of experimentation we found that a better convergence in the calculation of breakpoint currents can be achieved when the breakpoint impedance matrix is calculated neglecting the shunt components (mainly capacitors). Unfortunately this important conclusion is not reflected in the paper and we would like to thank the discussers for providing us with this opportunity.

Dr. Dromey:

The method presented in the paper can handle any load characteristics, since the load current is calculated at each step as a function of the prevailing voltage. We did not use however any convergence (accelerating) factors. We feel that such factors (not necessarily uniform and real) could improve the convergence of the method and appreciate the suggestion implied in the question.

We have developed our methodology and the accompanying program mainly for primary distribution networks. Primary distribution networks are, in almost all cases, either radial or weakly meshed. The program is, however, capable of representing up to 5000 nodes and 300 loops which is adequate for almost all practical cases including secondary distribution networks in downtown metropolitan areas. Even with such large dimensions, the memory requirement for the program is less than 500 Kbytes which makes it ideal for microcomputer applications. The largest network studied using this program consisted of 2411 nodes and 183 loops. It took the program a total CPU time of 12 seconds to process the input data, solve the power flow and print the results for this network on an IBM 3090-200 computer, which is very reasonable considering the size of the network.

Partitioning of distribution networks, or even of transmission networks, involves network equivalencing. Reference [A] is pertinent to this topic.

[A] F.F. Wu, A. Monticelli, "A Critical Review on External Network Modelling for On-Line Security Analysis", Electrical Power and Energy Systems, Vol. 5, October 1983, pp. 222-235.