IEEE Transactions on Power Delivery, Vol. 5, No. 1, January 1990

NEW METHOD FOR THE ANALYSIS OF DISTRIBUTION NETWORKS

Renato Cespedes G., Senior Member, IEEE
Systems Control - Division of Power Automation, Cra. 4 No.69-45
Bogota, Colombia

ABSTRACT

This paper presents a new method for the solution of load flow in radially operated distribution networks. The method is based on an electric equivalent and in the elimination of the voltage phase angle in the equations to be solved which permits to obtain the exact solution working only with voltage magnitudes. In this way, a simple and efficient method for the exact load flow solution for this type of networks is obtained, allowing the modelling of voltage dependent loads and the formulation of related problems like the radial load flow with voltage constraints. The method is easy to be programmed and has good convergency characteristics as shown by examples.

I. INTRODUCTION

The analysis of distribution systems is a matter of actual interest. In effect, recent papers are dedicated to several topics in this field among which:

- Single phase radial load flow.
- Three phase radial load flow.
- Optimal sizing and location of shunt capacitors.
- Calculation of feeder and transformer losses.

The previous is the result of a growing interest of the electrical utilities to increase their efficiency in what is related to the operation and management of electric energy distribution.

In teaching, the analysis of distribution systems has been given importance since the models used at medium voltage levels do not allow, in most cases, to make assumptions and simplifications that are valid in models for higher voltages covering in consequence a broad type of cases from which the balanced case is a particular case only. Phase unbalance together with the load models, including their variation with voltage and frequency has been receiving particular attention by universities [1],[6].

A distribution network is normally integrated by several primary feeders at voltage levels mostly in the range of 4 to 35 kV. A number of utilities design these feeders to be operated radially, that is, each feeder with only one point of power supply at a specific moment.

89 SM 604-0 PWRD A paper recommended and approved by the IEEE Transmission and Distribution Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1989 Summer Meeting, Long Beach, California, July 9 - 14, 1989. Manuscript submitted August 25, 1988; made available for printing April 21, 1989.

However, the possibility of transfering loads from one feeder to another normally exist in case of failures. Although a general load flow program can solve in most cases radially operated networks, a specific radial load flow is considered more suitable for this particular application due to the following reasons:

-High resistance/reactance ratio not found in the high voltage networks [3]

- Better efficiency and simplicity of alternate algorithms for radially configured networks than the Newton-Raphson method [4]

This work presents a new method to solve the radial load flow. The basic goals defined for the method were:

- The radial load shall use the voltage magnitude as the variable of most interest. The voltage phase angle is not important in most studies related to distribution voltage levels. Besides, the differences among voltage phase angles in a feeder do not exceed a few degrees.

- The method shall allow to define the voltage magnitude at any node being it possible to calculated the remaining voltage magnitudes in the feeder from the defined one.

- The load shall be represented with its variation with respect to the voltage magnitude.

- The method shall be applicable to single and three phase radial load flows.

- The algorithm shall be comparable in terms of speed, convergency and computer resources used to other methods reported by the technical literature [4],[6].

The theoretical bases and the practical results of the proposed method are presented in the following sections.

II. DISTRIBUTION NETWORK MODEL

Triphase Balanced Network

This model assumes that the triphase system can be represented by its equivalent one line system. The model comprises the following elements:

- Distribution lines: represented by the resistance and reactance in per unit. Line shunt capacitance (different to shunt capacitor banks that are considered as loads) is negligible at the distribution voltage levels as found in most practical cases. For special cases like in long radial lines the line capacity reactive injection can also be considered in the reactive loads.

0885-8977/90/0100-0391$01.00

NINE NODES EXAMPLE

One line diagram

```
_______________________________ 0
              |
              | 1
     ------------------- 1
     |             |
     | 2           | 6
     ------- 2   ------------------- 6
     |             |           |
     | 3           | 7         | 8
     ------- 3   ------- 7   ------- 8
     |                         |
     | 4                       | 9
     ------- 4               ------- 9
     |
     | 5
     ------- 5
```

Connectivity

branches		nodes
1	->	0
2	->	1
3	->	2
4	->	3
5	->	4
6	->	1
7	->	6
8	->	6
9	->	8

FIGURE 1

- Loads: all loads, including shunt capacitors for reactive power compensation, are represented by their active (Po) and reactive (Qo) component at 1.0 per unit (pu). The effect of voltage variation is represented as follows:

$$P = Po * V^k \quad (1)$$
$$Q = Qo * V^k \quad (2)$$

where:

V is the voltage magnitude
k = 0 for constant power loads (type 0)
k = 1 for constant current loads (type 1)
k = 2 for constant impedance loads (type 2)

The value of k may be different according to the load characteristics.

Triphase Unbalanced Network Model

The adopted unbalanced triphase network model is the one presented in [6]. This model is based on the following:

- Any distribution line is formed by one, two or three phases and the neutral conductor. The neutral is considered to be connected directly to ground thus at the same voltage (however neutral conductors may have different current flows according to the amount of phases unbalance). This model limitation is found valid for a great number of practical cases.

- The triphase loads are considered to be integrated by three single phase loads connected in a star configuration. Every single phase load is represented by an equation similar to (1) and (2). Other load configurations can be easly converted to the star configuration for example by a delta-star transformation.

Distribution Network Description

The branch and node nomenclature suggested by Rajagopalan [4] is adopted. The nomenclature gives the branches a number that coincide with one of the end nodes of the same, which allows the representation of the network by a single vector. The nine node example of figure 1 illustrates the method.

The previous, only employs a single vector to represent a feeder and can be extended to represent a complete distribution network as follows.

A branch can be considered to be "oriented" towards the source node; this is illustrated by an arrow in the list of nodes and branches of the previous example. In order to solve the load flow with the proposed method, as explained in the next section, the tree formed by all the branches shall be followed in a systematic form, starting with the last node and ending at the source node. In case of only one source node this can be achieved in a "once through" calculation without requiring the "orientation" concept for the programming.

However, in case of more than one source node, the previous concept is important together with the "end" node concept. "End" node is defined as any node that has only one branch connected to it. In the nine node example, nodes 0, 5, 7 and 9 are "end" nodes (they can be recognized easily since they do not appear in the node's vector of the right hand). It becomes evident that starting from all end nodes and following the orientation given by the current, the tree can be followed systematically.

The previously presented method allows to analyze a network that is fed by more than one source node. In the previous example assume that branch 1 to 6 is opened and that an alternate source is connected to node 9. In this case the branches orientation are given in the following way:

	branches		nodes	
	1	->	0	(Source)
	2	->	1	
	3	->	2	
	4	->	3	
	5	->	4	
	6	Open	1	
	7	->	6	
	8	<-	6	
(Source)	9	<-	8	

III. RADIAL LOAD FLOW

Basic Aspects

The solution proposed for the load flow problem is to solve for every branch the following basic equation (see details in appendix A):

```
    Source Side                    Load Side

          Vs                            Vr
     |____________->_______________|______
     |          R + jX             |      |
    node s                      node r    |
                                          |
              FIGURE 2                   P,Q
```

$$Vr^4 + [2*(PR + QX) - Vs^2] * Vr^2 + (P^2 + Q^2)*(R^2 + X^2) = 0 \quad (3)$$

where:

s: source side node
r: load side or receiving node
Vs: source side voltage magnitude
Vr: load side voltage magnitude
P , Q: active and reactive load, calculated according to (1) and (2)
R , X: branch resistance and reactance

Equation (3) has a straightforward solution and does not depend on the phase angle which simplifies the problem formulation as shown latter. It is to be noted that from the two solutions for Vr^2 only the one considering the positive sign of the square root of the solution of the quadratic equation gives a realistic value. Same is applicable when solving for Vr.

P,Q as required by the proposed solution is the total load fed by node r, comprising the load of the node itself plus all other loads fed through it, including losses. In other words, P,Q is the exact equivalent of the network connected to node r. Also the sum of all active, reactive loads fed through node r is a good estimate to the equivalent, being the difference originated by the -not in advance known- losses and the dependance of P,Q on the voltage magnitude, also to be calculated.

Active and reactive losses as required latter can be calculated as follows:

$$Lp = R * (P^2 + Q^2)/Vr^2 \quad (4)$$
$$Lq = X * (P^2 + Q^2)/Vr^2 \quad (5)$$

where:

Lp: active losses of the branch
Lq: reactive losses of the branch

Proposed Solution Method: Balanced System

The proposed method can be summarized in the following algorithm:

1) Read network data including parameters, topology description, voltage magnitude at the source(s) node(s), loads at nominal voltage.

2) Assume a voltage magnitude for each node for the initial load estimation and calculation of loads dependent on the voltage magnitude.

3) Calculate the equivalent for each node summing all loads of the network fed through the node, including losses. This is the "upstream" iteration, form end nodes to source nodes.

4) Starting from the source(s) node(s) and using equation (3), calculate the load voltage (Vr) for all loads. This is the "downstream" iteration, from source nodes to end nodes.

5) With the new voltages recalculate the losses. If the total losses variation with respect to the previous calculated value is greater than a specified error, go to step 3). Otherwise, calculate all other required results, such as currents for example.

29 Node Example

The network of 29 nodes of [4] and shown in figure 3 is taken as an example to show the characteristics of the proposed method. The basic data for this case are given in per unit:

br. nod.	P	Q	R	X	V
1 -> 0	0.00	0.00	0.0236	0.0233	0.926474
2 -> 1	0.00	0.00	0.0003	0.0002	0.925186
3 -> 2	0.16	0.08	0.0051	0.0005	0.924259
4 -> 2	0.16	0.08	0.0062	0.0006	0.924060
5 -> 2	0.00	0.00	0.0032	0.0011	0.918671
6 -> 5	0.16	0.08	0.0030	0.0003	0.918127
7 -> 5	0.16	0.08	0.0030	0.0003	0.918127
8 -> 5	0.16	0.08	0.0079	0.0008	0.917229
9 -> 5	0.00	0.00	0.0013	0.0008	0.916616
10-> 9	0.16	0.08	0.0033	0.0003	0.916010
11-> 9	0.16	0.08	0.0050	0.0005	0.915693
12-> 9	0.16	0.08	0.0027	0.0003	0.916115
13-> 9	0.00	0.00	0.0008	0.0005	0.915899
14->13	0.16	0.08	0.0025	0.0003	0.915441
15->13	0.16	0.08	0.0026	0.0003	0.915418
16->13	0.16	0.08	0.0065	0.0007	0.914711
17->13	0.16	0.08	0.0041	0.0004	0.915146
18-> 2	0.00	0.00	0.0012	0.0007	0.923047
19->18	0.16	0.16	0.0011	0.0001	0.922839
20->18	0.16	0.16	0.0061	0.0006	0.921936
21->18	0.00	0.00	0.0012	0.0008	0.921415
22->21	0.16	0.16	0.0008	0.0003	0.921253
23->21	0.16	0.16	0.0034	0.0003	0.920792
24->21	0.00	0.00	0.0009	0.0005	0.920597
25->24	0.16	0.16	0.0030	0.0003	0.920056
26->24	0.16	0.16	0.0032	0.0003	0.920015
27->24	0.00	0.00	0.0009	0.0006	0.920176
28->27	0.16	0.16	0.0060	0.0007	0.919065
29->27	0.16	0.16	0.0016	0.0002	0.919886

The last column in the previous table also presents the voltage magnitude results for case No. described below.

Test Cases

The following test cases for this example were run:

Case No.1: all loads of type 0, that is constant P and Q, independent to voltage magnitude of every node.

Case No.2: all loads of type 2, that is P and Q of the impedance type, depending on the square of the voltage.

The convergency criterion for these cases was established as follows:

{ABS(delta P) + ABS(delta Q)} less than Error

where

ABS	indicates absolute value
delta P	total active losses difference computed between two iterations
delta Q	total reactive losses difference computed between two iterations
Error	convergence error limit selected as 0.001 pu

The following are some additional data or results for case No.1:

Source Voltage: 1.05 pu.
Total active losses: 0.384895 pu.
Total reactive losses: 0.365985 pu.

For case 2, excerpts of the results are the following:

br. nod.	P	Q	V
1 -> 0	0.00	0.00	0.943755
2 -> 1	0.00	0.00	0.942645
3 -> 2	0.141893	0.070947	0.941839
.....			
18-> 2	0.00	0.00	0.940798
19->18	0.141525	0.070762	0.940617
20->18	0.141289	0.070645	0.939835
.....			
27->24	0.00	0.00	0.938320
28->27	0.140546	0.070273	0.937363
29->27	0.140761	0.070645	0.938080

Source Voltage: 1.05 pu.
Total active losses: 0.285297 pu.
Total reactive losses: 0.271338 pu.

The following table summarizes the convergency found for these cases:

	Case No.1		Case No.2	
Iter.	delta P	delta Q	delta P	delta Q
1	0.298269	0.283743	0.329253	0.313220
2	0.086390	0.082021	0.055621	0.052996
3	0.000232	0.000220	0.014802	0.014094
4			0.004037	0.003844
5			0.001104	0.001051
6			0.000302	0.000287

The convergence of case No.1 compares well with the same test run with the method presented in [4] which takes 6 iterations for solving the same case. However the solution by the coupled Newthon Raphson method is found to take only two iterations for this case.

The slower convergence for case No.2 is due to the variation of the active and reactive loads with the voltage which are recalculated each iteration with the last available voltage magnitudes.

Regarding the time taken by the computation the following times are indicative for case No.1 with the proposed method, considering only CPU time taken to solve the case, and not taken into account the time required for Input/output:

- Digital MicroVax I, 2 Mbites of main memory: 0.1250 seconds of CPU time.

- Digital MicroVax III, 8 Mbites of main memory: 0.007810 seconds of CPU time.

It has also be found that the number of iterations is dependant on the voltage profile start. A good start is found to be putting all voltages equal to the source voltage, instead of the "flat start" with voltages equals to 1.0 pu. For repetitive cases in which only some input data is modified it was found better to start from a previously calculated voltage profile for the same feeder.

Extension of the Method to Unbalanced Systems

The proposed method can be extended to the load flow calculation of unbalanced networks considering the following aspects:

- Each branch is represented by a 3x3 matrix which is full or partially empty depending on its number of phases [6].

- The loads of the system are calculated in similar form to those of the balanced system (equations 1,2) applied to the load of each one of the phases.

IV. ADDITIONAL APPLICATIONS OF THE PROPOSED METHOD

In radial load flows problems, voltages magnitudes are generally subject to upper and/or lower limit constraints depending on the load level. In order to maintain the voltage within limits, without having to disconnect load or switch loads among feeders, two types of controls are commonly used:

- Adjust the source voltage level

- Connect or disconnect capacitors modifying the reactive load of the feeder.

For the first type of control it is necessary to know the source voltage that is required in case that one or more node voltage magnitudes or out of limits. The required source node shall maintain all voltages within limits.
The algorithm proposed for the constrained load flow is based on the following:

a. All network loads have net lagging power factor, that is, they may include capacitors but which compensate the reactive load to maximum a power factor equal to 1.0.

b. The lowest voltage magnitude, under the assumption a. above, is found in one of the end nodes.

c. If the voltage magnitude of the end node with the lowest value is set at its lower limit by adjusting the source voltage, all node voltage magnitudes will be within the end node and source voltage values. If in addition the source voltage is less than the upper permissible limit the voltage constraints are satisfied.

The proposed algorithm for this case is as follows:

1) Solve the load flow according to the basic proposed algorithm

2) Determine the lowest node of the network by comparing the voltages of the extreme nodes. Lets call this voltage the "lowermost" voltage and the corresponding node the "lowermost" node.

3) If all voltages are higher than the lower permissible limit, end.

4) Set the lowermost node voltage in the lower limit.

5) Use equation (3) to calculate the source voltage using the path between the lowermost node and the source node only. For this calculation the unknown is Vs. In case this voltage is higher than the upper limit, issue the message that other additional control actions are required and stop.

6) Recalculate all voltage magnitudes with the new defined source node.

The proposed solution is an approximation only since step 5) is proposed to be performed without recalculating the node equivalents. However, for practical cases this approximation is valid and the error found is minimum as shown in the following example.

Consider the same 29 node example presented previously. Assume that all voltages shall be comprised within 0.95 and 1.1 pu. The results for this case are the following:

node	V not adjusted	V with adjustment
0	1.05	1.08033
...	....	....
4	0.924960	0.959086
...	....	....
8	0.917227	0.952509
...	....	....
12	0.916115	0.951436
...	....	....
14	0.915441	0.950788
15	0.915418	0.950765
16	0.914711	0.950085
17	0.915146	0.950503
...	....	....
20	0.921936	0.957040
...	....	....
29	0.919886	0.955065

It is observed that the lowermost voltage is found in node 16. The source voltage is adjusted and all recalculated voltages are within limits. The small error found in the adjusted voltage of node 16 (0.000085 pu.) is due to the approximation explained previously. However, this error is considered negligible. If desired, this approximation can be avoided by recalculating the complete losses value in step 5. This has been found to not be necessary in practical cases.

The previous analysis was based on the adjustment of the source voltage to increase the voltages. The opposite case, that is, the reduction of the source voltage is easier to solve since under the stated assumptions the higher voltage is found at the source node only. In case this voltage is higher than the upper limit it is enough to solve the load flow with this voltage set at the limit and verify that the remaining node voltages are still higher than the lower limit.

Finally, it is also possible to define any node voltage in the network and using equation (3) solve for the remaining nodes. This is found for instance in case more strict voltage limits are imposed to certain nodes than to others, being it necessary to fix a voltage in an intermediate node in the network.

Other applications derived from the main algorithm proposed and based on the formulation of equation (3) have been found including reactive compensation to define the optimum bank capacitors to be installed as a complementary control action for voltage control, losses control related problems, etc.

V. CONCLUSIONS

This work presents a new method for the solution of radial load flows. The method is based on the use of voltage magnitudes only, being it possible to calculate a load flow with a formulation not based on complex quantities, by eliminating the voltage phase angle from the equations. No simplifications besides the ones resulting from the adopted power system modeling are made. The proposed method can be easly implemented in microcomputers due to its very simple formulation. The proposed method includes the possibility to define loads dependent on voltage, which is a realistic form of representing loads at distribution voltage levels. It has been found from the cases with which the method was tested, good and fast convergence characteristics. Also, for each iteration the configuration tree is followed only twice, once upwards and a second time downwards, without having to employ subiterations like in [6] for certain branch ramifications. As shown, a load flow with voltage constraints can be solved easily with the proposed formulation; it is considered that other applications can be found based on the proposed method opening new research possibilities in a field - distribution - that has perhaps has not received the required attention in the past.

BIBLIOGRAPHY

[1] Sun D. et all, "Calculation of energy losses in distribution systems" , presented to the PES Summer Meeting of the IEEE, Vancouver, Canada, July 1979.

[2] Stott B., "Review of Load Flow calculation methods", IEEE Proceedings Vol. 62 No.7, July 1984.

[3] Tripaty S.C. et all, "Load Flow solutions for ill conditioned power systems by a newton like method" IEEE PAS-101, October 1982.

[4] Rajagopalan S., "A new computational algorithm for load flow study of radial distribution system", Computer and Electr. Eng. Vol.5, pp. 225-231, Pergamon Press, 1978.

[5] Cespedes R., Gallego R. "Optimizacion de la configuracion de redes de distribucion para su operacion economica", conference paper presented at the CIER meeting, Brasil 1986.

[6] Kersting W.H., "A method to teach the design and operation of a distribution system", IEEE PAS-103, No.7, pp.1945-1952, July 1984.

[7] Kaplan M., "Optimization of number, location, size, control type , and control setting of shunt capacitors on radial distribution feeders", IEEE PAS-103, No.9, pp.2659--2665, September 1984.

[8] G.W Stagg, A.H. El-Abiad, "Computer Methods in Power System Analysis", Mc Graw Hill, 1968.

APPENDIX A

DEDUCTION OF THE BASIC EQUATION

For any branch of the network (see figure 2), and being the letters in boldface and with a upper bar complex quantities, the following equations are applicable:

$$\bar{\mathbf{V}}\mathbf{s} - \bar{\mathbf{V}}\mathbf{r} = \bar{\mathbf{I}} * (R + jX) \qquad (A.1)$$

$$\bar{\mathbf{V}}\mathbf{s} - \bar{\mathbf{V}}\mathbf{r} = (P - jQ) * (R + jX) / \bar{\mathbf{V}}\mathbf{r}^* \qquad (A.2)$$

$$Vs\ Vr\ [\cos(p_s - p_r) + j \sin(p_s - p_r)] - Vr^2 = (P - jQ)*(R + jX) \qquad (A.3)$$

where

p_s, p_r are the phase angles of $\bar{\mathbf{V}}\mathbf{s}$ and $\bar{\mathbf{V}}\mathbf{r}$ respectively and $\bar{\mathbf{V}}\mathbf{r}^*$ is the notation for complex conjugate.

Separating real and imaginary parts the following equations are obtained:

$$Vs\ Vr \cos q - Vr^2 = (PR + QX) \qquad (A.4)$$

$$Vs\ Vr \sin q = (PX - QR) \qquad (A.5)$$

where: $q \equiv p_s - p_r$

Taking cos q from (A4) and sin q from (A.5) and summing the square of these two quantities the following equation is obtained:

$$Vr^4 + [2*(PR + QX) - Vs^2]*Vr^2 + (P^2 + Q^2)*(R^2 + X^2) = 0 \qquad (3)$$

Renato Cespedes G. is Engineer of the University of the Andes, Bogota - Colombia, and diplom Engineer and Doctor Engineer of the National Politechnique Institute of Grenoble, France.

He has worked as consultant to different electrical utilities in South and Central America and is currently staff engineer of Systems Control of Palo Alto a division of Power Automation of the Combustion Engineering group at the Bogota office. Dr. Cespedes is also lecturer at the National University of Colombia in Bogota. His main interests are the control of utility systems and the power system analysis in the area of control centers.

Dr. Cespedes is Senior Member of the IEEE and a member of the Association of Electric and Mechanical Engineers of Colombia.