1^o Grado en Informática.Matemáticas I.

Hoja 2. Resolución

1. a) $32 = 2^5 = \left(\frac{1}{2}\right)^{-5} \Rightarrow log_{\frac{1}{2}} 2^5 = log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-5} = -\frac{1}{5}$

   b) $\frac{1}{3} = 3{-}1 \Rightarrow log_3 \frac{1}{3} = log_3(3^{-1}) = -1$

   c) $0{,}001 = 10^{-3} \Rightarrow log_{10} 0{,}001 = log_{10}(10^{-3}) = -3$

   d) $\frac{1}{\sqrt{3}} = 3^{\frac{-1}{2}} \Rightarrow log_3 \frac{1}{\sqrt{3}} = log_3(3^{-\frac{1}{2}}) = -\frac{1}{2}$

2. a) $4 = 2^2; 8 = 2^3; 81 = 3^4 \Rightarrow \sqrt[4]{4} \cdot \sqrt[6]{8} \cdot \sqrt[8]{81} = 2^{\frac{2}{4}} \cdot 2^{\frac{3}{6}} \cdot 3^{\frac{4}{8}} = 2^{\frac{1}{2}+\frac{1}{2}} \cdot 3^{\frac{1}{2}} = 2\sqrt{3}$

   b) $12 = 3{\cdot}2^2; 27 = 3^3; 75 = 3{\cdot}5^2 \Rightarrow 5\sqrt{12} + \sqrt{27} - 8\sqrt{75} = 5\sqrt{3 \cdot 2^2} + \sqrt{3^3} - 8\sqrt{3 \cdot 5^2} = 5{\cdot}2\sqrt{3} + 3\sqrt{3} - 8{\cdot}5\sqrt{3} = -27\sqrt{3}$

   c) $\frac{(4 \cdot 6^{-2})^2}{2^3 \cdot 3^{-2}} = \frac{(2^2 \cdot (3 \cdot 2)^{-2})^2}{2^3 \cdot 3^{-2}} = \frac{(2^{(2-2)} \cdot 3^{-2})^2}{2^3 \cdot 3^{-2}} = \frac{(3^{-2})^2}{2^3 \cdot 3^{-2}} = \frac{3^{-2}}{2^3} = 2^{-3} \cdot 3^{-2}$

   d) $\sqrt[4]{x+1} \cdot \sqrt{(x+1)^3} = (x+1)^{\frac{1}{4}} \cdot (x+1)^{\frac{3}{2}} = (x+1)^{\frac{1}{4}+\frac{3}{2}} = (x+1)^{\frac{7}{4}} = \sqrt[4]{(x+1)^7} = (x+1)\sqrt[4]{(x+1)^3}$

   e) $\log_3\left(\frac{1}{3}\right) + \log_2\left(\frac{1}{8}\right) - \log_4\left(\frac{1}{16}\right) = \log_3\left(3^{-1}\right) + \log_2\left(2^{-3}\right) - \log_4\left(4^{-2}\right) = -1 - 3 - (-2) = -2$

   f) $3\sqrt{2} - 5\sqrt{8} + 2\sqrt{18} = 3\sqrt{2} - 5\sqrt{2^3} + 2\sqrt{2 \cdot 3^2} = 3\sqrt{2} - 5 \cdot 2\sqrt{2} + 2 \cdot 3\sqrt{2} = -\sqrt{2}$

   g) $\frac{10^3 \cdot 3^{-2}}{(6^2 \cdot 5)^3} = \frac{2^3 \cdot 5^3 \cdot 3^{-2}}{(2^2 \cdot 3^2 \cdot 5)^3} = \frac{2^3 \cdot 5^3 \cdot 3^{-2}}{2^6 \cdot 3^6 \cdot 5^3} = 2^{3-6} \cdot 3^{-2-6} \cdot 5^{3-3} = 2^{-3} \cdot 3^{-8}$

3. Realizamos el cambio $2^x = y$, con lo que la ecuación queda:

   $$y^2 - 3y + 2 = 0 \Rightarrow y = \frac{3 \pm \sqrt{9 - 4 \cdot 2}}{2} = \frac{3 \pm 1}{2} = \begin{array}{l} \nearrow 2 \\ \searrow 1 \end{array}$$

   Si $y = 2 \Rightarrow 2^x = 2 \Rightarrow x = 1$; si $y = 1 \Rightarrow 2^x = 1 \Rightarrow x = 0$, por lo que las soluciones son $x = 0$ y $x = 1$.

4. Despejando y en la primera ecuación: $y = 5 - x$, y sustituyendo en la segunda:

   $\log_2 x + \log_2(5-x) = 2 \Longleftrightarrow \log_2\left(x(5-x)\right) = 2 \Longleftrightarrow 2^{\log_2\left(x(5-x)\right)} = 2^2 \Longleftrightarrow x(5-x) = 4 \Longleftrightarrow 5x - x^2 = 4$

   Resolvemos la ecuación de segundo grado:

   $$x^2 - 5x + 4 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25 - 4 \cdot 4}}{2} = \frac{5 \pm \sqrt{9}}{2} = \frac{5 \pm 3}{2} = \begin{array}{l} \nearrow 4 \\ \searrow 1 \end{array}$$

   Si $x = 4 \Rightarrow y = 5 - 4 = 1$, y si $x = 1 \Rightarrow y = 5 - 1 = 4$.

5. a) $\frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}} \cdot \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{(\sqrt{2} - \sqrt{3})^2}{(\sqrt{2})^2 - (\sqrt{3})^2} = \frac{(\sqrt{2})^2 + (\sqrt{3})^2 - 2\sqrt{2}\sqrt{3}}{2 - 3} = \frac{2 + 3 - 2\sqrt{6}}{-1} = -5 + 2\sqrt{6}$

b) $\dfrac{7\sqrt{2}}{3-\sqrt{2}} \cdot \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = \dfrac{21\sqrt{2}+7(\sqrt{2})^2}{3^2-(\sqrt{2})^2} = \dfrac{21\sqrt{2}+14}{9-2} = \dfrac{7(3\sqrt{2}+2)}{7} = 2+3\sqrt{2}$

c) $\dfrac{4}{3\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{4\sqrt{2}}{3(\sqrt{2})^2} = \dfrac{4\sqrt{2}}{3\cdot 2} = \dfrac{2\sqrt{2}}{3}$

d) $\dfrac{2}{\sqrt[3]{4}} \cdot \dfrac{\sqrt[3]{4^2}}{\sqrt[3]{4^2}} = \dfrac{2\sqrt[3]{2^4}}{4^1} = \dfrac{2\cdot 2\sqrt[3]{2}}{4} = \sqrt[3]{2}$

e) $\dfrac{a+b}{\sqrt{a^2-b^2}} \cdot \dfrac{\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}} = \dfrac{(a+b)\sqrt{a^2-b^2}}{(\sqrt{a^2-b^2})^2} = \dfrac{(a+b)\sqrt{a^2-b^2}}{a^2-b^2} = \dfrac{(a+b)\sqrt{a^2-b^2}}{(a+b)(a-b)} = \dfrac{\sqrt{a^2-b^2}}{a-b}$

6. a) $\log_2(x) = \dfrac{-1}{2} \iff 2^{\log_2(x)} = 2^{\frac{-1}{2}} \iff x = 2^{\frac{-1}{2}} \iff x = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

b) $3^x = \dfrac{1}{27} \iff 3^x = \dfrac{1}{3^3} = 3^{-3} \iff x = -3$

c) $\log_x(2\sqrt{2}) = \dfrac{3}{2} \iff x^{\log_x(2\sqrt{2})} = x^{\frac{3}{2}} \iff 2\sqrt{2} = x^{\frac{3}{2}} \Rightarrow (2\sqrt{2})^{\frac{2}{3}} = (x^{\frac{3}{2}})^{\frac{2}{3}} \Rightarrow x = (2^{1+\frac{1}{2}})^{\frac{2}{3}} \Rightarrow x = 2$

d) $3\log_2 x + \log_2 1000 = \log_2 x^3 + \log_2 10^3 = \log_2 (x^3 \cdot 10^3) = \log_2 ((10x)^3) = 3\log_2 (10x)$

Teniendo en cuenta lo anterior:

$3\log_2 x + \log_2 1000 = 3 \iff 3\log_2 (10x) = 3 \iff \log_2 (10x) = 1 \iff 2^{\log_2 (10x)} = 2^1 \iff 10x = 2 \iff x = \dfrac{2}{10} = \dfrac{1}{5}$