1º Worksheet // 1º order ODE // BDSA   Raúl López González

① a) $(x^2y^2 - y)dx + (2x^3y + x)dy = 0$

$$\frac{dy}{dx} = \frac{-x^2y^2 + y}{2x^3y + x}$$

$x^2y^2 - y = M \} \Rightarrow M/dy = 2x^2y - 1$ No exact

$2x^3y + x = N \} \Rightarrow N/dx = 4x^2y + 1$

$\mu(x) \cdot [(x^2y^2 - y)dx + (2x^3y + x)dy] = 0$

$M = \mu(x)(x^2y^2 - y) \qquad M/dy = \mu(x)(2x^2y - 1)$

$N = \mu(x)(2x^3y + x) \qquad N/dx = \mu'(x)(2x^3y + x) + \mu(x)(6x^2y + 1)$

$M/dy = N/dx \Rightarrow 2x^2y \cdot \mu(x) - \mu(x) = \mu'(x)(2x^3y + x) + \mu(x) + 6x^2y\,\mu(x)$

$\| \mu'(x) = d\mu(x)/dx \| \quad \dfrac{d\mu(x)}{\mu(x)} = \dfrac{(-2 - 4x^2y)}{2x^3y + x}dx$

$$\ln|\mu(x)| = -2\int \frac{1 + 2x^2y}{x(1 + 2x^2y)} = -2\ln|x|$$

$$\mu(x) = x^{-2}$$

$[(x^2y^2 - y)dx + (2x^3y + x)dy]\,x^{-2} = 0 \Rightarrow \left(y^2 - \frac{y}{x^2}\right)dx + \left(2xy + \frac{1}{x}\right)dy = 0$

$M = y^2 - y/x^2 \Rightarrow M/dy = 2y - 1/x^2 \;//\; N = 2xy + 1/x \Rightarrow N/dx = 2y - \frac{1}{x^2}$

$\psi(x,y) = \int M dx = xy^2 + y/x + h(y) \rightarrow \dfrac{\psi(x,y)}{dy} = N \rightarrow h(y) = 0$

i) $(x^2-y^2+y)dx + (2yx - x)dy = 0$

$$\frac{dy}{dx} = \frac{-x^2+y^2-y}{2yx-x} \Rightarrow \begin{array}{l} M_0 = x^2-y^2+y \to M/dy = -2y+1 \\ N_0 = 2yx - x \to N/dx = 2y-1 \end{array} \quad \text{no exact}$$

$$\mu(x)\cdot[(x^2-y^2+y)dx + (2yx-x)dy] = 0$$

$$M = \mu(x)\cdot(x^2-y^2+y) \to M/dy = \mu(x)\cdot(-2y+1)$$

$$N = \mu(x)\cdot(2yx-x) \to N/dx = \mu(x)'\cdot(2yx-x) + \mu(x)(2y-1)$$

$$\frac{dM}{dy} = \frac{dN}{dx} \Rightarrow -2y\,\mu(x) + \mu(x) = \mu'(x)(2yx-x) + 2\mu(x)\,y - \mu(x)$$

$$\ln|\mu(x)| = \int \frac{(-4y+2)}{2yx-x}dx = -2\int \frac{2y-1}{x(2y-1)} = -2\ln(x)$$

$$\mu(x) = x^{-2}$$

$$\int M dx = \varphi(x,y) = x + \frac{y^2-y}{x} + h(y) \;//\; \frac{d\varphi(x,y)}{dy} = N \to h(y) = 0$$

g) $\frac{1}{2}y^2dx + (e^x - y)dy = 0$

$$\frac{dy}{dx} = \frac{-y^2}{2(e^x-y)} \Rightarrow \begin{array}{l} M_0 = y^2/2 \to M/dy = y \\ N_0 = e^x - y \to N/dx = e^x \end{array} \quad \text{no exact}$$

$$\mu(x)\cdot[\tfrac{1}{2}y^2dx + (e^x-y)dy] = 0 \to \begin{array}{l} M = \mu(x)\,y^2/2 \\ N = \mu(x)(e^x-y) \end{array}$$

$$M/dy = N/dx \Rightarrow \mu(x)\,y = \mu'(e^x-y) + \mu(e^x)$$

$$\ln|\mu(x)| = \int \frac{y-e^x}{e^x-y}dx = -\int \frac{e^x-y}{e^x-y} = -x$$

$$\mu(x) = e^{-x}$$

$$\varphi(x,y) = \int M dx = -e^{-x}\,y^2/2 + h(y) \;//\; d\varphi/dy = N \Rightarrow -ye^{-x} + h'(y) = N$$

$$h'(y) = 1 \to h(y) = y$$

$$\varphi(x,y) = (-e^{-x})\frac{y^2}{2} + y$$

1 g) $(5-6y+2^{-2x})dx = dy$ $\qquad M = 5-6y+2^{-2x} \to M/dy = -6$ No exact

$dy/dx = 5-6y+2^{-2x}$ $\qquad N = -1 \to N/dy = 0$

$\mu(x)\cdot[\,] \Rightarrow M = \mu(x)[5-6y+2^{-2x}]$ $\qquad dM/dy = -6\mu(x)$

$N = -\mu(x)$ $\qquad dN/dx = -\mu(x)'$

$dM/dy = dN/dx \Rightarrow \int -6dx = \int -d\mu/\mu(x) \Rightarrow e^{6x} = \mu(x)$

$\varphi(x,y) = \int M dx = \frac{5e^{6x}}{6} - y e^{6x} + \int 2^{-2x} e^{6x} dx$ $\quad \Big\| \; U = \frac{e^{6x}}{2^{2x}} \quad \frac{dU}{dx} = \frac{6e^{6x}2^{2x} - e^{6x}\ln(2)\cdot 2^{2x-1}}{2^{4x}} \Big\|$

$-\int \frac{1}{2(\ln(2)-3)} dU = \frac{-U}{2\ln(2)-6}$

$= \frac{-e^{6x}}{(2\ln(2)-6)\,2^{2x}}$

$\int M dx = \varphi(x,y) = \frac{5e^{6x}}{6} - ye^{6x} - \frac{e^{6x}}{(2\ln(2)-6)2^{2x}} + h(y) \to \| h(y) = c \|$

$dM/dy = N \to h'(y) = 0 \to h(y) = C$

k) $(x^2-3y)dx + xdy = 0$ $\qquad M = x^2-3y \to dM/dy = -3$ No exact

$N = x \to N/dx = 1$

$dy/dx = \frac{-x^2+3y}{x}$

$\mu(x)\cdot[\,] \Rightarrow M = \mu(x)(x^2-3y)$ $\qquad dM/dy = -3\mu(x)$

$N = \mu(x)x$ $\qquad \to dN/dx = x\mu'(x) + \mu(x)$

$dM/dx = dN/dy \Rightarrow d\mu/\mu(x) = -4/x \, dx \Rightarrow \mu(x) = x^{-4}$

$\varphi(x,y) = \int M dx = \frac{-1}{x} + y/x^3 + h(y) \to d\varphi/dy = N \to h'(y) = 0$

$h(y) = C$

$\varphi(x,y) = \frac{-1}{x} + \frac{y}{x^3} + C$

e) $(e^{2x}+3y-5)\,dx = dy$ $\quad M = e^{2x}+3y-5 \rightarrow \partial M/\partial y = 3$

$dy/dx = \underline{\underline{e^{2x}+3y-5}}$ $\quad \Rightarrow \quad N=-1 \rightarrow \partial N/\partial x = 0$ $\quad$ No exact

$\mu(x)\cdot[\;] \Rightarrow M = \mu(x)(e^{2x}+3y-5) \rightarrow \partial M/\partial y = 3\mu(x)$

$N = -\mu(x) \rightarrow \partial N/\partial x = -\mu(x)'$

$\partial M/\partial y = \partial N/\partial x \Rightarrow -3dx = d\mu/\mu(x) \rightarrow \underline{\underline{e^{-3x}}} = \mu(x)$

$\varphi(x,y) = \int M\,dx = \frac{-1}{e^x} - \frac{y}{e^{3x}} + \frac{5}{3e^{3x}} + h(y)$ // $\partial\varphi/\partial y = N \rightarrow h'(y)=0$

$h(y) = C$

$\underline{\underline{\varphi(x,y) = -e^{-x} - y\,e^{-3x} + \frac{5}{3}e^{-3x} + C}}$

a) $y' + y(1+x) = 0$ $\quad \rightarrow \quad -\int \frac{+dy}{dx\,y} = (1+x) \rightarrow -\ln|y| = x + \frac{x^2}{2} + C \Rightarrow$

$\frac{dy}{dx} = \underline{\underline{-y(1+x)}}$

$e^{\cdot\cdot} \Rightarrow \frac{1}{y} = e^{x+\frac{x^2}{2}} + e^c \Rightarrow \| e^c = k \| \Rightarrow$

$\underline{\underline{k = y\cdot e^{x+\frac{x^2}{2}}}}$ $\quad \mu(x) = \underline{\underline{e^{x+x^2/2}}}$

c) $ay\,dx + bx\,dy = 0$ $\quad c - \int\frac{dx}{bx} = \int\frac{dy}{ay} \Rightarrow c - \frac{1}{b}\ln|x| = \frac{1}{a}\ln|y|$

$dy/dx = \underline{\underline{\frac{-ay}{bx}}}$ $\quad \rightarrow$

$\| c = \ln k \| \Rightarrow (\ln|x|)^{-a/b} + \ln k = \ln|y| \Rightarrow$

$y = k/x^{-a/b} \rightarrow \underline{\underline{k = y\cdot x^{-a/b}}}$ $\quad \underline{\underline{\mu = x^{-a/b}}}$

① h) $xy^2 dy - (x^2+y^3)dx = 0$

$dy/dx = \frac{x^2+y^3}{xy^2} \to M = -x^2 - y^3 \quad N = xy^2 \to M/dy \neq N/dx$ no exact

$\mu(x) \cdot xy^2 = N \to N/dx = \mu' xy^2 + \mu y^2$

$(-x^2-y^3)\mu(x) = M \quad M/dy = -3y^2\mu$

$N/dx = M/dy \Rightarrow \mu' xy^2 = -4y^2\mu \to \frac{d\mu}{\mu} = \frac{-4dx}{x} \to \ln\mu = -4\ln|x| \quad \mu = x^{-4}$

$\varphi(x,y) = \int M dx = \int \frac{-x^2-y^3}{x^4} dx = \int \frac{-1}{x^2}dx - \frac{y^3}{x^4} dx \Rightarrow$

$\varphi(x,y) = \frac{1}{x} + \frac{y^3}{3x^3} + h(y) \to \varphi/dy = N \Rightarrow h(y) = 0$

b) $x^3 y' = xy + x \quad dM/dy = -x \neq \frac{dN}{dx} = 3x^2$ No exact

$dy/dx = \frac{xy+x}{x^3}$

$\frac{dM}{dy}\mu(x) = \frac{dN}{dx}\mu(x) \Rightarrow -x\mu = 3x^2\mu + x^3\mu' \Rightarrow$

$\ln\mu = \int \frac{-x-3x^2}{x^3} dx \Rightarrow$

$\ln\mu(x) = -3\ln|x| + \frac{1}{x} \to \mu(x) = e^{1/x} x^{-3}$

$\varphi(x,y) = \int M\mu dx = \int (e^{1/x} x^{-2} y + e^{1/x} x^{-1}) dx = -ye^{1/x} + \int x^{-1} e^{1/x} dx$

d) $5dx - e^{y-x} dy = 0 \quad dM/dy \neq dN/dx$ No exact

$dy = \frac{+5}{e^{y-x}}$

$\frac{dM\mu}{dy} = \frac{dN\mu}{dx} \Rightarrow 0 = \mu e^{y-x} - e^{y-x}\mu'(x)$

$\mu(x) = e^x$

$\varphi(x,y) = \int M\mu dx = 5e^x + h(y) \to \varphi/dy = N \to h'(y) = -e^{y-x} \cdot e^x$

$h(y) = -e^y$

$\varphi(x,y) = 5e^x - e^y$ //

e) $(yx^3e^{xy} - 2y^3)dx + (x^4e^{xy} + 3xy^2)dy = 0$

$$dy/dx = \frac{-yx^3e^{xy} + 2y^3}{x^4e^{xy} + 3xy^2} \quad // \quad \frac{dM}{dy} = x^3e^{xy} + x^4ye^{xy} - 6y^2$$

No good

$$\frac{dN}{dx} = 4x^3e^{xy} + x^4ye^{xy} + 3y^2$$

$\mu(x) = x^{-3}$ by inspection $\Rightarrow M\mu = ye^{xy} - 2y^3/x^3$

$N\mu = xe^{xy} + 3y^2/x^2$

$\psi(x,y) = \int M\mu\, dx = e^{xy} - y^3/x^2 + h(y) \Rightarrow e^{xy} - y^3/x^2 + C$

$d\psi/dy = N\mu \Rightarrow h'(y) = 0 \Rightarrow h(y) = C$

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② a) $(3x+y-1)dx = (y-x-1)dy$

$\frac{dy}{dx} = \frac{3x+y-1}{y-x-1}$ $M = 3x+y-1$ $M/dy = 1$

$N = -y+x+1$ $N/dx = 1$

$\psi(x,y) = \int M\,dx = \int (3x+y-1)dx = \frac{3x^2}{2} + yx - x + h(y)$

$\frac{d\psi(x,y)}{dy} = 0 + x - 0 + h'(y) = N \Rightarrow x + h'(y) = -y+1+\cancel{x}$

$h'(y) = -y+1 \Rightarrow h(y) = \frac{-y^2}{2} + y$

$\psi(x,y) = \frac{3x^2}{2} + yx - x - \frac{y^2}{2} + y = \frac{3x^2-y^2}{2} + yx - x + y$

b) $(x+2y+2)dx + (2x+3y+2)dy = 0$

$\frac{dy}{dx} = \frac{-x-2y-2}{2x+3y+2}$ $M = x+2y+2$ $M/dy = 2$

$N = 2x+3y+2$ $N/dx = 2$

$\psi(x,y) = \int M\,dx = \int (x+2y+2)dx = \frac{x^2}{2} + 2yx + 2x + h(y)$

$\frac{d\psi(x,y)}{dy} = 0 + 2x + 0 + h'(y) = N \Rightarrow h'(y) = +2+3y$

$h(y) = +2y + 3y^2/2$

$\psi(x,y) = \frac{x^2}{2} + 2yx + 2x + 2y + 3y^2/2 \Rightarrow \frac{x^2+3y^2}{2} + 2xy + 2x + 2y$

c) $(4x+2y-8)dx + (2x-y)dy = 0$

$dy/dx = \frac{-4x-2y+8}{2x-y}$ $M = 4x+2y-8$ $M/dy = 2$

$N = 2x-y$ $N/dx = 2$

$\psi(x,y) = \int M\,dx = \int (4x+2y-8)dx = 2x^2 + 2yx - 8x + h(y)$

$d\psi(x,y)/dy = 0 + 2x - 0 + h'(y) = N \Rightarrow h'(y) = -y \Rightarrow h(y) = -y^2/2$

$\psi(x,y) = 2x^2 - 2yx - 8x + (-y^2/2) \Rightarrow 2x^2 - 2yx - 8x - \frac{y^2}{2}$

d) $(x-4y-9)dx + (4x+y-2)dy = 0$

$dy/dx = \dfrac{-x+4y+9}{4x+y-2} \Rightarrow \begin{array}{l} M = x-4y-9 \\ N = 4x+y-2 \end{array} \quad \begin{array}{l} M/dy = -4 \\ N/dx = 4 \end{array}$ No exact

$\alpha + (-4\beta) - 9 = 0 \rightarrow \alpha = 9 + 4\beta$

$4\alpha + \beta \quad -2 = 0 \rightarrow \hookrightarrow 4(9+4\beta) + \beta = 2$ $\quad \alpha = \underline{\underline{1}}$

$\beta = -34/17 = \underline{\underline{-2}}$

$\begin{array}{l} X + \alpha = x \\ Y + \beta = y \end{array} \quad // \quad \dfrac{dX}{dY} = \dfrac{dx}{dy} \Rightarrow \dfrac{-(x+1)+4(Y-2)+9}{4(x+1)+(Y-2)-2} = \dfrac{dy}{dx} \Rightarrow$

$\dfrac{-X+4Y}{4X+Y} = \left\| \dfrac{dy}{dx} = \dfrac{dY}{dX} \right\| \rightarrow$ C.V $Y = vX \rightarrow \dfrac{dY}{dX} = v + X\dfrac{dv}{dX} \mapsto$

$\dfrac{-X+4(Xv)}{4X+Xv} = v + X\dfrac{dv}{dX} \Rightarrow \dfrac{-1+4v}{4+v} - v = X\dfrac{dv}{dX} \Rightarrow$

$\ln|X| + C = \dfrac{(4+v)dv}{-v^2-1} \Rightarrow \left\| \dfrac{4}{-v^2-1} + \dfrac{v}{-v^2-1} \right\| \Rightarrow \ln|X| + \ln C = -4\arctg(v) - \frac{1}{2}\ln|v^2+1|$

$\ln|x-1| + \ln k = -4\arctg\left(\frac{y+2}{x-1}\right) - \frac{1}{2}\ln\left|\left(\frac{y-2}{x-1}\right)^2 + 1\right|$

$\ln|x-1| + \ln k = -8\arctg\left(\frac{y+2}{x-1}\right) - \ln\left|\left(\frac{y-2}{x-1}\right)^2 + 1\right|$

$-C = 8\arctg\left(\frac{y+2}{x-1}\right) + \ln\left|\left(\left(\frac{y-2}{x-1}\right)^2+1\right)(x-1)\right|$

$\shortparallel$

constant $= 8\arctg\left(\frac{y+2}{x-1}\right) + \ln|(y-2)^2 + (x-1)^2|$

e) $(5y-10)dx + (2x+4)dy = 0$ No exact [$M/dy \neq N/dx$]

$dy/dx = \dfrac{-5y+10}{2x+4} \Rightarrow \dfrac{dy}{-5y+10} = \dfrac{dx}{2x+4} \Rightarrow -\frac{1}{5}\ln|-5y+10| = \frac{1}{2}\ln|x+2| + C \Rightarrow$

$-\frac{1}{5}\ln|-5y+10| = \frac{1}{2}\ln|(x+2)k| \Rightarrow \ln|-5y+10| = \ln|(x+2)k|^{-5/2} \Rightarrow e^{\circ\circ}$

$-5y+10 = (x+2)k^{-5/2} \Rightarrow y = \dfrac{(x+2)k^{-5/2}}{-5} + 2$

② f) $(2x+y-8)dx = (-2x+9y-12)dy$

$$\frac{dy}{dx} = \frac{2x+y-8}{-2x+9y-12}$$

$M = 2x+y-8 \rightarrow M/dy = 1$ No exact

$N = +2x-9+12 \quad N/dx = 2$

$x = X+\alpha$
$y = Y+\beta$
$dx = dX$
$dy = dY$

$$\rightarrow \begin{matrix} -8+2\alpha+\beta=0 \\ -12-2\alpha+9\beta=0 \end{matrix} \rightarrow \frac{(-8+2\alpha+\beta)}{+-12-2\alpha+9\beta} = -20+10\beta=0$$

$\beta = 2$
$\rightarrow \alpha = 3$

$$\frac{2(X+3)+(Y+2)-8}{-2(X+3)+9(Y+2)-12} = \frac{2X+Y}{-2X+9Y} \rightarrow \left\| \; Y = X\cdot v \quad dy/dx = \frac{dv}{dx}X+v \; \right\|$$

$$\frac{dv}{dx}X+v = \left(\frac{2X+Xv}{-2X+9Xv}\right) = \frac{2+v}{-2+9v} \Rightarrow \frac{dv}{dx}X = \frac{2+v}{-2+9v}-v = \frac{2+3v-9v^2}{-2+9v} \Rightarrow$$

$$\int\frac{dX}{X} = \int\frac{-2+9v}{2+3v-9v^2}dv \Rightarrow \ln|x|+C = \int\frac{-2+9v}{(v-2/3)(v+1/3)} \Rightarrow$$

$$\ln|X|+C = \int\frac{4}{v-2/3}+\int\frac{5}{v+1/3} \quad \left\| \; 9v-2 = A(v+1/3)+B(v-2/3) = \cdots \; \right\|$$

$$\ln|x|+C = 4\ln|v-2/3|+5\ln|v+1/3| \Rightarrow \left\| \; v = Y/X \; \right\| \Rightarrow$$

$X = x-3$
$Y = y-2$

$$C = -\ln|x-3| + 4\ln\left|\frac{3y-2x}{3x-9}\right| + 5\ln\left|\frac{3y+x-9}{3x-9}\right|$$

$$e^C \Rightarrow C\cdot|x-3| = \left(\frac{3y-2x}{3x-9}\right)^4\cdot\left(\frac{3y+x-9}{3x-9}\right)^5$$

n) $(3x+y+1)dx + (x+3y+11)dy = 0$

$dy/dx = \dfrac{-3x-y-1}{x+3y+11}$ $\quad M = 3x+y+1 \to M/dy = 1$ $\quad$ Exact

$N = x+3y+11 \to N/dx = 1$

$x = X+\alpha$

$y = Y+\beta \Rightarrow$ $\quad 3\alpha+\beta+1=0$, $\alpha+3\beta+11=0$ $\to$ $\beta = -1-3\alpha \to \alpha+3(-1-3\alpha)+11=0$

$dx = dX$ $\qquad \alpha - 3 - 9\alpha + 11 = 0$

$dy = dY$ $\qquad -8\alpha = -8$

$\alpha = +1 \to \beta = -4$

$$\frac{-3(X+1)-(Y-4)-1}{(X+1)+3(Y-4)+11} = \frac{-3X-Y}{X+3Y} \Rightarrow \left\| \begin{array}{l} Y = vX \\ dY/dx = \frac{dv}{dx}X + v \end{array} \right\| \Rightarrow$$

$$\frac{dv}{dX}X + v = \frac{-3X - vX}{X+3Xv} = \frac{-3-v}{1+3v} \Rightarrow \int \frac{dX}{X} = \int \frac{1+3v}{-3-2v-3v^2}\,dv \Rightarrow$$

$$\ln|x| + C = -\frac{1}{2}\int \frac{+2+6v}{+3+2v+3v^2} = -\frac{1}{2}\ln\left|+3+2v+3v^2\right|$$

$$C = -\ln|X| - \frac{1}{2}\ln\left|3+2v+3v^2\right| \Rightarrow \left\| \begin{array}{l} X = x-1 \\ Y = y+4 \\ v = Y/X \end{array} \right\|$$

$$C = -\ln|x-1| - \frac{1}{2}\ln\left|3+2\left(\frac{y+4}{x-1}\right)+3\left(\frac{y+4}{x-1}\right)^2\right|$$

[doing it as exact equation ... $\psi(x,y) = \frac{3x^2}{2} + yx + x + h(y) \to$

$\psi(x,y)/dy = N \to h(y) = \frac{3y^2}{2} + 11y \Rightarrow \psi(x,y) = \frac{3x^2}{2} + yx + x + \frac{3y^2}{2} + 11y$]

② g) $(x-1)dx - (3x-2y-5)dy = 0$

$$dy/dx = \frac{-x+1}{-3x+2y+5} \Rightarrow dM/dy = 0, \; dN/dx = -3 \quad \text{No exact}$$

$$-1+\alpha = 0 \Rightarrow \alpha = 1$$
$$-5 - 2\beta + 3\alpha = 0 \Rightarrow \beta = -1$$

$$\Rightarrow \begin{array}{l} x = X+\alpha \\ y = Y+\beta \\ dx = dX \\ dY = dy \end{array} \Rightarrow \frac{dY}{dX} = \frac{X}{3X-2Y} \Rightarrow$$

$$\| Y = Xv \Rightarrow \frac{dY}{dX} = \frac{dv}{dX}X + v \| \Rightarrow \frac{X}{3X-2Xv} = \frac{dv}{dX}X + v \Rightarrow \frac{dv}{dX}X + v = \frac{1}{3-2v} \Rightarrow$$

$$\frac{dv}{dX}X = \frac{1-3v+2v^2}{3-2v} \Rightarrow \ln|X| + C = \int \frac{(3-2v)\,dv}{2v^2-3v+1} \quad \| 2v^2-3v+1 = (v-1)(2v-1) \|$$

$$\ln|X| + C = \left\| \begin{array}{l} 3-2v = A(2v-1)+B(v-1) \\ A = 1 \quad B = -4 \end{array} \right\| = \ln|v-1| - \frac{4}{2}\ln|2v-1|$$

$$\ln|x-1| + C = \ln\left|\frac{y+1}{x-1} - 1\right| - 2\ln\left|2\left(\frac{y+1}{x-1}\right) - 1\right| \quad \left\| \begin{array}{l} v = Y/X \\ Y = y-\beta \\ X = x-\alpha \end{array} \right\|$$

j) $(2x-y-3)dx + (3x+y+3)dy = 0$

$$dy/dx = \frac{2x-y-3}{-3x-y-3} \Rightarrow \begin{array}{l} 2\alpha - \beta - 3 = 0 \\ 3\alpha + \beta + 3 = 0 \end{array} \Rightarrow \beta = 2\alpha - 3 \Rightarrow \beta = -3$$
$$\Rightarrow 2\alpha + 3\alpha - 3 + 3 = 0$$
$$\alpha = 0$$

$$\frac{2X-Y}{-3X-Y} \Rightarrow \frac{-2X-Xv}{-3X-Xv} = \frac{2-v}{-3-v} = \frac{dv}{dX}X + v \Rightarrow$$

$$\frac{2-v+3v+v^2}{-3-v} = \frac{dv}{dx}x \Rightarrow \int \frac{dx}{x} = \int \frac{-3-v}{2+2v+v^2} \Rightarrow$$

$$\ln|x| + C = \frac{-\ln|v^2+2v+2|}{2} - \operatorname{arctg}(v+1) \Rightarrow$$

$$C = -\ln|x| - \frac{\ln}{2}\left|\left(\frac{y+3}{x}\right)^2 + 2\left(\frac{y+3}{x}\right) + 2\right| - \operatorname{arctg}\left(\frac{y+3}{x} + 1\right) \quad //$$

h) $(2x+y-5)\,dx = (x-2y+4)\,dy$

$$dy/dx = \frac{2x+y-5}{x-2y+4} \rightarrow \begin{matrix} 2\alpha+\beta-5=0 \\ -\alpha+2\beta-4=0 \end{matrix} \rightarrow \beta = 5-2\alpha \rightarrow -\alpha+10-4\alpha-4=0 \quad \alpha = 6/5 \rightarrow \beta = 13/5$$

$$\frac{2x+y-5}{x-2y+4} = dy/dx \rightarrow \frac{2X+Y}{X-2Y} = \frac{dv}{dx}x + v \rightarrow \| y = xv \| \Rightarrow$$

$$\frac{2x+xv}{x-2xv} = \frac{2+v}{1-2v} = \frac{dv}{dx}x + v \rightarrow \frac{2+v-v+2v^2}{1-2v} = \frac{dv}{dx}x \rightarrow$$

$$\int \frac{dx}{x} = \int \frac{1-2v}{2+2v^2} \rightarrow \ln|x| + C = -\frac{\ln}{2}|2-2v^2| + \frac{1}{2}\operatorname{arctg}(v)$$

$$C = -\ln|x-6/5| - \frac{\ln}{2}\left|1+\left(\frac{y-13/5}{x-6/5}\right)^2\right| + \frac{1}{2}\operatorname{arctg}\left(\frac{y-13/5}{x-6/5}\right)$$

i) $(3x+2y-5)\,dx + (-2x+3y-1)\,dy = 0$

$$\frac{dy}{dx} = \frac{3x+2y-5}{2x-3y+1} \quad \begin{matrix} 3\alpha+2\beta-5=0 \\ -2\alpha+3\beta-1=0 \end{matrix} \rightarrow \beta = \frac{5}{2} - \frac{3}{2}\alpha \rightarrow \alpha = 1 \rightarrow \beta = 1$$

$$\frac{3X+2Y}{2X-3Y} = \frac{dY}{dX} \rightarrow \frac{3X+2Y}{2X-3Y} = \frac{dv}{dx}x + v \rightarrow \frac{3+3v^2}{2-3v} = \frac{dv}{dx}x \rightarrow$$

$$\int \frac{dx}{x} = \int \frac{2-3v}{3+3v^2} \rightarrow \ln|x| + C = \frac{-1}{2}\ln|1+v^2| + \frac{2}{3}\operatorname{arctg}(v)$$

$$C = -\ln|x-1| - \frac{\ln}{2}\left|1+\left(\frac{y-1}{x-1}\right)^2\right| + \frac{2}{3}\operatorname{arctg}\left(\frac{y-1}{x-1}\right)$$

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③ d) $y' - 2y = 4$ $y' + a(x)\,y = f(x)$

$\frac{dy}{dx} = 4 + 2y$ $\rightsquigarrow$ $y(x) = e^{-\int a(x)dx} \cdot \int e^{\int a(x)dx} \cdot f(x) + Ce^{-\int a(x)dx}$

$a(x) = -2 \;\rightarrow\; e^{-\int a(x)dx} = e^{2x}$ $\Rightarrow$ $\int e^{-2x}\,4 = -2e^{-2x}$

$f(x) = 4$ $e^{\int a(x)dx} = e^{-2x}$

$\underline{\underline{y = Ce^{2x} - 2}} = e^{2x} \cdot (-2e^{-2x}) + Ce^{2x}$

e) $y' - 2y = 2 + 4x$ $y' - 2y = 2 + 4x \Leftrightarrow y'(x) + a(x)\,y(x) = f(x)$

$\frac{dy}{dx} = 2 + 4x + 2y$ $\rightsquigarrow$ $y(x) = e^{-\int a(x)dx} \cdot \int e^{\int a(x)dx} \cdot f(x) + Ce^{-\int a(x)dx}$

y_P y_H

$a(x) = -2 \Rightarrow e^{-\int a(x)dx} = e^{2x}$, $e^{\int a(x)dx} = e^{-2x}$

$f(x) = 2 + 4x$

$\int e^{-2x}(2 + 4x) = -e^{-2x} + (-2e^{-2x} \cdot x - e^{-2x}) \Rightarrow$

$e^{2x} \cdot (\dots) = -2 - 2x$ $\rightsquigarrow$ $\underline{\underline{y(x) = -2 - 2x + Ce^{2x}}}$

f) $y' - 2y = 3e^{-x}$ $a(x) = -2$ // $f(x) = 3e^{-x}$

$\frac{dy}{dx} = 3e^{-x} + 2y$ $e^{-\int a(x)dx} = e^{2x}$ // $e^{\int a(x)dx} = e^{-2x}$

$e^{2x} \cdot \int e^{-2x} \cdot 3e^{-x} + Ce^{2x} \Rightarrow$

$y(x) = e^{2x} \cdot (-e^{-3x}) + Ce^{2x} = \underline{\underline{-e^{-x} + Ce^{2x}}}$

i) $y' + 2y = 4$ $\qquad y(x) = e^{-\int 2dx} \cdot \left[\int e^{\int 2x} \cdot 4 dx \right] + Ce^{-\int 2dx}$

$dy/dx = 4 - 2y$ $\qquad y(x) = e^{-2x} \cdot 2e^{2x} + Ce^{-2x} = 2 + Ce^{-2x}$

j) $y' + 2y = 4e^{2x}$ $\qquad y(x) = e^{-\int 2dx} \cdot \left[\int e^{\int 2k} \cdot 4e^{2x} dx \right] + Ce^{-\int 2dx}$

$dy/dx = 4e^{2x} - 2y$ $\qquad y(x) = e^{-2x} \cdot e^{4x} + Ce^{-2x} = e^{2x} + Ce^{-2x}$

k) $y' + 2y = e^{-2x}$ $\qquad y(x) = e^{-2x} \cdot \left[\int e^{2x} \cdot e^{-2x} dx \right] + Ce^{-2x}$

$dy/dx = e^{-2x} - 2y$ $\qquad y(x) = e^{-2x} \cdot x + Ce^{-2x}$

e) $y' + 2y = 3\cosh(x)$

$dy/dx = 3\cosh(x) - 2y = \frac{3}{2}(e^{x} + e^{-x}) - 2y$

$$y(x) = e^{-2x}C + e^{-2x}\left[\frac{3}{2}\int \underbrace{e^{3x}}_{e^{2x} \cdot e^{x}} dx + \frac{3}{2}\int \underbrace{e^{x}}_{e^{2x} \cdot e^{-x}} \right]$$

$$y(x) = e^{-2x}C + e^{-2x}\left(\frac{e^{3x}}{2} + \frac{3}{2}e^{x} \right) = e^{-2x}C + \frac{e^{x}}{2} + \frac{3}{2}e^{-x}$$

g) $y' - 2y = e^{2x}$ $\rightarrow$ $a(x) = -2 \quad f(x) = e^{2x}$

$dy/dx = e^{2x} + 2y$ $\qquad y(x) = e^{-\int a(x)dx} \cdot \left[\int e^{\int a(x)dx} \cdot f(x) + C \right]$

$$y(x) = e^{2x} \cdot \left[\int e^{-2x} \cdot e^{2x} dx + C \right]$$

$y(x) = Ce^{2x} + xe^{2x}$

③ k) $y' - 2y = 5\,sen(x)$ $\quad a(x) = -2 \quad f(x) = 5\,sen(x)$

$dy/dx = 5\,sen(x) + 2y$ $\rightarrow$ $y(x) = e^{-\int a(x)dx} \cdot \left[\int e^{\int a(x) dx} \cdot f(x) + C \right]$

$$y(x) = e^{2x} \cdot \left[5 \cdot \int e^{-2x}\, sen(x) + C \right]$$

$\left\| \begin{matrix} u = sen(x) & du = cos(x) \\ dv = e^{-2x} & v = \frac{-e^{-2x}}{2} \end{matrix} \right\| \rightarrow \dfrac{-e^{-2x} sen(x)}{2} - \int \dfrac{-e^{-2x} cos(x)}{2}\, dx$

$\left\| \begin{matrix} u = cos(x) & du = -sen(x) \\ dv = \frac{-e^{-2x}}{2} & v = \frac{e^{-2x}}{4} \end{matrix} \right\| \rightarrow \dfrac{-e^{-2x} sen(x)}{2} - \left(\dfrac{e^{-2x} cos(x)}{4} - \int - \dfrac{e^{-2x} sen(x)}{4} dx \right) \Rightarrow$

$$\frac{-2e^{-2x} sen(x) - e^{-2x} cos(x)}{5} + C$$

$\Rightarrow$

$$y(x) = e^{2x} \cdot \left[5 \cdot \left(\frac{-e^{-2x} \cdot (2\,sen\,x + cos(x))}{5} \right) + C \right]$$

$$y(x) = Ce^{2x} - 2\,sen(x) - cos(x)$$

m) $y' + 2y = 3\,senh(x)$ $\quad a = 2 \quad f(x) = 3\,senh(x)$

$dy/dx = 3\,senh(x) - 2y$ $\rightarrow$ $y(x) = e^{-\int a(x)} \cdot \left[\int e^{\int a(x)} \cdot f(x) + C \right]$

$$e^{-2x} \cdot \left[\int e^{2x} \cdot 3\,senh(x) + C \right]$$

$\left\| senh(x) = \dfrac{e^{x} - e^{-x}}{2} \right\| \rightarrow e^{-2x} \cdot \left[\frac{3}{2} \int e^{3x} - e^{x} + C \right] \Rightarrow$

$$y(x) = e^{-2x} \cdot \left(\left(\frac{3e^{3x}}{6} - \frac{9e^{x}}{6} \right) + C \right) = Ce^{-2x} + \frac{e^{x}}{2} - \frac{3e^{-x}}{2}$$

a) $y' + 4y = 17\sin(x)$ // $a(x) = 4 \quad f(x) = 17\sin(x)$

$dy/dx = 17\sin(x) - 4y$ $\quad y(x) = e^{-4x} \cdot \left(\int 17\sin(x)\, e^{4x}\, dx + C \right)$

$$y(x) = \frac{C}{e^{4x}} + \int 17\sin(x) e^{4x} dx \quad \Big\| \begin{matrix} dv = e^{4x} & v = \frac{e^{4x}}{4} \\ u = \sin(x) & du = \cos x \end{matrix} \Big\| \Rightarrow$$

$$y(x) = \frac{C}{e^{4x}} + 17 \cdot \left(\frac{e^{4x}}{4} \sin(x) - \int \cos(x) \frac{e^{4x}}{4} dx \right) \Rightarrow$$

$$y(x) = \frac{C}{e^{4x}} + \cancel{17} \cdot \left(\frac{e^{4x}}{4} \sin(x) - \left(\frac{e^{4x}\cos(x)}{4} - \int - \frac{e^{4x}\sin(x)}{16} dx \right) \right) \Rightarrow$$

$$\left(\frac{e^{4x}}{4} \sin(x) - \frac{e^{4}\cos(x)}{16} + \frac{1}{16} \int e^{4x} \sin(x) \right)$$

$$\cancel{17} I = \frac{4 e^{4x}\sin(x) - e^{4}\cos(x)}{16} \Rightarrow$$

$$y(x) = \frac{C}{e^{4x}} + 4e^{4x}\sin(x) - e^{4x}\cos(x)$$

b) $y' + 4y = 2e^{-2x}$ // $a(x) = 4 \quad f(x) = 2e^{-2x}$

$dy/dx = 2e^{-2x} - 4y$ $\quad y(x) = e^{-4x} \cdot \left(\int e^{4x} \cdot 2e^{-2x} dx + C \right)$

$$y(x) = \frac{C}{e^{4x}} + \int e^{4x} 2e^{-2x} dx \Rightarrow \quad \Big\| e^{4x} \cdot e^{-2x} = e^{2x} \Big\|$$

$$y(x) = e^{-2x} + \frac{C}{e^{4x}}$$

c) $y' + 4y = e^{-4x}$ // $a(x) = 4 \quad f(x) = e^{-4x}$

$dy/dx = e^{-4x} - 4y$ $\quad y(x) = e^{-4x} \cdot \left(\int e^{4x} \cdot e^{-4x} + C \right)$

$$y(x) = \frac{x}{e^{4x}} + \frac{C}{e^{4x}}$$

③ p) $y' = 2y + x^2 + 3$ $\quad a(x) = -2 \quad f(x) = x^2+3$

$dy/dx = 2y + x^2 + 3$ $\quad // \quad y(x) = e^{2x} \cdot \left(\int e^{-2x} \cdot (x^2+3) + C \right) \Rightarrow$

$y(x) = e^{2x} \cdot C + e^{2x} \int e^{-2x} (x^2+3) dx \quad \left\| \begin{matrix} u = x^2+3 & du = 2x \\ dv = e^{-2x} & v = -e^{-2x}/2 \end{matrix} \right\| \Rightarrow$

$y(x) = e^{2x} C + e^{2x} \left(\frac{-e^{-2x}}{2} (x^2+3) + \int \frac{2x e^{-2x}}{2} dx \right) \Rightarrow$

$\left\| \begin{matrix} x = u & du = 1 \\ e^{-2x} = dv & v = \frac{-e^{-2x}}{2} \end{matrix} \right\|$

$y(x) = e^{2x} C + \cancel{e^{2x}} \left(\frac{-\cancel{e^{-2x}}}{2} (x^2+3 + \left(\frac{-x \cancel{e^{-2x}}}{2} + \int \frac{e^{-2x}}{2} dx \right) \right) \Rightarrow$

$y(x) = e^{2x} C + \frac{(-x^2-3)}{2} - \frac{x}{2} - \frac{1}{4}$

n) $y' = y + 4 \operatorname{senh}(x)$ $\quad a(x) = -1 \quad f(x) = 4 \operatorname{senh}(x)$

$dy/dx = y + 4 \frac{(e^x - e^{-x})}{2}$ $\quad // \quad y(x) = e^x C + e^x \int e^{-x} 2 (e^x - e^{-x}) dx$

$y(x) = e^x C + e^x (2x + e^{-2x})$

o) $y' - y = 4 \cosh(x)$ $\quad a(x) = -1 \quad f(x) = 2 \cosh(x)$

$dy/dx = y - \frac{4}{2} (e^x + e^{-x})$ $\quad // \quad y(x) = e^x C + e^x \left(\int e^{-x} \cdot 2 (e^x + e^{-x}) dx \right)$

$y(x) = e^x C + e^x (2x - e^{-2x})$

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4 c) $xy' + (3x+1)y = e^{-3x} \to \cdot \frac{1}{x} \Rightarrow y' + (3 + \frac{1}{x})y = \frac{e^{-3x}}{x}$

$\frac{dy}{dx} + \frac{e^{-3x}}{x} = y(3+\frac{1}{x})$

$a(x) = 3 + \frac{1}{x} \to e^{-\int a(x)dx} = e^{-3x - \ln(x)} = \frac{e^{-3x}}{e^{\ln x}} = \frac{e^{-3x}}{x}$

$f(x) = e^{-3x}/x$

$e^{\int a(x)dx} = e^{3x + \ln|x|} = e^{3x} \cdot e^{\ln|x|} = e^{3x} \cdot x$

$$y(x) = \frac{e^{-3x}}{x} \int e^{3x} \cdot x \cdot \frac{e^{-3x}}{x} + \frac{Ce^{-3x}}{x}$$

$$\frac{e^{-3x}}{x} \cdot \int e^{0} + \frac{Ce^{-3x}}{x} = e^{-3x} + \frac{Ce^{-3x}}{x}$$

d) $xy' + (2x+1)y = 4x \to \frac{1}{x} \Rightarrow y' + (2 + \frac{1}{x})y = 4$

$\frac{dy}{dx} = \frac{4x - y(2x+1)}{x}$ //

$a(x) = 2 + \frac{1}{x} \to e^{-\int a(x)dx} = e^{2x}/x$

$f(x) = 4$

$e^{\int a(x)dx} = e^{2x} \cdot x$

$$y(x) = \frac{e^{-2x}}{x} \int e^{2x} \cdot x \cdot 4 + \frac{Ce^{-2x}}{x} \Rightarrow \frac{e^{-2x}}{x} \cdot (2e^{2x}x - e^{2x}) + C\frac{e^{-2x}}{x}$$

$$y(x) = 2 - \frac{1}{x} + C\frac{e^{-2x}}{x}$$

f) $y' + \cot(x)y = x$

$a(x) = \cot(x) \to e^{-\int a(x)dx} = e^{-\ln|\sin(x)|}$

$f(x) = x$

$e^{\int a(x)dx} = e^{\ln|\sin(x)|}$

$\frac{dy}{dx} = x - \cot(x)y$

$$y(x) = e^{-\int \cot(x)dx} \cdot \left[\int e^{\int \cot(x)dx} x\,dx + C \right]$$

$$y(x) = \frac{1}{\sin(x)} \cdot \left[\int x\sin(x)\,dx + C \right] \quad \| \begin{matrix} u = x & v = \cos(x) \\ du = 1 & dv = \sin(x) \end{matrix} \|$$

$$= \frac{1}{\sin(x)} \cdot \left[\int \cos(x)\,dx + (-x)\cos(x) + C \right]$$

$$= 1 - x\cos(x)/\sin(x) + C/\sin(x)$$

e) $(1-x^2)y' - xy + x(1-x^2) = 0$

$$dy/dx = \frac{xy - x(1-x^2)}{1-x^2} \Rightarrow y' = -x - \frac{xy}{x^2-1}$$

$$y(x) = e^{-\int \frac{x}{x^2-1}dx}\left[\int e^{\int \frac{x}{x^2-1}dx} \cdot (-x)\,dx + C\right]$$

$$y(x) = \frac{1}{\sqrt{(x^2-1)}} \cdot \left[\int -x\sqrt{x^2-1}\,dx + C\right]$$

$$y(x) = \frac{C}{\sqrt{x^2-1}} - \frac{\sqrt{x^2-1}\,(x^2-1)}{3} \cdot \frac{1}{\sqrt{x^2-1}} = \frac{C}{\sqrt{x^2-1}} - \frac{x^2}{3} + \frac{1}{3}$$

g) $x\ln|x|\,y' + y = 9x^3\ln(x)$ // $a(x) = \frac{1}{x\ln(x)}$ $f(x) = 9x^2$

$$dy/dx = \frac{-y + 9x^3\ln(x)}{x\ln(x)}$$

$$y(x) = e^{-\int \frac{1}{x\ln|x|}} \cdot \left(\int e^{\int \frac{1}{x\ln|x|}} \cdot 9x^2 + C\right) \Rightarrow$$

|| $\int \frac{1}{x\ln|x|} \Rightarrow \ln|\ln|x|| \Rightarrow e^{\cdot\cdot} = \ln|x|$ // $e^{-\cdot\cdot} = \frac{1}{\ln|x|}$ //

$$y(x) = \frac{1}{\ln|x|} \cdot \left(\int \ln|x|\,9x^2 + C\right) \Rightarrow \frac{C}{\ln|x|} + \frac{1}{\ln|x|}\int \ln|x|\,9x^2\,dx \Rightarrow$$

|| $\int \ln|x|\,9x^2 \Rightarrow$ $u = \ln|x|$, $v = x^3/3$ $\Rightarrow 9\left(\frac{x^3\ln|x|}{3} - \int \frac{x^3}{3x}\,dx\right) = 3x^3\ln|x| - x^3$ ||

$$y(x) = \frac{C}{\ln|x|} + \frac{1}{\ln|x|} \cdot \left(3x^3\ln|x| - x^3\right) = \frac{C}{\ln|x|} + 3x^3 - \frac{x^3}{\ln|x|}$$

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④ h) $(2-y\sin(x))dx = \cos(x)dy$

$$dy/dx = \frac{2-y\sin(x)}{\cos(x)}$$

$M = 2-y\sin(x) \to M/dy = -\sin(x)$

$N = -\cos(x) \to N/dx = \sin(x)$

NO Exact

$$dM\mu/dy = dN/dx \cdot \mu \Rightarrow -\sin(x)\mu = \sin(x)\mu - \cos(x)\mu' \Rightarrow$$

$$2\tan(x) = \mu' \Rightarrow \ln|\mu| = -2\ln|\cos(x)|$$

$$\mu = \frac{1}{(\cos(x))^2}$$

$$\psi(x,y) = \int M\mu\,dx = \frac{2\sin(x)}{\cos(x)} - \frac{y}{\cos(x)} + h(y) \; ; \; \psi/dy = N\mu \Rightarrow$$

$$h'(y) = 0 \to h(y) = C$$

$$\psi(x,y) = 2\tan(x) - \frac{y}{\cos(x)} + C$$

a) $y' + xy = x$

$$dy/dx = x - xy$$

$f(x) = x$

$a(x) = x$

$$y(x) = e^{-x^2/2} \cdot \left(\int x \cdot e^{x^2/2} + C\right)$$

$$y(x) = 1 + Ce^{-x^2/2}$$

b) $x^2y' + xy = 1$

$$dy/dx = \frac{1-xy}{x^2}$$

$$y' + \frac{y}{x} = \frac{1}{x^2}$$

$f(x) = 1/x^2$

$a(x) = 1/x$

$$y(x) = e^{-\ln|x|} \cdot \left(\int e^{\ln x} \cdot \frac{1}{x^2} + C\right)$$

$$y(x) = \frac{\ln|x|}{x} + \frac{C}{x}$$

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⑤ a) $xy' = y + x^2e^x$

$$dy/dx = \frac{y + x^2e^x}{x} \Rightarrow xy' - y = x^2e^x \rightarrow y' - y/x = xe^x \quad \begin{cases} a(x) = -1/x \\ f(x) = xe^x \end{cases}$$

$$y(x) = e^{-\int -1/x\,dx} \cdot \left[\int e^{-\int 1/x\,dx} \cdot xe^x \right] + Ce^{-\int -1/x\,dx}$$

$$y(x) = x \cdot \left(\int 1/x \cdot x e^x\,dx \right) + Cx = \underline{\underline{xe^x + Cx}}$$

b) $y' + 2xy = 4x$ $dy/dx = 4x - 2xy$

$$a(x) = 2x \quad f(x) = 4x \rightarrow y(x) = e^{-\int 2x\,dx} \cdot \left[\int e^{\int 2x\,dx} \cdot 4x \right] + Ce^{-\int 2x\,dx}$$

$$y(x) = e^{-x^2} \cdot \left[\int e^{x^2} \cdot 4x\,dx \right] + Ce^{-x^2}$$

$$y(x) = e^{-x^2} \cdot 2e^{x^2} + Ce^{-x^2} = \underline{\underline{e^{-x^2}(2e^{x^2} + C)}}$$

e) $x\ln(x)y' + y = x \rightarrow \frac{1}{x\ln(x)}$ $a = \frac{1}{x\ln(x)}$ $f(x) = \frac{1}{\ln(x)}$

$$\underline{\underline{dy/dx = \frac{x - y}{x\ln(x)}}}$$

$$y(x) = e^{-\int a(x)} \cdot \left[\int e^{\int a(x)} \cdot f(x) + C \right]$$

$$y(x) = e^{-\ln(\ln(x))} \cdot \left[\int e^{\ln(\ln(x))} \cdot \frac{1}{\ln(x)}\,dx + C \right]$$

$$y(x) = \underline{\underline{\left(1/\ln(x)\right) \cdot [x + C]}}$$

c) $y' = (y-1)\tan x$ // $a(x) = -\tan x$ $f(x) = -\tan(x)$

$dy/dx = \tan x\,(y-1)$ $e^{-\int a(x)} = \frac{1}{\cos(x)}$ $e^{\int a(x)} = \cos(x)$

$$y(x) = \frac{1}{\cos(x)} \cdot \left(\int -\tan(x)\cos(x)\,dx + c \right) \Rightarrow \left\| \tan(x) = \frac{\sin}{\cos}(x) \right\|$$

$$y(x) = \frac{c}{\cos(x)} + \frac{\cos(x)}{\cos(x)} = \frac{c}{\cos(x)} + 1$$

d) $(1+x)y' = xy + x^2$ // $a(x) = \frac{-x}{1+x}$ $f(x) = \frac{x^2}{x+1}$

$dy/dx = \frac{xy + x^2}{1+x}$ $e^{-\int a(x)} = e^{x - \ln|x+1|} = \frac{e^x}{x+1}$

$e^{\int a(x)} = e^{-x + \ln|x+1|} = e^{-x} \cdot (x+1)$

$$y(x) = \frac{e^x}{x+1} \cdot \left(\int \frac{x+1}{e^x} \cdot \frac{x^2}{x+1}\,dx + c \right) = \frac{ce^x}{x+1} + \frac{e^x}{x+1} \int \frac{x^2}{e^x}\,dx \Rightarrow$$

$$y(x) = \left\| \begin{matrix} u = x^2 & du = 2x \\ dv = e^{-x} & v = -e^{-x} \end{matrix} \right\| \Rightarrow \frac{ce^x}{x+1} + \frac{e^x}{x+1}\left(-x^2 e^{-x} - \int -2x e^{-x}\,dx \right) \Rightarrow$$

$$y(x) = \left\| \begin{matrix} u = x & du = 1 \\ dv = e^{-x} & v = -e^{-x} \end{matrix} \right\| \Rightarrow \frac{ce^x}{x+1} + \left(-x^2 e^{-x} - 2xe^{-x} - 2e^{-x} \right) \frac{e^x}{x+1}$$

$$y(x) = \frac{(-x^2 - 2x - 2)}{x+1} \cdot \cancel{e^{-x}} \cdot \cancel{e^x} + \frac{ce^x}{x+1}$$

(5) g) $y' = y/x + 4x^2 \ln(|x|)$ // $a(x) = \frac{-1}{x}$ $f(x) = 4x^2 \ln(x)$

$$dy/dx = \frac{y}{x} + 4x^2 \ln(x) \qquad y(x) = e^{\ln|x|} \cdot \left(\int e^{-\ln|x|} 4x^2 \ln(x) + C \right) \Rightarrow$$

$$y(x) = xC + x \int \frac{1}{x} \ln|x| \, 4x^2 \, dx \Rightarrow$$

$$\left\| \begin{matrix} u = \ln(x) & du = 1/x \\ dv = x & v = x^2/2 \end{matrix} \right\| \Rightarrow y(x) = xC + x\left(\frac{4x^2}{2} \ln|x| - 4 \int \frac{x}{2} dx \right) \Rightarrow$$

$$y(x) = xC + 2x^3 \ln|x| - x^3$$

h) $y' + \frac{2xy}{1-x^2} + 4x = 0 \quad (|x| < 1)$

$a(x) = \frac{2x}{1-x^2}$ $f(x) = -4x$

$dy/dx = -4x - \frac{2xy}{1-x^2}$ //

$$e^{-\int \frac{2x}{1-x^2}} = \ln|1-x^2|$$

$$e^{\int \frac{2x}{1-x^2}} = -\ln|1-x^2|$$

$$y(x) = e^{\ln|1-x^2|} \cdot \left(\int e^{-\ln|1-x^2|} \cdot -4x + C \right) \Rightarrow$$

$$y(x) = (1-x^2) \cdot \left(C + \int \frac{-4x}{1-x^2} dx \right) \Rightarrow$$

$$y(x) = C(1-x^2) + 2\ln|1-x^2| \cdot (1-x^2)$$

1° Worksheet // 1° order ODE // BDSA Raul [illegible]

6 a) $y' + 2y = 10 \quad y(0) = 8$

$dy/dx = 10 - 2y \rightarrow a(x) = 2 \rightarrow e^{\int -a(x)dx} = e^{-2x}$

$\rho(x) = 10 \quad e^{\int a(x)dx} = e^{2x}$

$$y(x) = e^{-2x}\int e^{2x}\cdot 10 + e^{-2x}C \Rightarrow 5e^{-2x}e^{2x} + e^{-2x}C \Rightarrow \underline{\underline{5 + e^{-2x}C}}$$

$$y(0) = 8 \rightarrow 5 + e^{-2\cdot 0}C = 8 \rightarrow C = 8 - 5 = \underline{\underline{3}}$$

d) $x^2y' + 4xy = 2e^{-x} \quad y(1) = 1$

$y' = -4y/x + \frac{2}{x^2e^x} \quad dy/dx = y'$

$$y(x) = e^{-\int \frac{4}{x}dx}\cdot\left(\int e^{\int 4/x\,dx}\cdot\frac{2}{x^2e^x}\,dx + C\right) = x^{-4}\cdot\left[\int \frac{x^4\,2}{x^2e^x}\,dx + C\right]$$

$$y(x) = x^{-4}\left[2\int\frac{x^2}{e^x}\,dx\right] + \frac{C}{x^4} = x^{-4}\cdot\left(\frac{-2x^2}{e^x} + 2\int\frac{2x}{e^x}\,dx\right) + \frac{C}{x^4}$$

|| $u = x^2 \quad du = 2x$
$dv = 1/e^x \quad v = -1/e^x$

$$y(x) = \underline{\underline{\left(\frac{-2x^2}{e^x} - \frac{4}{x^3e^x} - \frac{4}{x^4e^x} + \frac{C}{x^4}\right)}}$$

|| $y(1) = 1 \rightarrow 1 = \frac{-2}{e} - \frac{4}{e} - \frac{4}{e} + C$

$C = 1 + \frac{10}{e}$ ||

$$y(x) = \underline{\underline{\frac{-2x^2 - 4x - 4}{x^4e^x} + \frac{1 + \frac{10}{e}}{x^4}}}$$

e) $xy' = y + 2x^2 \quad y(5) = 5 \qquad dy/dx = y/x + 2x$

$y' = 2x + y/x$

$y(x) = e^{\int 1/x\, dx} \cdot \left[\int e^{-\int 1/x\, dx} \cdot 2x\, dx + C \right] = x \int 2\, dx + xC = 2x^2 + xC$

$\| y(5) = 5 \rightarrow 5 = 50 + 5C \rightarrow C = -9 \| \quad y(x) = 2x^2 - 9x$

f) $xy' + (x+2)y = 2\sin(x) \qquad y(\pi) = 1 \qquad dy/dx = \frac{2}{x}\sin(x) - y\,\frac{x+2}{x}$

$y(x) = e^{-\int \frac{x+2}{x}} \cdot \left[\int e^{\int x+2/x} \cdot 2/x \sin(x)\, dx + C \right] = \left\| \int \frac{x+2}{x} = x + 2\ln(x) \right\|$

$y(x) = \frac{1}{e^x \cdot x^2} \cdot \left[\int \frac{e^x \cdot x^2}{x} \cdot 2\sin(x)\, dx + C \right] = \frac{1}{e^x x^2} \cdot \left[2 \int x e^x \sin(x)\, dx + C \right]$

$y(x) = \frac{1}{e^x x^2} \left[C + 2 \left[-xe^x \cos(x) + \int \cos(x) e^x + \int x e^x \cos(x)\, dx \right] \right]$

$\| \int \cos(x) e^x dx = e^x \sin(x) - \int \sin(x) e^x = e^x \sin(x) - \left(-e^x \cos(x) + \int e^x \cos(x)\, dx\right)$

$\int \cos(x) e^x dx = \frac{e^x \sin(x) + e^x \cos(x)}{2} \|$

$\| \int x e^x \cos(x)\, dx = \left\| \begin{matrix} u = xe^x & du = e^x + xe^x \\ dv = \cos(x) & v = \sin(x) \end{matrix} \right\| = xe^x \sin(x) - \int e^x \sin(x)\, dx - \int \sin(x) e^x x\, dx$

$\int x e^x \cos(x)\, dx = xe^x \sin(x) + \frac{e^x \cos(x) - e^x \sin(x)}{2} - \int \sin(x) x e^x \Rightarrow \left\| \begin{matrix} u = e^x x & du = e^x + xe^x \\ dv = \sin(x) & v = -\cos(x) \end{matrix} \right\|$

$\int x e^x \cos(x)\, dx = \dots - \left(-xe^x \cos(x) + \int \cos(x)\, dx\, e^x + \int \cos(x)\, x e^x\, dx \right)$

$\int x e^x \cos(x)\, dx = xe^x(\sin(x) + \cos(x)) - e^x \sin(x) \,/2 \|$

$y(x) = \frac{C}{e^x x^2} + \frac{2}{e^x x^2} \left(\frac{e^x \sin(x) + e^x \cos(x)}{2} + \frac{-xe^x \cos(x) + xe^x \sin(x) - e^x \sin(x)}{2} \right)$

$y(x) = \frac{C}{e^x x^2} + \frac{\cos(x) - x\cos(x) + x\sin(x)}{x^2} \rightarrow y(\pi) = 1 \rightarrow C = e^{\pi}(\pi^2 - \pi + 1)$

$y(x) = \left(e^{-\pi - x}(\pi^2 - \pi - 1) + \cos(x) - x\cos(x) + x\sin(x) \right) / x^2 \;//$

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6 g) $xy' + 2y = x$ $y' = 1 - \frac{2y}{x}$ // $a(x) = 2/x$ $f(x) = 1$ $y(3) = 1$

$dy/dx = \frac{x - 2y}{x}$ $\to$ $y(x) = e^{\int -a(x)} \cdot \left[\int e^{\int a(x)} \cdot f(x) + C \right]$

$y(x) = e^{-2\ln(x)} \cdot \left[e^{2\ln(x)} dx + C \right] = e^{-2\ln|x|} \cdot \left[x^3/3 + C \right] =$

$y(x) = \frac{1}{x^2} \cdot \left[x^3/3 + C \right] = x/3 + \frac{C}{x^2}$ $\to$ $\frac{x}{3} = y(x)$

$y(3) = 1 = \frac{3}{3} + \frac{C}{x^2} \Rightarrow \frac{C}{x^2} = 0 \to C = 0$

h) $xy' + 2y = 2\operatorname{sen}(x)$ $a(x) = 2/x$ $f(x) = \frac{2\operatorname{sen}(x)}{x}$ $y(\pi) = 1$

$dy/dx = \frac{2\operatorname{sen}(x) - 2y}{x}$ $y(x) = e^{-\int 2/x} \cdot \left[\int e^{\int 2/x} \cdot \frac{2\operatorname{sen}(x)}{x} + C \right]$

$y(x) = \frac{1}{x^2} \cdot \left[\int x^2 \cdot \frac{2\operatorname{sen}(x)}{x} + C \right] = \frac{1}{x^2} \cdot \left[\int 2x\operatorname{sen}(x) + C \right]$

$y(x) = \frac{1}{x^2} \cdot C + \frac{1}{x^2} \cdot 2 \int x\operatorname{sen}(x) =$ // $u = x$ $du = 1$; $dv = \operatorname{sen}(x)$ $v = -\cos(x)$ // $\Rightarrow$

$y(x) = \frac{C}{x^2} + \frac{2}{x^2} \cdot \left[-x\cos(x) - \int -\cos(x) \right] = \frac{C}{x^2} + \frac{2}{x^2}\left(-x\cos(x) + \operatorname{sen}(x) \right) =$

$y(x) = \frac{C}{x^2} + \frac{2\operatorname{sen}(x)}{x^2} - \frac{2x\cos(x)}{x^2}$ $\to$ $y(\pi) = 1$ $\to$ $\frac{C}{\pi^2} - \frac{2\pi \cdot 1}{\pi^2} = 1$

$C = \pi^2 - 2\pi$

$y(x) = \frac{2\operatorname{sen}(x)}{x^2} - \frac{2x\cos x}{x^2} + \frac{\pi^2 - 2\pi}{x^2}$

i) $y' + y = e^x$ $a(x) = 1$ $f(x) = e^x$ $y(0) = 1$

$dy/dx = e^x - y$ $y(x) = e^{-x} \cdot \left[\int e^x \cdot e^x dx + C \right]$

$y(x) = Ce^{-x} + \int e^{2x} dx = Ce^{-x} + \frac{e^x}{2}$

$y(0) = 1 \to 1 = C + \frac{1}{2} \Rightarrow C = \frac{1}{2}$

$y(x) = \frac{e^{-x}}{2} + \frac{e^x}{2}$ //

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b) $x^2y' + 2yx - x + 1 = 0$ // $y(1) = 0$ $\qquad a(x) = \frac{2}{x}$

$dy/dx = \frac{-1 + x - 2yx}{x^2}$ $\rightarrow$ $y' + \frac{2}{x}y = \frac{x-1}{x^2}$ $\Rightarrow f(x) = \frac{x-1}{x^2}$

$e^{-\int a(x)} = e^{-2\ln|x|} = \frac{1}{x^2}$ // $e^{\int a(x)} = x^2$

$y(x) = \frac{1}{x^2} \cdot \left(\int \frac{x-1}{x^2} \cdot x^2 dx + C \right) \Rightarrow \frac{C}{x^2} + \frac{1}{x^2} \cdot \left(\int (x-1) dx \right) \Rightarrow$

$y(x) = \frac{1}{2} - \frac{1}{x} + \frac{C}{x^2}$ $\rightarrow$ $y(1) = 0$ $\rightarrow$ $C = \frac{1}{2}$ $\rightarrow$ $\frac{1}{2} - \frac{1}{x} + \frac{1}{2x^2}$

c) $y' = y + 6x^2$ // $y(0) = -2$ $\qquad a(x) = -1 \quad f(x) = 6x^2$

$dy/dx = y + 6x^2$ $\qquad y(x) = e^{x} \cdot \left(\int 6x^2 e^{-x} dx + C \right) \Rightarrow$

$y(x) = e^x C + e^x 6 \int x^2 e^{-x} dx$ || dans la S.d || $\Rightarrow$

$y(x) = 6(-x^2 - 2x - 2) + e^x C = -6x^2 - 12x - 12 + e^x C$

$y(0) = -2$ $\rightarrow$ $C = -2 + 12$ $\rightarrow$ $C = 10$ $\rightarrow$ $y(x) = -6x^2 - 12x - 12 + 10e^x$

g) $x^2y' - 4x^3 = 0$ $\qquad y(1) = 1$

$dy/dx = 4x^3/x^2 = 4x$ $\rightarrow$ $\int dy = \int dx\, 4x \Rightarrow y = \frac{4x^2}{2} + C$

$y(1) = 1 \rightarrow C = 1 - 2 = -1$

$y(x) = 2x^2 - 1$

⑥ k) $y' + y = e^{-x}$ $(y(0) = 1)$ $a(x) = 1$ $f(x) = e^{-x}$

$dy/dx = e^{-x} - y \rightsquigarrow e^{-x}\left(c + \int e^{x} \cdot e^{-x}\right) = y(x)$

$y(x) = e^{-x}c + xe^{-x} \rightsquigarrow y(0) = 1 \rightsquigarrow c = 1 \rightsquigarrow y(x) = e^{-x} + xe^{-x}$

l) $y' + \dfrac{y}{x^2-1} = 1 + x$ $(y(0) = 1)$

$dy/dx = 1 + x - \dfrac{y}{x^2-1}$ $a(x) = \dfrac{1}{x^2-1}$ $f(x) = 1 + x$

$e^{-\int a(x)} = e^{-\frac{1}{2}(\ln|-x+1| - \ln|x+1|)} = e^{-\frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|} \Rightarrow$

$e^{-\int a(x)} = \sqrt{\dfrac{x+1}{x-1}}$ $e^{\int a(x)} = \sqrt{\dfrac{x-1}{x+1}}$

$y(x) = \sqrt{\dfrac{x+1}{x-1}} \cdot \left(\int \sqrt{\dfrac{x-1}{x+1}} \cdot 1 + x + c\right)$

$y(x) = \sqrt{\dfrac{x+1}{x-1}} \cdot \left(c + \int \sqrt{\dfrac{(x-1)(1+x)^2}{(\cancel{x+1})}}\right) \Rightarrow$

$y(x) = c\sqrt{\dfrac{x+1}{x-1}} + \sqrt{\dfrac{x+1}{x-1}} \int \sqrt{(x-1)(x+1)}\, dx \Rightarrow$ || Wolfr. alpha ||

$y(x) = c\sqrt{\dfrac{x+1}{x-1}} + \sqrt{\dfrac{x+1}{x-1}} \cdot \left(\dfrac{1}{2}\left(x\sqrt{x^2-1} - \ln\left|\sqrt{x^2-1} + x\right|\right)\right)$

$y(x) = c\sqrt{\dfrac{x+1}{x-1}} + \dfrac{x}{2} \cdot \sqrt{\dfrac{(x+1)(x^2-1)}{(x-1)}} - \dfrac{1}{2}\sqrt{\dfrac{x+1}{x-1}} \ln\left|\sqrt{x^2-1} + x\right|$