

Llei d'Ohm

Impedància

fasor intensitat

$$\bar{I} = I_0 \angle -\varphi$$

$$V_{ef} = \frac{1}{\sqrt{2}} V_0$$

$$V_{ef} = Z_{ef} \cdot I_{ef}$$

$$P = R_{ef} \cdot I_{ef}^2 = R_{ef} \cdot \left(\frac{V_{ef}}{Z_{ef}} \right)^2 \Rightarrow V_{ef} = \sqrt{\frac{P \cdot Z_{ef}^2}{R_{ef}}}$$

$$P = V_{ef} \cdot I_{ef} \cdot \cos \varphi$$

$$P = Z \cdot I_{ef} \cdot \cos \varphi$$

$$P = \frac{V_{ef}^2}{R}$$

$$S^2 = P^2 + Q^2$$

$$f_b = \frac{1}{T}$$

$$f_b = 2V$$

$$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

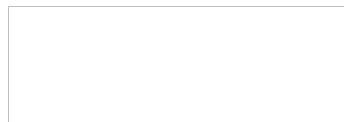
$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$

$$-\sin(x) = \sin(x + \pi)$$

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$

$$\cos\left(\varphi + \frac{\pi}{2}\right) = \sin(\varphi + \pi)$$

$$\cos(\alpha) = \sin\left(\alpha + \frac{\pi}{2}\right)$$



Nombres complexos

suma disjunta nombres reals i imaginaris purs. part real i una d'imaginària,

pla complex l'eix real i imaginària unitat imaginària $i = \sqrt{-1}$

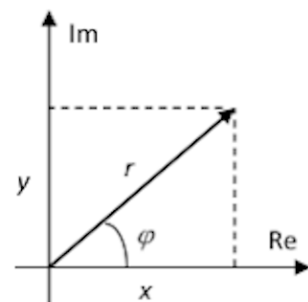
complex conjugat $\bar{z}_2^* = x_2 - iy_2 = r_2 \angle -\varphi_2$)

$$\bar{z}_1 = x_1 + iy_1 = r_1 \angle \varphi_1$$

$$\bar{z}_2 = x_2 + iy_2 = r_2 \angle \varphi_2$$

- Notació cartesiana $\bar{z} = x + iy$

$$x = r \cos \varphi \quad y = r \sin \varphi$$



- Notació polar $\bar{z} = r_\varphi$ o $r \angle \varphi$

$$r = \sqrt{x^2 + y^2} \quad \varphi = \tan^{-1} \left(\frac{y}{x} \right)$$

- fórmula d'Euler

$$e^{i\varphi} = \cos \varphi + i \sin \varphi$$

$$\bar{z} = x + iy = r(\cos \varphi + i \sin \varphi) = r e^{i\varphi} = r \angle \varphi$$

Euler

Exemples:

- $4 + 3i$

$$r = \sqrt{4^2 + 3^2} = 5$$

$$\varphi = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$$

- $5 - 12i$

$$\sqrt{5^2 + 12^2} = 13$$

$$\varphi = \tan^{-1}\left(\frac{-12}{5}\right) = -67.4^\circ = 292.6^\circ$$

- $6\angle 120^\circ$

$$6 \cos 120^\circ + i \sin 120^\circ = -3 + 5.2i$$

Exemples: $(1 + 2i) \cdot (3 - 2i) = (3 + 4) + i(6 - 2) = 7 + 4i$

$$2\angle 150^\circ \cdot 1\angle 30^\circ = 2\angle 180^\circ$$

$$(3 - 2i) \cdot (2 + 3i) = (6 + 6) + i(9 - 4) = 12 + 5i$$

$$\begin{aligned}(3 - 2i) \cdot (2 + 3i) &= \sqrt{13}\angle -33.7^\circ \cdot \sqrt{13}\angle 56.3^\circ = 13\angle 22.6^\circ \\ &= 13 \cos 22.6^\circ + i 13 \sin 22.6^\circ = 12 + 5i\end{aligned}$$

Exemples:

$$\frac{(2+i)}{(1-2i)} = \frac{(2+i)(1+2i)}{(1-2i)(1+2i)} = \frac{(2-2) + i(4+1)}{(1+4)} = \frac{5i}{5} = i$$

$$4\angle 65^\circ / 2\angle 15^\circ = 2\angle 50^\circ$$

$$\frac{(18-i)}{(3+4i)} = \frac{(18-i)(3-4i)}{(3+4i)(3-4i)} = \frac{(54-4) + i(-72-3)}{(9+16)} = \frac{50-75i}{25} = 2-3i$$

Exemples: $(2 + 3i) + (4 - i) = 6 + 2i$

$$(3 + 3i) - (6 + 2i) = -3 + i$$

- Suma i resta **cartesiana:**

$$\bar{z}_1 + \bar{z}_2 = (x_1 + x_2) + i(y_1 + iy_2)$$

$$\bar{z}_1 - \bar{z}_2 = (x_1 - x_2) + i(y_1 - iy_2)$$

- Multiplicació

cartesiana:

$$\bar{z}_1 \cdot \bar{z}_2 = (x_1 + iy_1) \cdot (x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$$

polar

$$\bar{z}_1 \cdot \bar{z}_2 = r_1 \angle \varphi_1 \cdot r_2 \angle \varphi_2 = r_1 r_2 \angle (\varphi_1 + \varphi_2)$$

- Divisió

cartesiana:

$$\begin{aligned} \frac{\bar{z}_1}{\bar{z}_2} &= \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} = \\ &= \frac{(x_1x_2 + y_1y_2) + i(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2} \end{aligned}$$

polar

$$\bar{z}_1 / \bar{z}_2 = r_1 \angle \varphi_1 / r_2 \angle \varphi_2 = r_1 / r_2 \angle (\varphi_1 - \varphi_2)$$

$$\begin{aligned}\frac{(18-i)}{(3+4i)} &= \frac{\sqrt{325}\angle -3.2^\circ}{5\angle 53.1^\circ} = 3.6\angle -56.3^\circ = \\ &= 3.6 \cos -56.3^\circ + i 3.6 \sin -56.3^\circ = 2 - 3i\end{aligned}$$

Función Periódica Alterna Compleja

$$\bar{z}(t) = A \cos(\omega t + \theta) + iA \sin(\omega t + \theta)$$

$$= x(t) + iy(t)$$

$$x(t) = A \cos(\omega t + \theta)$$

$$iy(t) = iA \sin(\omega t + \theta)$$

$$= Ae^{i(\omega t + \theta)}$$

$$= A \angle (\omega t + \theta)$$

$$= A \angle \omega t \angle \theta$$