Studying bidimensional turbulence with Kelvin-Helmholtz instability

phenomenological description

by

Vítor Sudbrack
vitorsudbrack@gmail.com

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Turbulence

Present in all scales

turba-ulentus

Two fundamental,
yet opposing, aspects

Self-organization
and cascade effects

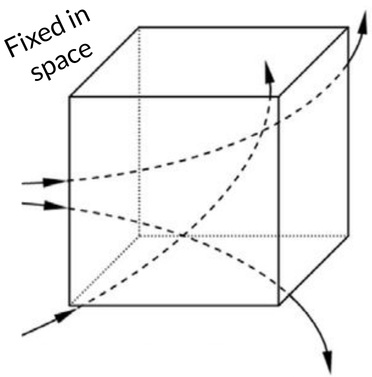
Forcing in all spectra

Many open questions
and open problems

Imagem: Turbulence Research Laboratory, Columbia University

Fluid mechanics

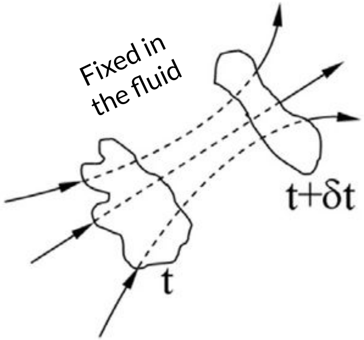
Eulerian description



What is the value of Z right here right now?

$$Z(\mathbf{r}, t)$$

Lagrangian description



What is the value of Z now, in the region of the fluid that was previously at this point?

$$Z(\mathbf{r}(t_0), t)$$

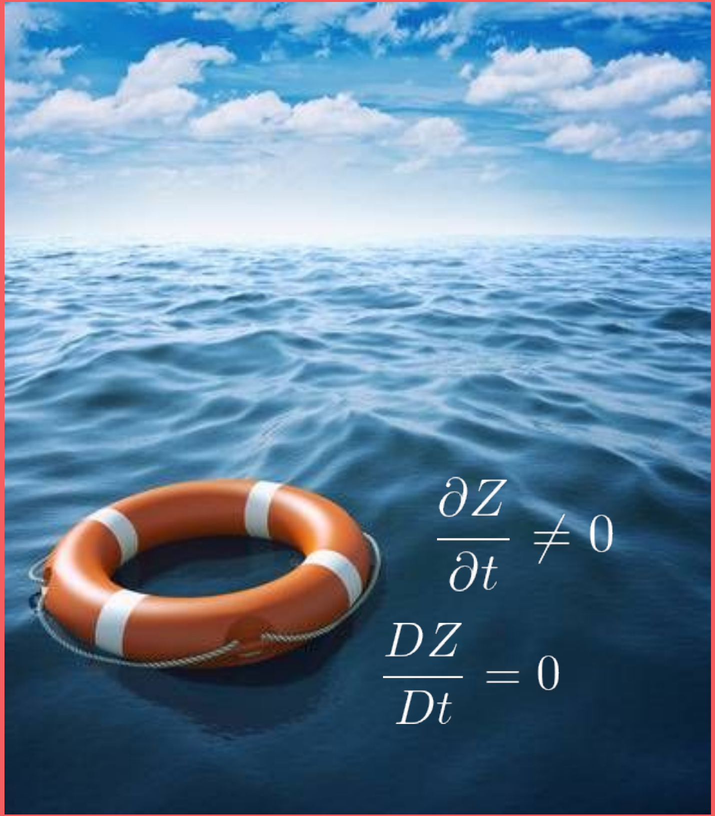
Tools

Material derivative

$$\frac{DZ}{Dt} = \frac{\partial Z}{\partial t} + (\mathbf{v} \cdot \nabla) Z$$

Vorticity

$$\omega = (\nabla \times \mathbf{v})_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$



Equations of motion

Stream function

$$\mathbf{v} = (u, v) = (\partial_y \psi, -\partial_x \psi)$$

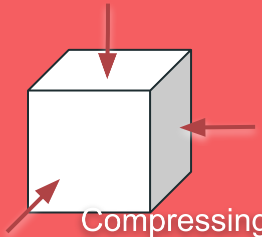
$$\nabla^2 \psi = -\omega$$

Incompressibility
$\rho = 1$

Continuity Eq.

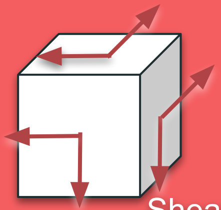
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Mass conservation



Compressing Forces

Fluid



Shearing Forces

Conservation of linear momentum

No viscosity

With viscosity

Euler Eq.

$\nu \to 0$

Navier-Stokes Eq.

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p$$

$$\frac{\partial \omega}{\partial t} = \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial \omega}{\partial t} = \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} + \nu \nabla^2 \omega$$

Reynolds Number

Given boundary conditions, the **Reynolds number is the only characterizing parameter of the fluid flow**

$$Re \sim \frac{[(\mathbf{v}\cdot\nabla)\mathbf{v}]}{[\nu\nabla^2\mathbf{v}]} \sim \frac{LV}{\nu}$$

In **Fully Developed Turbulence**
all symmetries of NS Eq. are restored
in a statistical sense

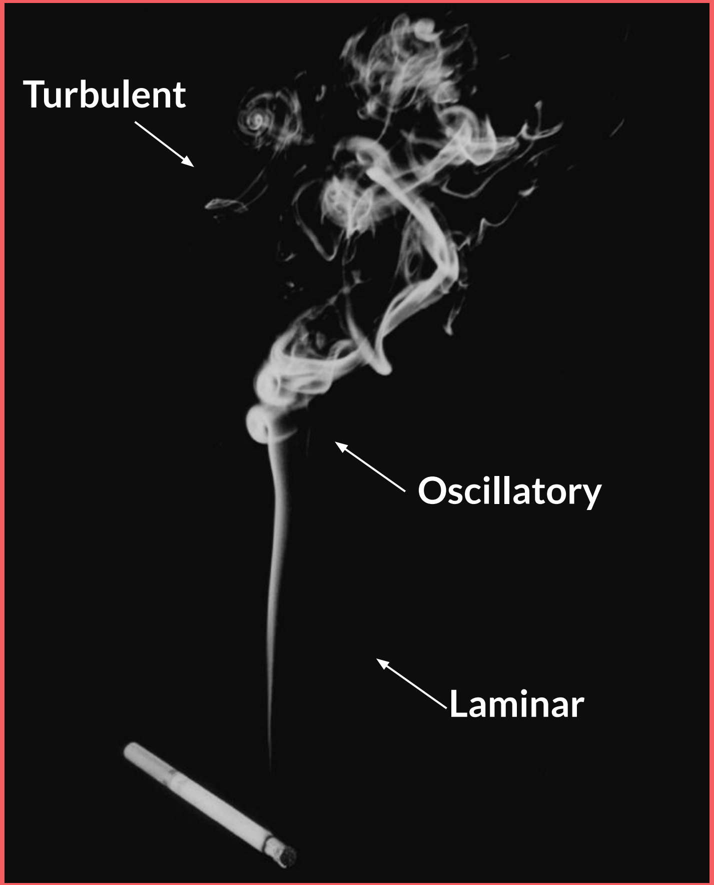


Imagem: IF - UFRJ

Conservation laws

In a boundaryless domain...

$$E = \frac{1}{2}\langle \psi\omega \rangle \qquad \frac{\partial E}{\partial t} = -2\nu\Omega$$

$$\Omega = \frac{1}{2}\langle \omega^2 \rangle \qquad \frac{\partial \Omega}{\partial t} = -\nu\left\langle |\nabla \times \omega|^2 \right\rangle$$

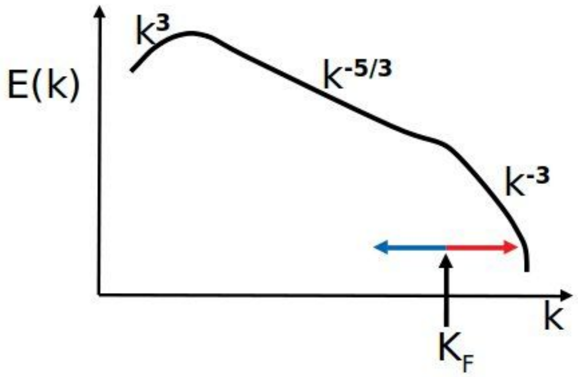
Ideal fluid $\longrightarrow$ Energy and enstrophy are strongly conserved

Viscous fluid
- $\nearrow$ Bidimensional $\longrightarrow$ Energy and enstrophy are weakly conserved
- $\searrow$ Tridimensional $\longrightarrow$ Anomalous dissipation!

Energy spectrum

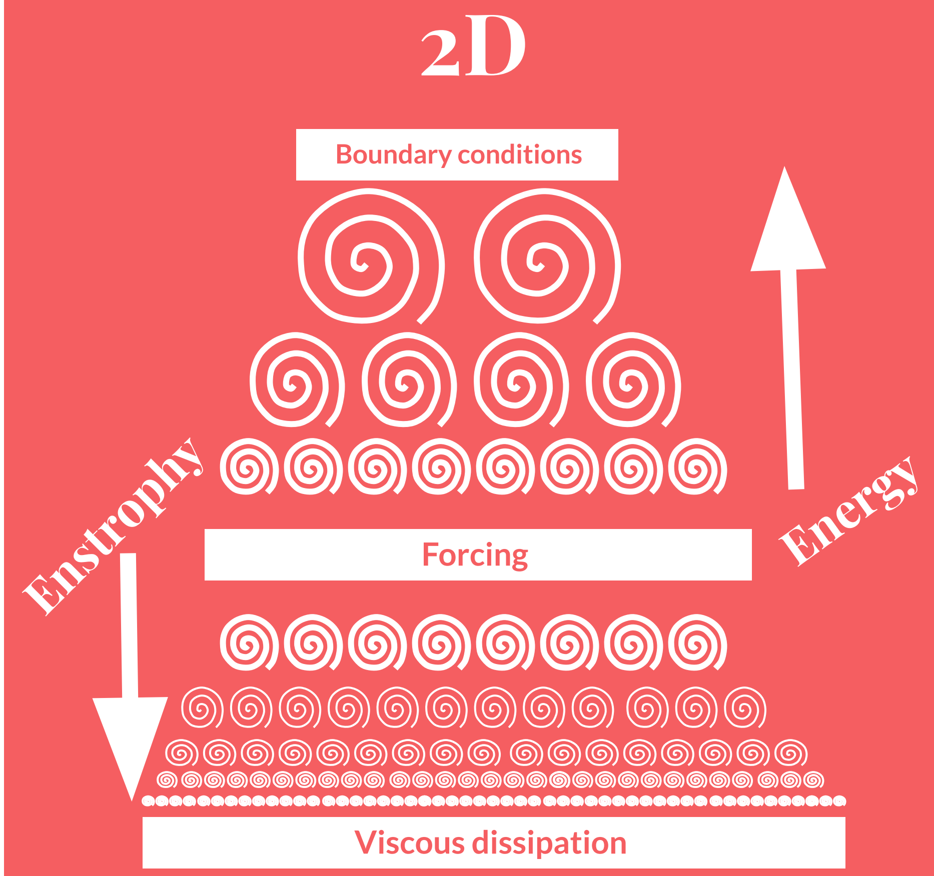
K41: $E(k) = F(\eta k)\varepsilon^{2/3}k^{-5/3}$

KLB:



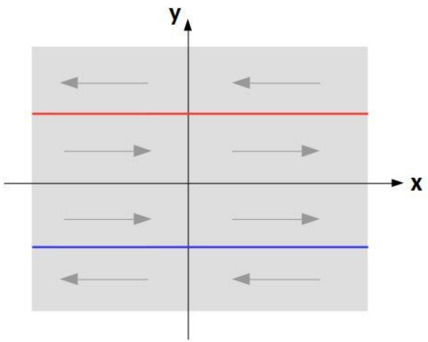
Integral scale | **Inertial range** | Viscous scale

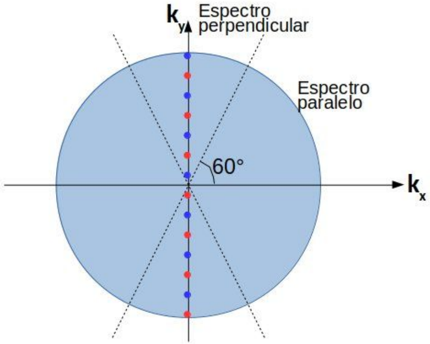
$$L \gg x \gg \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$$



Kelvin-Helmholtz instability

The interface of two fluid layers presenting relative motion is perturbed, generating turbulence





Euler: $\lambda \propto iku_0$ (via linear analysis)

N-S: ?

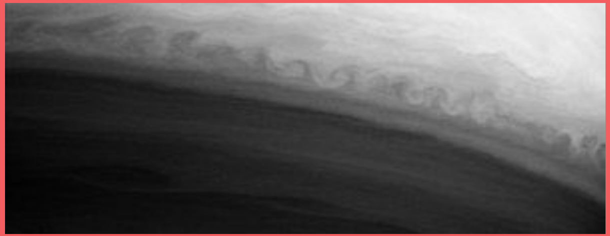


Imagem: NASA; DNSimulations;

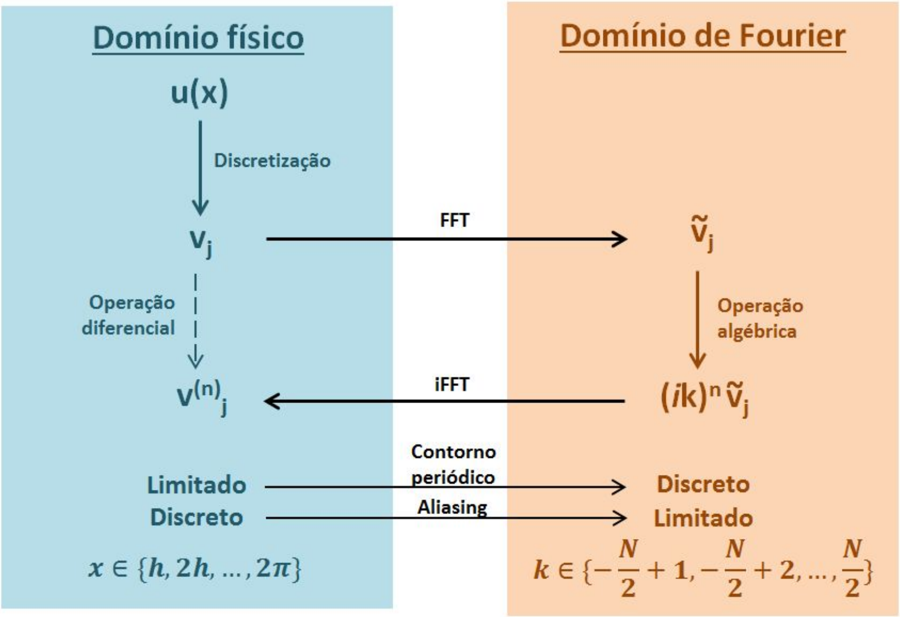
Spectral methods (DNS)

$$\tilde{\psi} = \frac{1}{k^2}\tilde{\omega}$$

$$\frac{\partial}{\partial t}\left(\tilde{\omega} e^{\nu k^2 t}\right) = e^{\nu k^2 t}\tilde{J}(\omega, \psi)$$

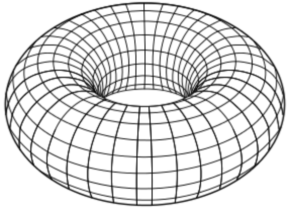
$$\begin{aligned}\tilde{J}(\psi, \omega) = \mathcal{F}\{&\mathcal{F}^{-1}\{ik_x\tilde{\psi}\}\mathcal{F}^{-1}\{ik_y\tilde{\omega}\} \\ &- \mathcal{F}^{-1}\{ik_y\tilde{\psi}\}\mathcal{F}^{-1}\{ik_x\tilde{\omega}\}\}\end{aligned}$$

$$\tilde{\omega}(\mathbf{k}, 0) = \frac{\omega_0}{2\pi}\left[i^{k_y} - (-i)^{k_y}\right]\delta(k_x) + \xi_{\mathbf{k}}$$



$$\nabla \longleftrightarrow i\mathbf{k}$$

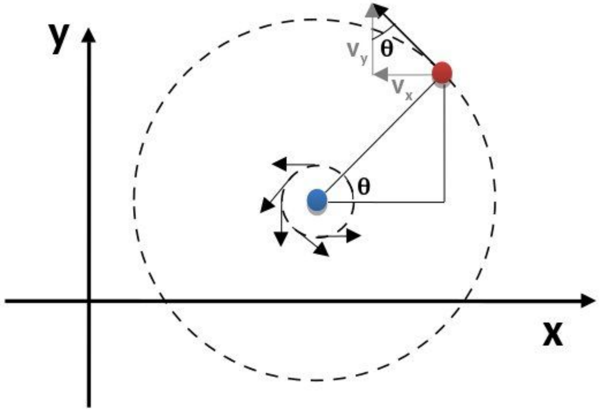
$$\nabla^2 \longleftrightarrow -k^2$$



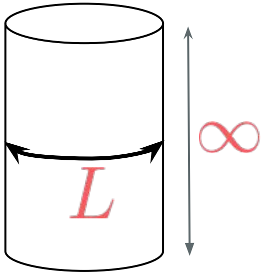
Point-vortex methods (PV)

$$z = x + iy$$

$$\frac{dx_\alpha}{dt} = -\frac{\Gamma_\beta}{2\pi r}\sin(\theta) = -\frac{\Gamma_\beta}{2\pi r}\frac{(y_\alpha - y_\beta)}{r}$$
$$\frac{dy_\alpha}{dt} = \frac{\Gamma_\beta}{2\pi r}\cos(\theta) = -\frac{\Gamma_\beta}{2\pi r}\frac{(x_\alpha - x_\beta)}{r}$$



$$\overline{\frac{dz_\alpha}{dt}} = \frac{-i}{2L}\sum_{\beta=1,\beta\neq\alpha}^{N}\Gamma_\beta \cot\left[\frac{\pi(z_\alpha - z_\beta)}{L}\right]$$

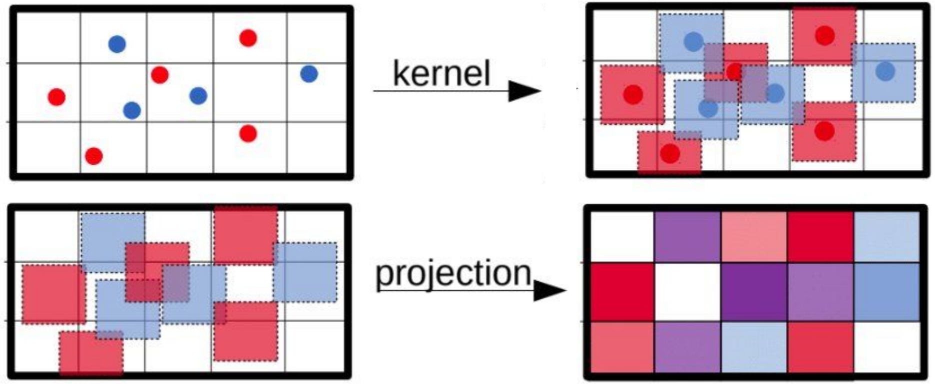


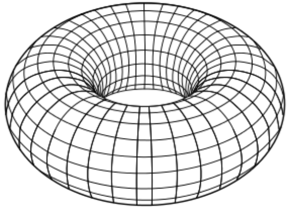
Vortex-in-cell method (ViC)

$$\omega(z) = \sum_{\alpha=1}^{N} \Gamma_\alpha \delta(z - z_\alpha)$$

$$(u, v) = \left(\mathcal{F}^{-1}\{ik_y\tilde{\psi}\}, \mathcal{F}^{-1}\{-ik_x\tilde{\psi}\}\right)$$

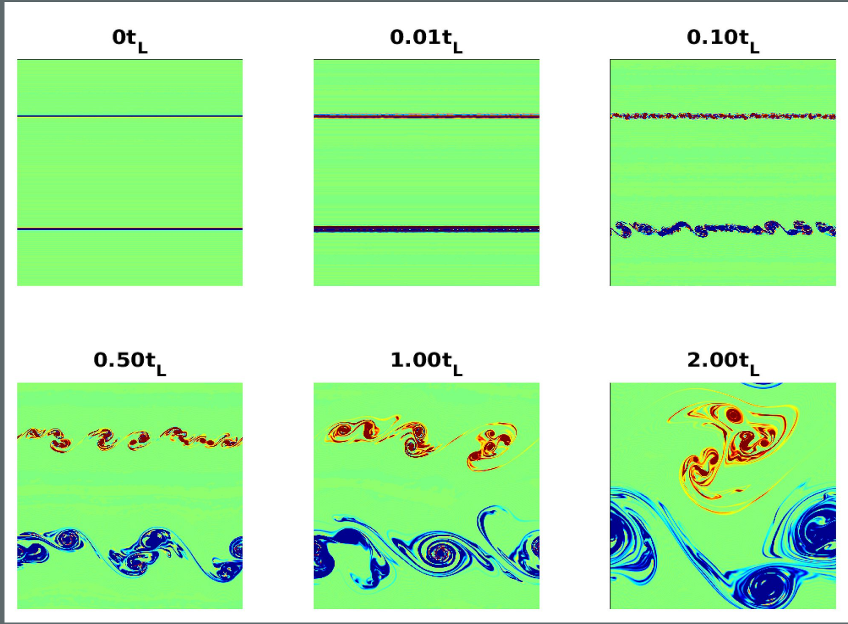
$$\frac{dz_\alpha}{dt} = u(z_\alpha, t) + iv(z_\alpha, t)$$





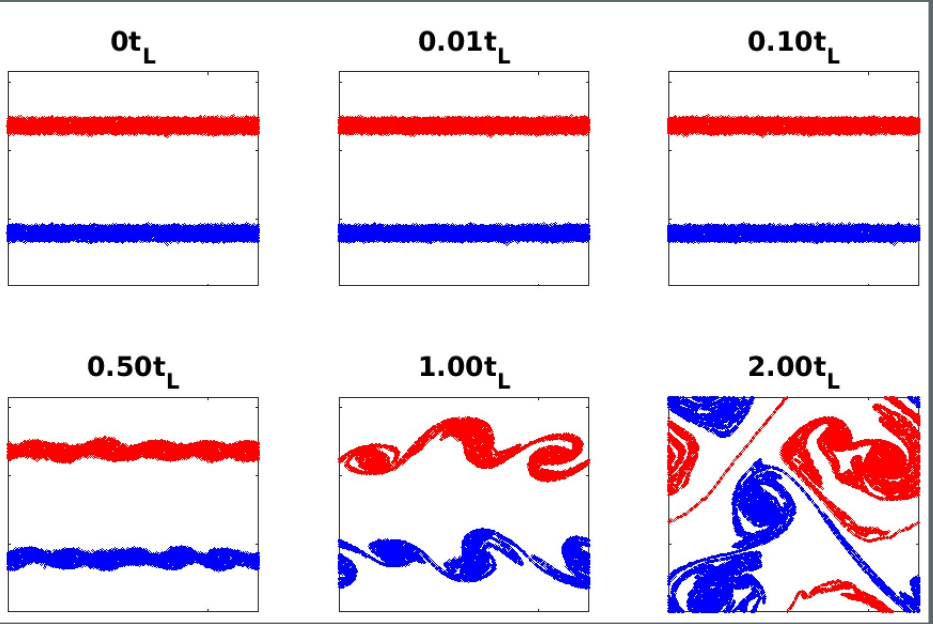
$$t_L = \frac{2\pi}{\sqrt{2E}}$$

DNS



Vorticity field for initial condition perturbed with 1%-amplitude white noise, non-local and equipartioned in enstrophy, resolution 2048^2. Color scale is non-linear and saturated. Green means zero vorticity and blue and red are high vorticities of different signs.

ViC

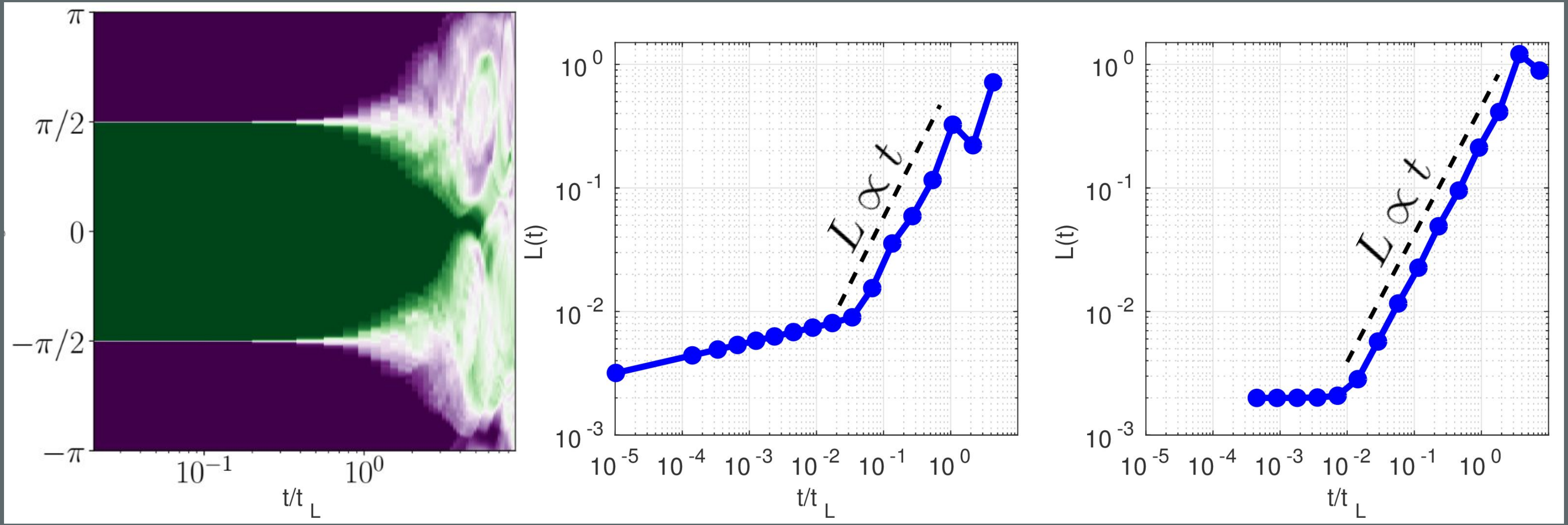


One million identical point-vortices, blue and red are opposite signs, imitating the previous case with vorticity lined perturbed with white noise of 0.01 SD.

Characteristic mixing length

DNS

ViC

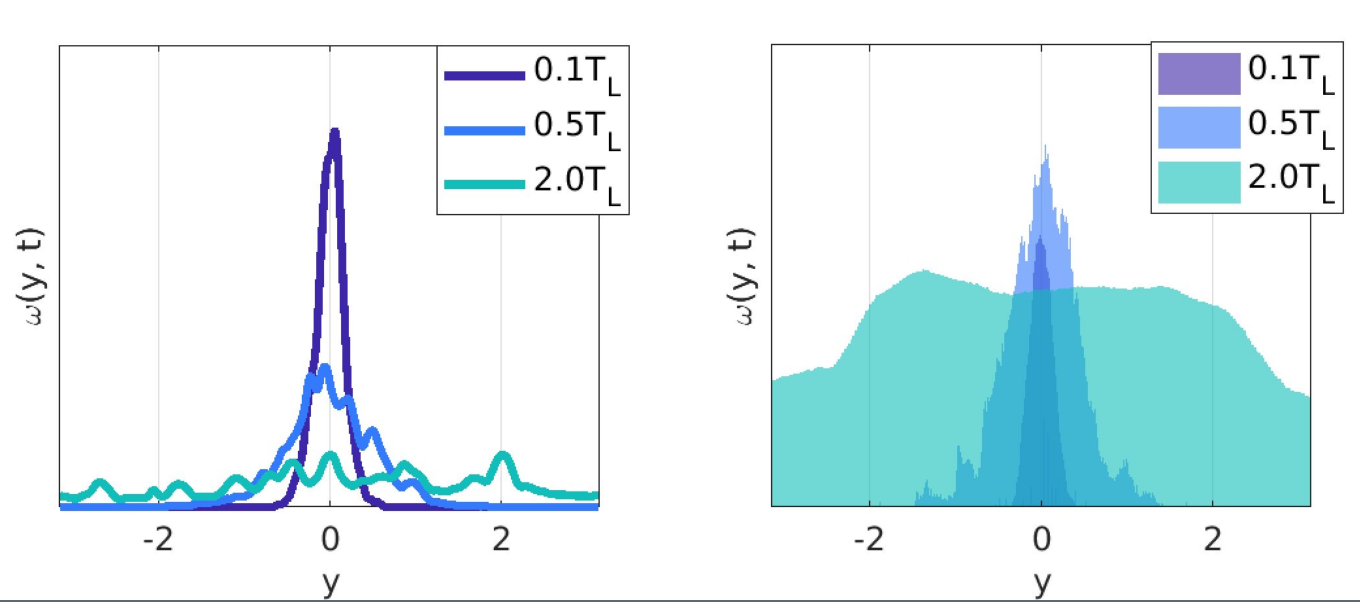


$$L(t) = \left(\frac{2\pi}{N}\right)\left(\sum_{i=1}^{N}\sum_{j=1}^{N/2}\left(y_j + \frac{\pi}{2}\right)^2 \omega_{ij}^2\right)^{1/2}$$

$$L(t) = \frac{1}{N}\left(\sum_{\alpha=1}^{N}(y_\alpha - y_0)^2\right)^{1/2}$$

Vorticity profile

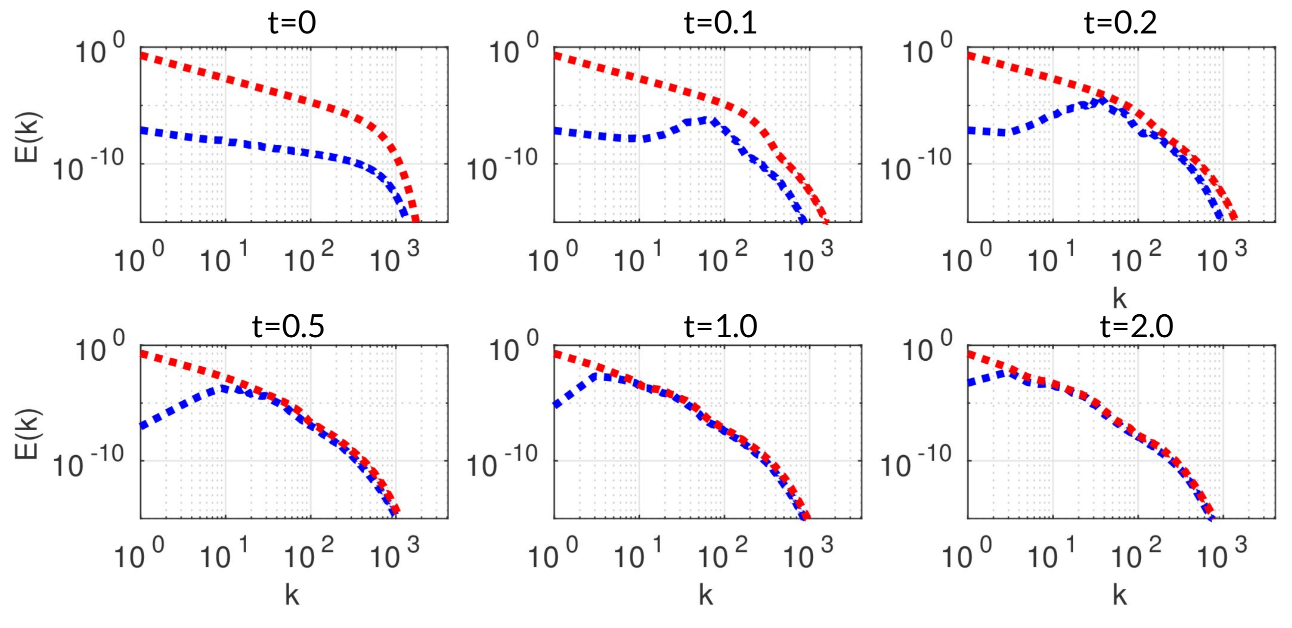
DNS



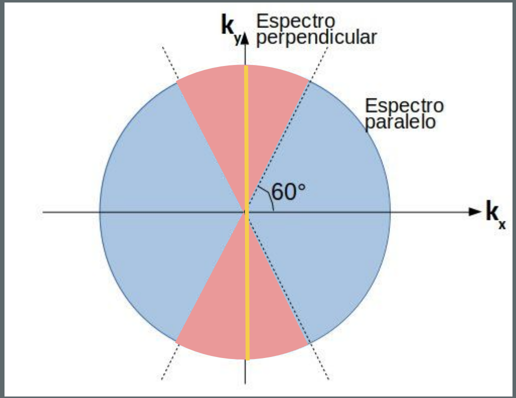
ViC

$$\langle \omega \rangle_x (y,t) = \frac{1}{\pi} \int_{-\pi}^{0} \omega(x,y,t)\,dx$$

Anisotropy



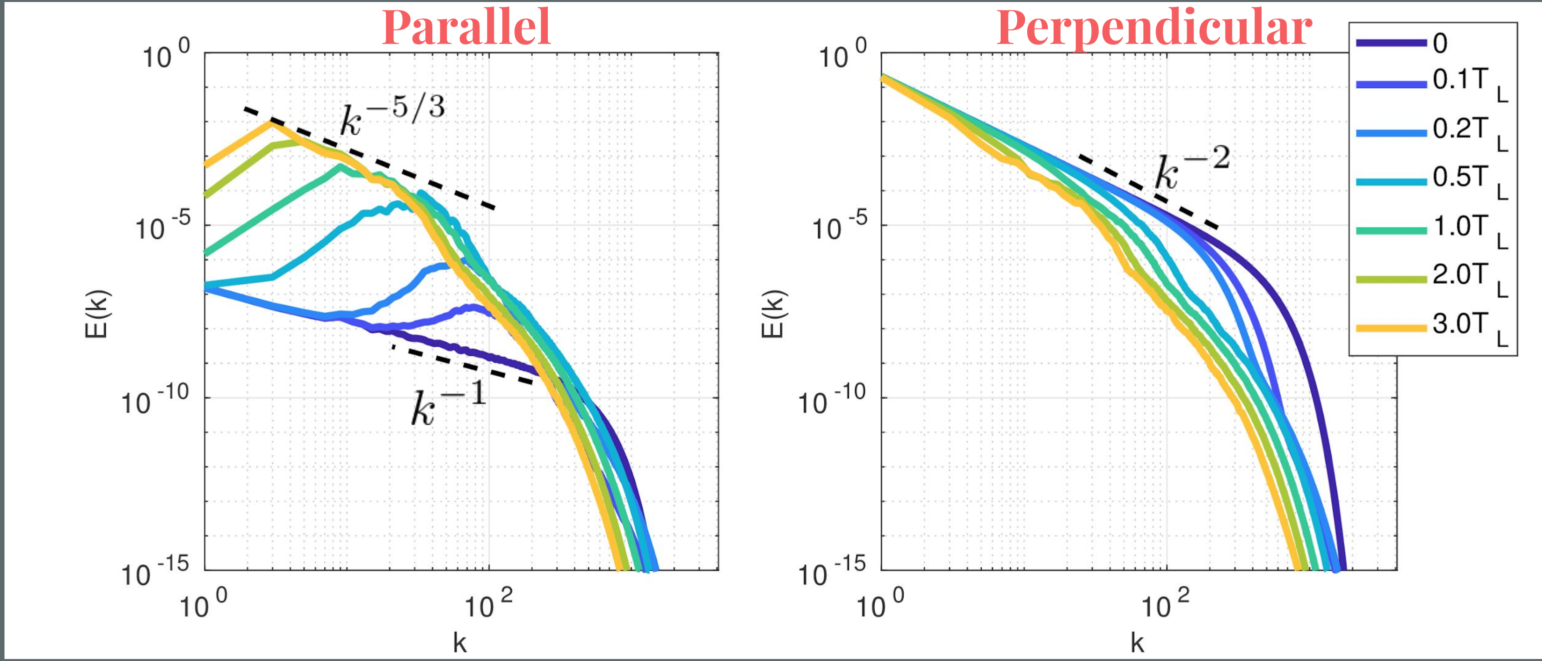
Comparing the parallel spectrum (espectro paralelo in blue) and the perpendicular spectrum (espectro perpendicular in red) in different instants in units of tL.



tangencial discontinuity

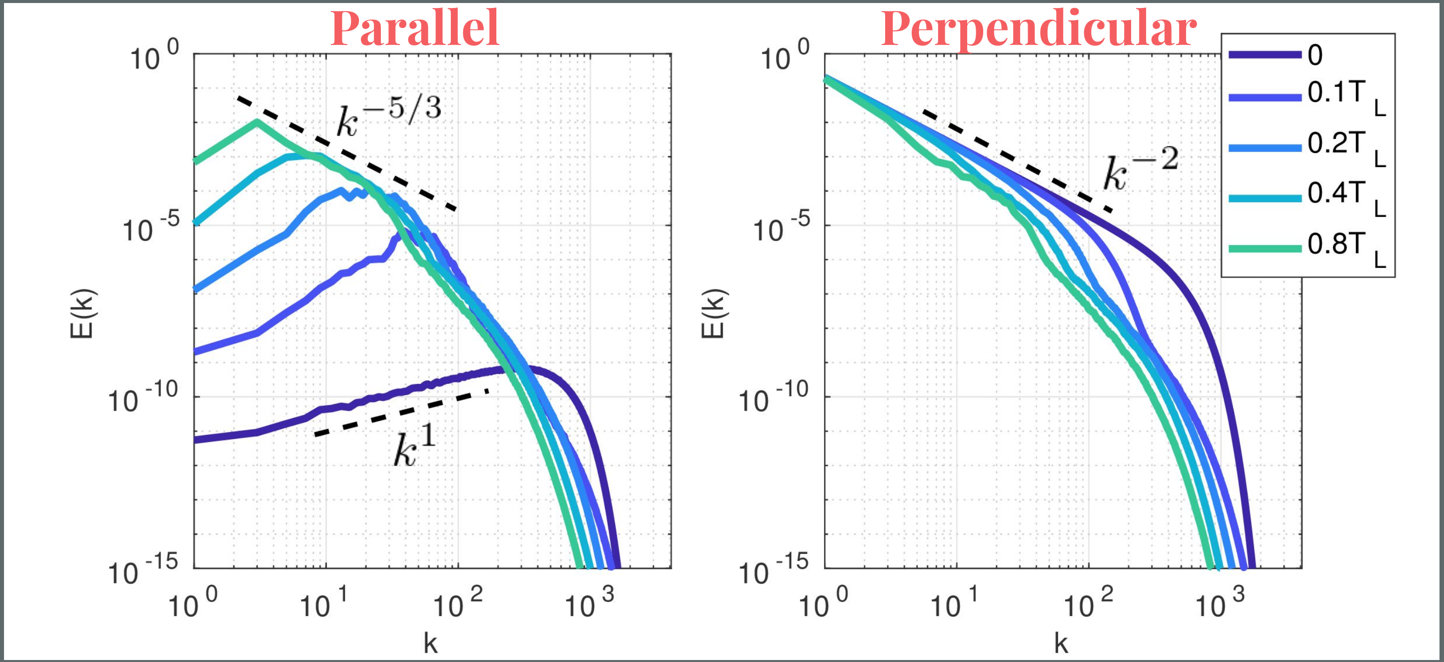
The **isotropy is restored progressively**, from the smaller to greater scales.

Energy spectrum - DNS



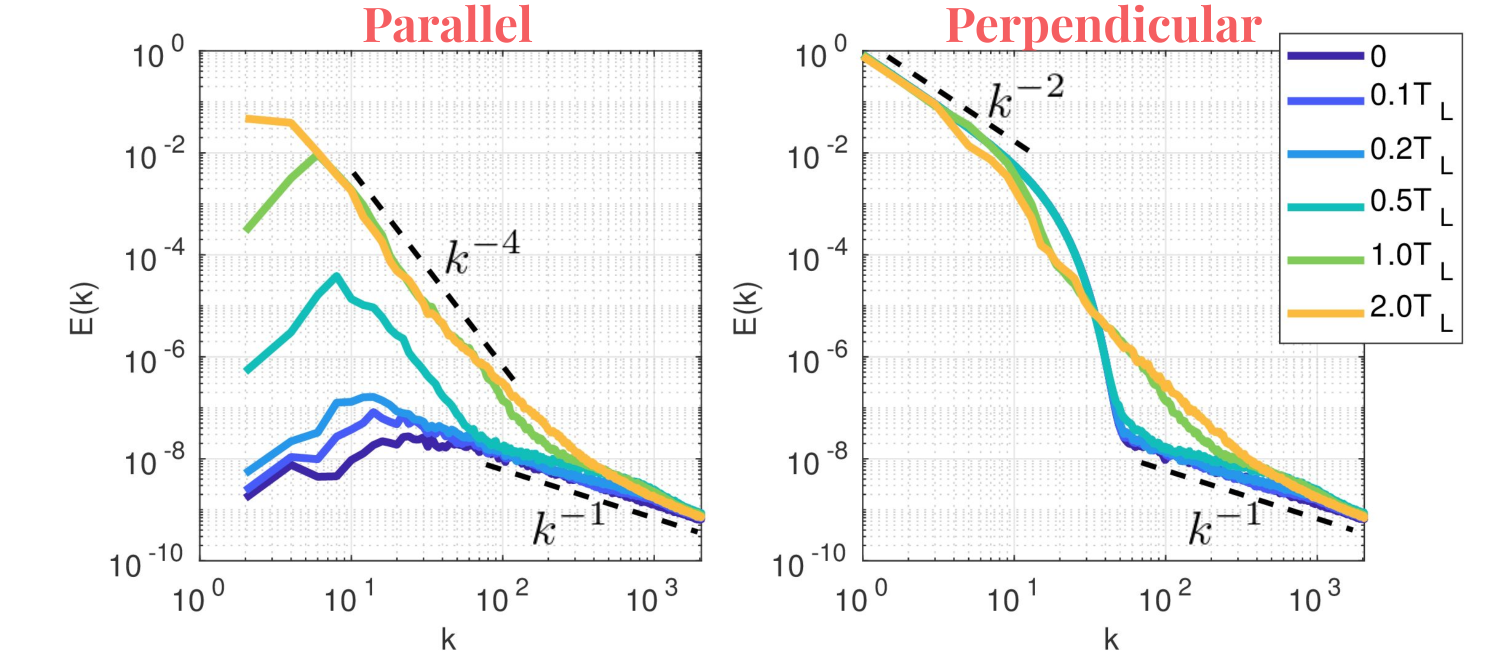
Parallel and perpendicular energy spectra, E(k), for DNS with resolution 8196^2 in different instants. **The initial noise is equipartitioned in enstrophy.** Average over 9 runs.

Energy spectrum - DNS



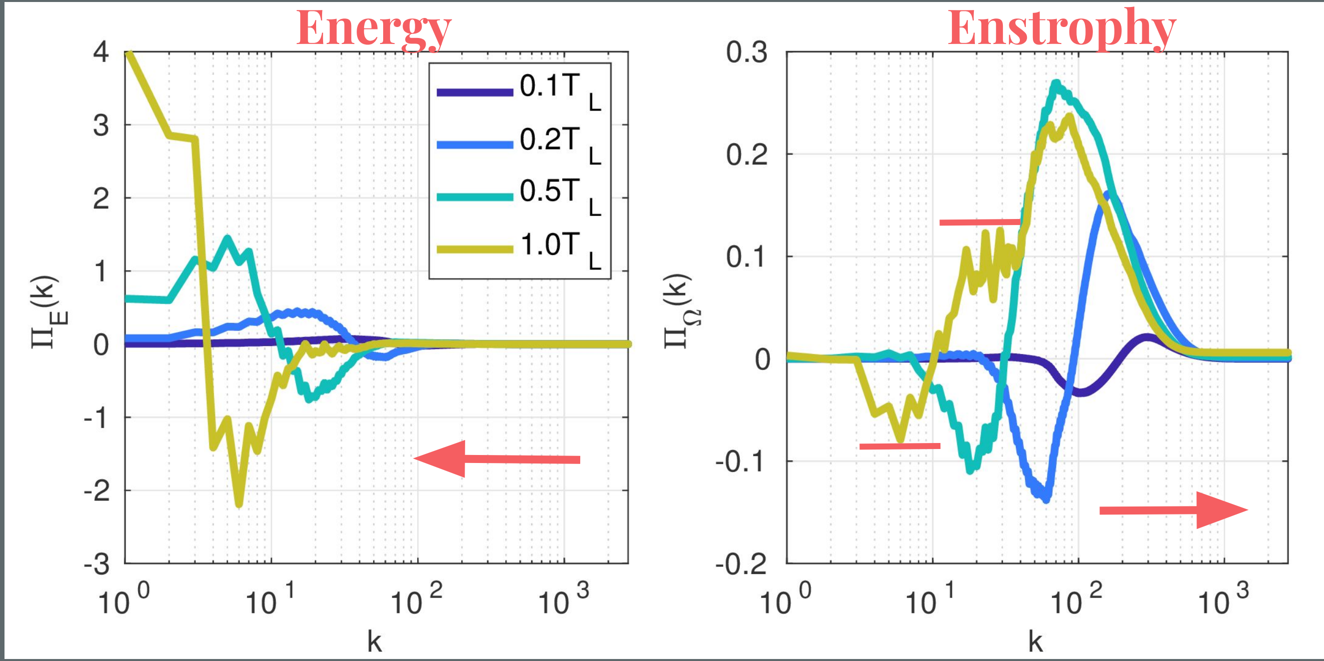
Parallel and perpendicular energy spectra, E(k), for DNS with resolution 8196^2 in different instants. **The initial noise is equipartitioned in energy.** Average over 9 runs.

Energy spectrum- ViC



Parallel and perpendicular energy spectra, E(k), for simulations with 1 million point vortices in ViC method, gridded with 2048^2 cells. The initial noise is on their vertical position, gaussian, with 0.06 SD.

Spectral flux- DNS



Energy and enstrophy fluxes for DNS with resolution 8196^2 in different instants. The initial noise is spectrally non-localized and equipartitioned in enstrophy. Average over 9 runs.

Conclusions

- ➔ Vorticity structured are created and spatially increasing in time
  - Qualitative images of both simulations are shown side-by-side with natural time scale
  - Characteristic mixing length and vorticity profile are compatible between vicious and ideal fluids

- ➔ Power laws in energy spectrum
  - Initially anisotropic in all scales, isotropy is progressively restored
  - Robust to resolution and changes in initial perturbation spectral distribution
  - Both current theories (K41 e KLB) do not meet these criteria
  - Different spectral behaviours between ideal and viscous fluids

- ➔ Fluxes
  - Not constant, neither stationary; occur between multiple scales
  - Energy flows from middle scales to integral scales
  - Increasing enstrophy flux in inertial range
  - Outbreak of enstrophy cascade (?)

Perspectives

- ➔ Phenomenological description
  - Defining/Calculating spectral flux in ViC simulations
  - Calculation correlation and structure functions
  - Dispersion of passive scalar

- ➔ Tridimensional simulations

- ➔ The acting mechanism itself needs to be further elucidated:

Toward analytic theory of
Kelvin-Helmholtz instability bidimensional turbulence

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To the examining committee and the audience!

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