PROJECTS 1 MATH 226-03, FALL 2021

RAUL PENAGUIAO

San Francisco State University

Rules and requirements

- Your project group should have between three and five people.
- 6 sections: Introduction, Development, Examples, Conclusion, References, Pictures.
- You should hand in a report, between 4 and 7 pages without references and pictures.
- **The projects have to be mathematical in nature**, so you should present a solution to one or several specific problems, that should be stated in the beginning of the report. A little bit of historical context and related problems adds value.
- These are just guidelines that help you start out with a writeout about the problems, and to give some structure for the grader to have a homogeneous work. If you see a good reason to deviate from these (for instance you want to study a particular example in depth in its own section) do so.
- Agree with your group mates on a timeline for the project ahead of time.
- Due until Thursday 14th of October, at 5pm.

Projects

The exponential function.

$$\exp(x)$$

Banking Analyse the function that assigns the value of a bank account that is acruing interest over time. You are to assume that this interest rate is constant. Study the following two cases: capitalization of interest at fixed intervals over time, and continuous capitalization of interest.

Resources: `https://en.wikipedia.org/wiki/Compound_interest`

E-mail address: `raulpenaguiao@sfsu.edu`.

Date: September 24, 2021.

Radioactive The presence of a radioactive in a given mass is constantly changing, and a common model assumes that each isotope decays in a given interval of time with the same independent probability. This leads to the notion of half-life, and a formula for the radioactive mass.

Resources: `https://courses.lumenlearning.com/waymakercollegealgebra/chapter/exponential-growth-and-decay/`

Geometry optimization.

$\circ$ $\square$

Isoperimetric Given a family of geometric figures with a fixed perimeter, which one maximizes the area? You are to specifically study among squares, rectangles, diamonds, triangles, pentagons and other regular polygons, which ones are the ones that maximize the area with a fixed perimeter. Can you come up with other more interesting shapes?

Resources: `https://math.stackexchange.com/questions/2293547/finding-maximum-area-of-a-pentagon`; `https://math.stackexchange.com/questions/128825/maximum-area-of-rectangle-with-fixed-perimeter`;

Three-dimentional packing You have a 100ft by 60ft bubble wrapper. What is the box with the biggest volume that you can enclose? Try out some particular shapes, like paralelipipedic boxes, or prismic boxes where the base is a pentagon or other regular polygons. Can you conjecture what is the shape of the *best* box?

Resources: none, but you can try to find some

Inscribed rectangle Find the largest rectangle that can fit inside the ellipse that satisfies the equations $\frac{1}{2}x^2 + y^2 = 1$.

Resources: none, but you can try to find some

Price optimization.

$$$

Price fixing You are selling an article that costs \$50 to produce, and you are paying $\$15,000$ in fixed monthly expenses. After a thourough study of the market, your analysts came up with the fact that for a given price p of the article, you will be able to sell $N(p) = \exp(-0.05p) \times 150$

Try to alter the model a bit, and see what other optimization problems can you solve.

Resources: Example 2 in `http://www2.gcc.edu/dept/math/faculty/BancroftED/buscalc/chapter2/section2-9.php`

Physics.  

Volcanic lava fountains Although the November 1959 Kilauea Iki eruption on the island of Hawaii began with a line of fountains along the wall of the crater, activity was later confined to a single vent in the crater's floor, which at one point shot lava 1900 ft straight into the air (a Hawaiian record). What was the lava's exit velocity in feet per second?

Resources: Early Transcendentals (source); https://openstax.org/books/university-physics-volume-1/pages/4-3-projectile-motion

The catenary This is the curve drawn by a hanging rope at rest between two poles. We can use calculus to find the equation that describes this curve.

Resources: https://en.wikipedia.org/wiki/Catenary

Mathematics.

$$1 + 1 = 2$$

Limits We keep using the limits $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ and $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$, but can you find why this is the case? Establish these facts from first principles. What is the relevance of these limits when computing the derivative of $\exp(x)$ and $\sin(x)$?

Resources:

Numerical Methods.

$$a_n = 1 + \frac{1}{n}$$

The Newton method Given a function f, we can try to compute the zeroes explicitly. However, sometimes that is not possible, and numerical methods help us to obtain approximations. With the access to derivatives, we are able to use Newton's method, a very fast and reliable method, that can be creatively explained in a few pictures. In this project you can apply the Newton method to a few functions and initial values, you can write a programming code that computes the Newton method and you can also compare for which initial values will the method converge.

Resources: `https://en.wikipedia.org/wiki/Newton's_method`; `https://openstax.org/books/calculus-volume-1/pages/4-9-newtons-method`