The discrete signature Veronese variety

Workshop on Probabilistic methods, Signatures, Cubature and Geometry

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Slides can be found at `raulpenaguiao.github.io/`
Joint work with Carlo Bellingeri.

Path signatures

Given a path $\mathbf{X} : [0,1] \to \mathbb{R}^d$, we can define its (continuous) signatures:

$$\sigma_{\omega_1 \dots \omega_k}(\mathbf{X}) = \int_{0<t_1<\dots<t_k<1} X'_{\omega_1}(t_1) \cdots X'_{\omega_k}(t_k) d\mathbf{t} \,,$$

defined for $\omega_i \in \{1, \dots, d\}$.
These satisfy the **shuffle relations**:

$$\sigma_\omega(\mathbf{X})\sigma_\tau(\mathbf{X}) = \sum_{\alpha \in \omega ш \tau} \sigma_\alpha(\mathbf{X}) \,.$$

Example: $\sigma_1(\mathbf{X})^2 = 2\sigma_{11}(\mathbf{X})$.

Discrete path signatures

Given a sequence of vectors $\mathbf{X} = (\mathbf{X}^0, \dots, \mathbf{X}^N) \in (\mathbb{R}^d)^{N+1}$, we can define its (discrete) signatures:

$$\Sigma_{p_1 \dots p_k}(\mathbf{X}) = \sum_{0<t_1<\cdots<t_k\leq N+1} p_1(\mathbf{X}^{t_1} - \mathbf{X}^{t_1-1}) \cdots p_k(\mathbf{X}^{t_k} - \mathbf{X}^{t_k-1}) \, ,$$

defined for p_i **non-constant monomials** in $\{x_1, \dots, x_d\}$.
These satisfy the **quasi-shuffle relations**:

$$\sigma_\omega(\mathbf{X})\sigma_\tau(\mathbf{X}) = \sum_{\alpha \in \omega \overline{\sqcup\!\sqcup} \tau} \sigma_\alpha(\mathbf{X}) \, .$$

Example: $\sigma_1(\mathbf{X})^2 = 2\sigma_{11}(\mathbf{X}) + \sigma_1(\mathbf{X})$.

1 Introduction

2 Universal varieties

3 Computation considerations

The universal discrete signature variety

The universal discrete signature variety

Some degree considerations

Biblio

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Thank you