California State Polytechnic University Pomona

Aerospace Engineering

ARO4080

FEM Documentation

for

Homework 24

by

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04 December 2018

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1.0 Introduction

1.1 Purpose

Purpose of this Report is to document applying a nonlinear gap to a structure with an applied thermal load in NX NASTRAN, comparing the output: deflection, axial loads and stress of each element and comparing it to a hand analysis using linear theory. And then making an observation of how the closed gap stiffness effects the output; deflection, axial load, and stress of each element.

2.0 F.E.M. Documentation

2.1 Sketch of Model

Finite Element Model (FEM) is shown below, with loading and constraints shown below

Note: Node elements numbers are in squares, □

Elements are number in a circle, ①

Constraints are in parentheses, (1,2,3,4,5,6)

Thermal load is ΔT, while T_i is the initial temperature of the system.

Nodes 1 and 2 are coincident (see GRID 1 and GRID 2 cards in Section 2.2)

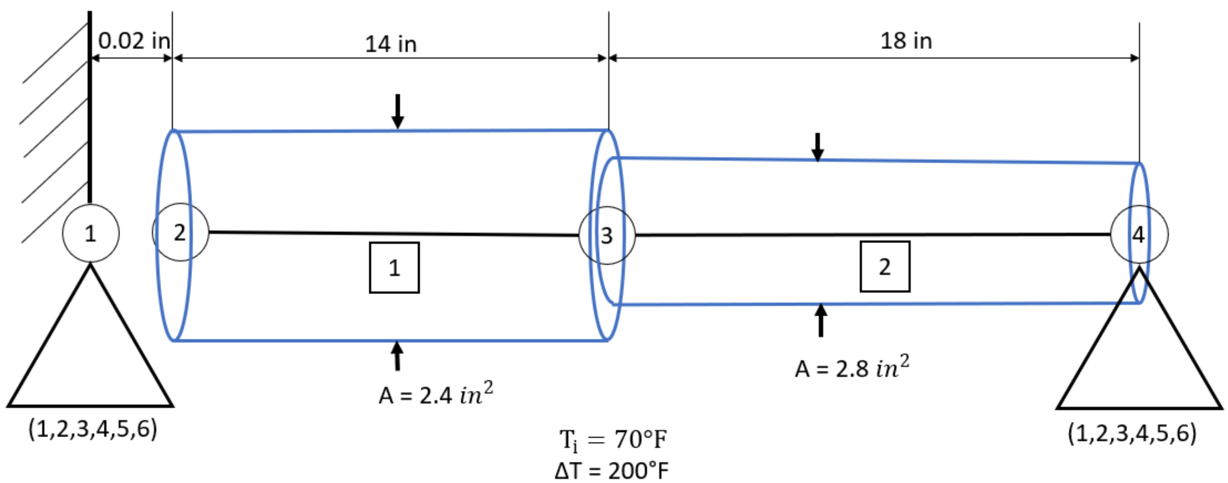

Figure 2.1.1: FEM Sketch

2.2 Bulk Data File

The FEM bulk data file is shown below

```
$-------------------------------------------------------
$ Prepared by: Raul Perez
$ Date: November 30, 2018
$ Revised:
$ Title: Homework 24 Thermal Gaps
$-------------------------------------------------------
SOL 106
TIME 5
CEND           $ Designates end of Executive Control
ECHO = SORT
TITLE = ARO4080: Thermal Gaps e=10^12
SUBTITLE = BY RAUL PEREZ, 11/30/18
SUBCASE 1
        SUBTITLE = Thermal Gaps
            TEMPERATURE(INITIAL) = 3
        TEMPERATURE(LOAD) = 7
        SPC = 1
        LOAD = 1
        DISPLACEMENT = ALL
        SPCFORCES = ALL
        STRESS = ALL
        FORCE = ALL
        GPFORCE = ALL
        NLPARM = 10
BEGIN BULK      $ Designates end of Case Control
NLPARM,10,20
PARAM,LGDISP,1

GRID, 1,,0.,0.,0.
GRID, 2,,0.,0.,0.
GRID, 3,,14.,0.,0.
GRID, 4,,32.,0.,0.

CROD, 1,1,2,3 $CROD, id, pid, grid 1, grid 2  %Bronze
CROD, 2,2,3,4 $CROD, id, pid, grid 1, grid 2  %Al

PROD, 1,1,2.4,,,.1 $PROD, pid, mid, A, J, C, NSM  %Bronze
PROD, 2,2,2.8,,,.1 $PROD, pid, mid, A, J, C, NSM  %Al

MAT1, 1,15.e6,,0.33,,12.0e-6     $ID,E,G,nu,rho,aplha %Bronze
MAT1, 2,10.6e6,,0.33,,12.9e-6    $ID,E,G,nu,rho,aplha %Al

CGAP,20,21,1,2,1.,0.,0.,0 $Ele_ID,PGAP_ID,G1,G2,G0,X1,X2,X3,CID
PGAP,21,0.02,,1.e12 $Prop_ID,Intial_Gap,Preload,Closed_Stifness,Open_Stiffness,

TEMPD,3,70. $ Initial T
TEMPD,7,270. $ delta T

SPC,1,1,123456
SPC,1,4,123456  $ID,Gridpt,constraints

ENDDATA $Designates end of the Bulk Data section
```

2.3 Material Properties

Each rod is made of a different material, the first rod element is made of bronze, with E = 15 Msi, and α = $12*10^{-6}$ 1/°F, thus the MAT1 card for the first rod element is entered as,

```
MAT1, 1,15.e6,,0.33,,12.0e-6     $ID,E,G,nu,rho,aplha %Bronze
```

The second rod element made of aluminum, with E = 10.6 Msi and α = $12.9*10^{-6}$ 1/°F, thus the MAT1 card for the second rod element is entered as,

```
MAT1, 2,10.6e6,,0.33,,12.9e-6      $ID,E,G,nu,rho,aplha %Al
```

Note: Poisson's ratio of 0.33 was entered for both MAT1 cards to avoid a warning, won't affect the output.

Property cards of each rod element is different since each is made of different material and each has a different cross-sectional area. For the first rod element being made of bronze and having a cross-section area of 2.4 in^2, thus the PROD card for first rod element is

```
PROD, 1,1,2.4,,,.1 $PROD, pid, mid, A, J, C, NSM  %Bronze
```

For the second rod element being made of aluminum and having a cross-sectional area of 2.8 in^2, thus the PROD card for second rod element is

```
PROD, 2,2,2.8,,,.1 $PROD, pid, mid, A, J, C, NSM  %Al
```

Note: NSM value of 0.01 was applied to both PROD cards to also avoid an output warning

The gap between the rod element 1 and the wall (See Figure 2.1.1) is modeled using a gap element using the CGAP card, with the gap being connected to coincident nodes 1 and 2 and since the nodes are coincident, the coordinate system has to be defined, thus the default coordinate system 0 was selected. With the displacement vector pointing in the positive x direction, thus the CGAP card is inputted as,

```
CGAP,20,21,1,2,1.,0.,0.,0 $Ele_ID,PGAP_ID,G1,G2,G0,X1,X2,X3,CID
```

The CGAP card calls the PGAP card (the CGAP property card), the gap has a length of 0.02 in and for the stiffness when the rod makes contact with the wall is 10^{12} lb$_f$/in, thus the PGAP card is inputted as

```
PGAP,21,0.02,,1.e12 $Prop_ID,Intial_Gap,Preload,Closed_Stifness,Open_Stiffness,
```

2.4 Applied Loads

A thermal load (temperature change) of 200 °F is applied to the whole structure from an initial temperature at 70 °F, thus the TEMPD used to specify the initial temperature and thermal load as shown below

```
TEMPD,3,70. $ Initial T
TEMPD,7,270. $ delta T + Initial T
```

Note: each is specified by the TEMPERATURE(INITIAL), TEMPERATURE(LOAD) card in the Case Control Deck (see Section 2.2)

2.5 Constraints

NASTRAN requires constraints be define for all nodes. Node 1 and 4 are fixed, while Nodes 2 and 3 are able to freely move, thus the SPC cards inputted as,

```
SPC,1,1,123456
SPC,1,4,123456  $ID,Gridpt,constraints
```

3.0 FEM Results

3.1 Nodal Displacements

Nodal displacements are shown below, units are in inches.

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	0.0	0.0	0.0	0.0	0.0	0.0
2	G	-2.000006E-02	0.0	0.0	0.0	0.0	0.0
3	G	-9.857819E-03	0.0	0.0	0.0	0.0	0.0
4	G	0.0	0.0	0.0	0.0	0.0	0.0

1 ARO4080: THERMAL GAPS E=10^12 DECEMBER 1, 2018 NX NASTRAN 5/23/18
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BY RAUL PEREZ, 11/30/18

3.2 SPC Forces

SPC forces are shown below, units are in pounds force

F O R C E S   O F   S I N G L E - P O I N T   C O N S T R A I N T

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	6.031995E+04	0.0	0.0	0.0	0.0	0.0
4	G	-6.031995E+04	0.0	0.0	0.0	0.0	0.0

1 ARO4080: THERMAL GAPS E=10^12 DECEMBER 1, 2018 NX NASTRAN 5/23/18
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THERMAL GAPS

3.3 Grid Point Force Balance

The grid point force balances are shown below (sums at each node is zero or approximately zero), units are in pounds force

G R I D   P O I N T   F O R C E   B A L A N C E

	POINT-ID	ELEMENT-ID	SOURCE	T1	T2	T3	R1	R2	R3
	1		F-OF-SPC	6.031995E+04	0.0	0.0	0.0	0.0	0.0
	1	20	GAP	-6.031995E+04	0.0	0.0	0.0	0.0	0.0
	1		*TOTALS*	0.0	0.0	0.0	0.0	0.0	0.0
0	2	20	GAP	6.031995E+04	0.0	0.0	0.0	0.0	0.0
	2	1	ROD	-6.031995E+04	0.0	0.0	0.0	0.0	0.0
	2		*TOTALS*	-1.192158E-06	0.0	0.0	0.0	0.0	0.0
0	3	1	ROD	6.031995E+04	0.0	0.0	0.0	0.0	0.0
	3	2	ROD	-6.031995E+04	0.0	0.0	0.0	0.0	0.0
	3		*TOTALS*	7.756171E-09	0.0	0.0	0.0	0.0	0.0
0	4		F-OF-SPC	-6.031995E+04	0.0	0.0	0.0	0.0	0.0
	4	2	ROD	6.031995E+04	0.0	0.0	0.0	0.0	0.0
	4		*TOTALS*	0.0	0.0	0.0	0.0	0.0	0.0

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3.4 Rod Element Forces & Stresses

Rod element forces & stresses are shown below

```
                              F O R C E S   I N   R O D   E L E M E N T S     ( C R O D )
       ELEMENT          AXIAL                                        ELEMENT          AXIAL
         ID.            FORCE          TORQUE                          ID.            FORCE          TORQUE
             1      -6.031995E+04      0.0                              2          -6.031995E+04      0.0
1    ARO4080: THERMAL GAPS E=10^12                                          DECEMBER   1, 2018  NX NASTRAN  5/23/18   PAGE    38
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                             S T R E S S E S   I N   R O D   E L E M E N T S      ( C R O D )
       ELEMENT       AXIAL       SAFETY      TORSIONAL     SAFETY       ELEMENT       AXIAL       SAFETY      TORSIONAL     SAFETY
         ID.        STRESS       MARGIN        STRESS      MARGIN         ID.        STRESS       MARGIN        STRESS      MARGIN
             1   -2.513331E+04                 0.0                        2       -2.154284E+04                 0.0
1    ARO4080: THERMAL GAPS E=10^12                                          DECEMBER   1, 2018  NX NASTRAN  5/23/18   PAGE    40
      BY RAUL PEREZ, 11/30/18
```

4.0 Evaluation

4.1 Evaluation of FE Results to Hand Analysis

Table 4.1.1, shows the results of displacements, axial load and stress of both rod elements, and how changing the gap stiffness by two magnitudes each time effects the output. Comparing the hand analysis to the output by NX NASTRAN at gap stiffness equal to 10^{12} lb_f/in, the answers are equal to each other that the difference is insignificant. Having a large stiffness six magnitudes greater than that of the stiffness of the rods (10^{12} lb_f/in vs $2.571*10^6$ lb_f/in of the bronze rod), makes the gap when closed, now a wall seem to a have a stiffness of infinity, this is assumed in linear theory, the wall won't be deformed and stops the bronze bar left side from deforming anymore.

	u_2 (in)	u_3 (in)	Axial Force (lb_f)	Stress of Brass rod (psi)	Stress of Aluminum rod (psi)
Hand Analysis	-0.02	-0.00985	-60,332.7	-25,133	-21,542
NASTRAN Gap Stiffness (lb/in)					
1.0E+12	-0.02000006	-0.00985782	-60,319.950	-25,133.310	-21,542.840
1.0E+10	-0.02000603	-0.00986146	-60,313.950	-25,130.810	-21,540.700
1.0E+08	-0.02059720	-0.01022166	-59,720.030	-24,883.340	-21,328.580
1.0E+06	-0.05008984	-0.02819144	-30,089.840	-12,537.430	-10,746.370
1.0E+04	-0.07944828	-0.04607946	-594.483	-247.701	-212.315

Table 4.1.1, Comparison of Hand Analysis to NX NASTRAN (at gap stiffness = 10^{12}) and how Gap Stiffness Effects the results

Now changing the gap stiffness by two magnitudes down each time, from 10^{12} to 10^4, we see from Figure 4.1.1, 4.1.2 and 4.1.3, that from 10^{12} to 10^8 the deflections, axial force and stress vary little, at 10^{12} to 10^{10}, virtually no difference, while 10^{12} and 10^8 starting to see a difference looking at Table 4.1.1 closer. This is because the gap stiffness is still much greater than any of the elements stiffnesses (gap stiffness: > 10^8 vs element stiffness: ~10^6).

From 10^6 and below, the gap stiffness is now equal or lesser than the rod elements stiffness, thus when the gap is closed, the now wall will now deflect as much or more than the rods, hence the higher deflections, and because of the higher deflections the axial force and stress of each rod is less as there is less compression. The same is true for gap stiffness of 10^4 where know the rod stiffness are two magnitudes higher than the gap stiffness, and because of this, axial force and stress have dropped by two magnitudes from a gap stiffness of 10^6. Thus gap stiffness should be four magnitudes higher than the highest element stiffness to get a good imitation of a node being fixed in the x direction.

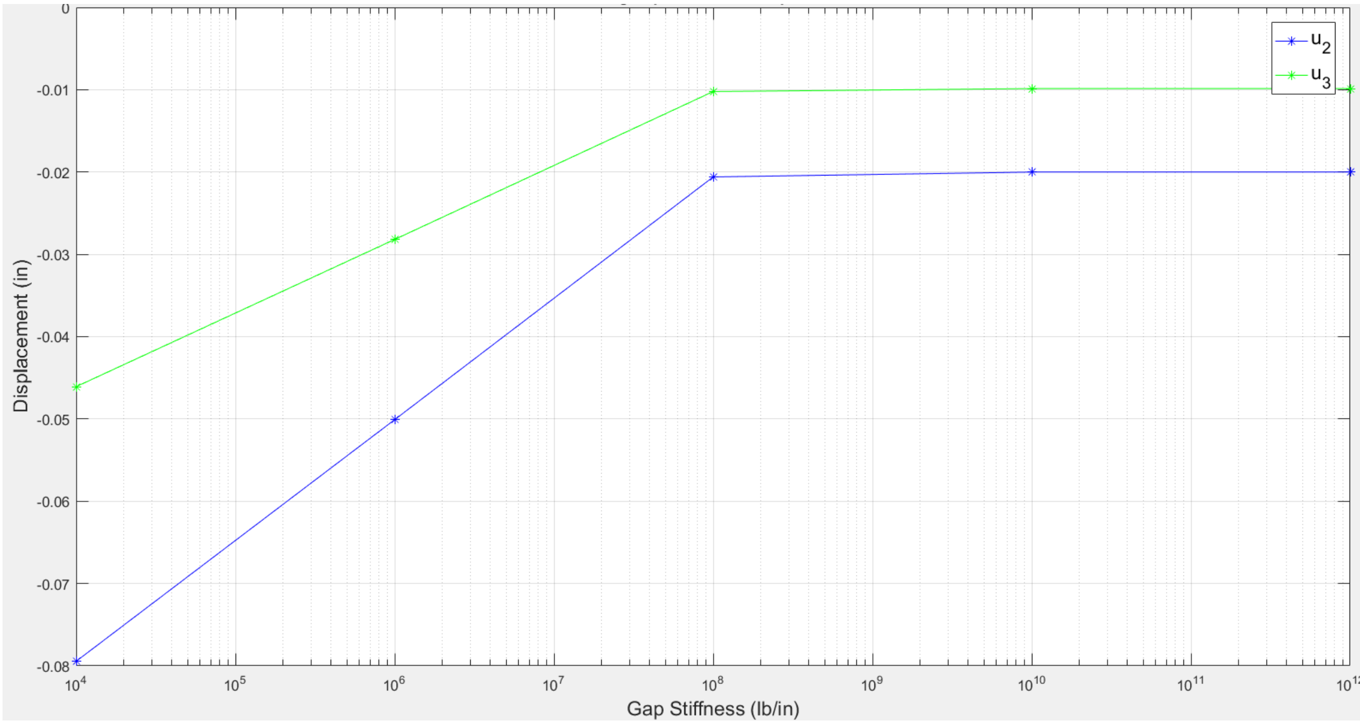


Figure 4.1.1 SemiLog Gap Stiffness vs Displacement

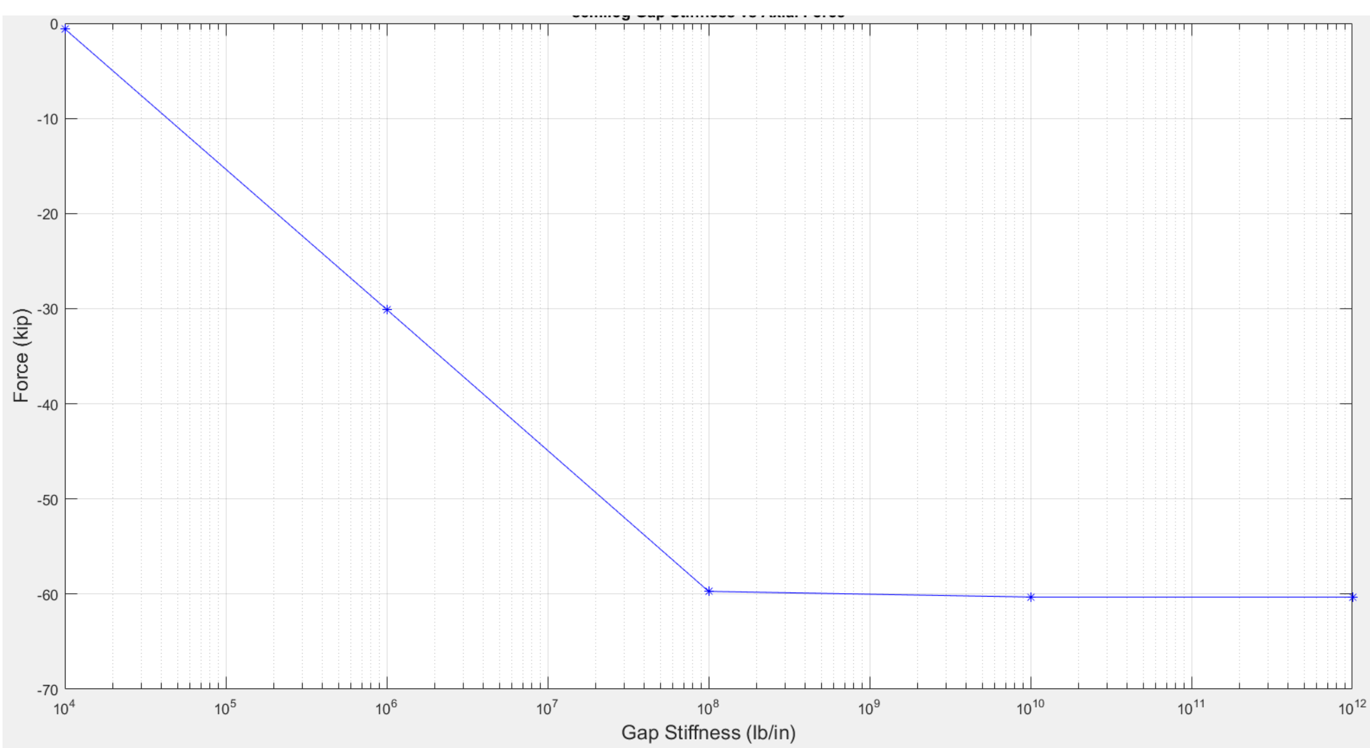


Figure 4.1.2 SemiLog Gap Stiffness vs Axial Force

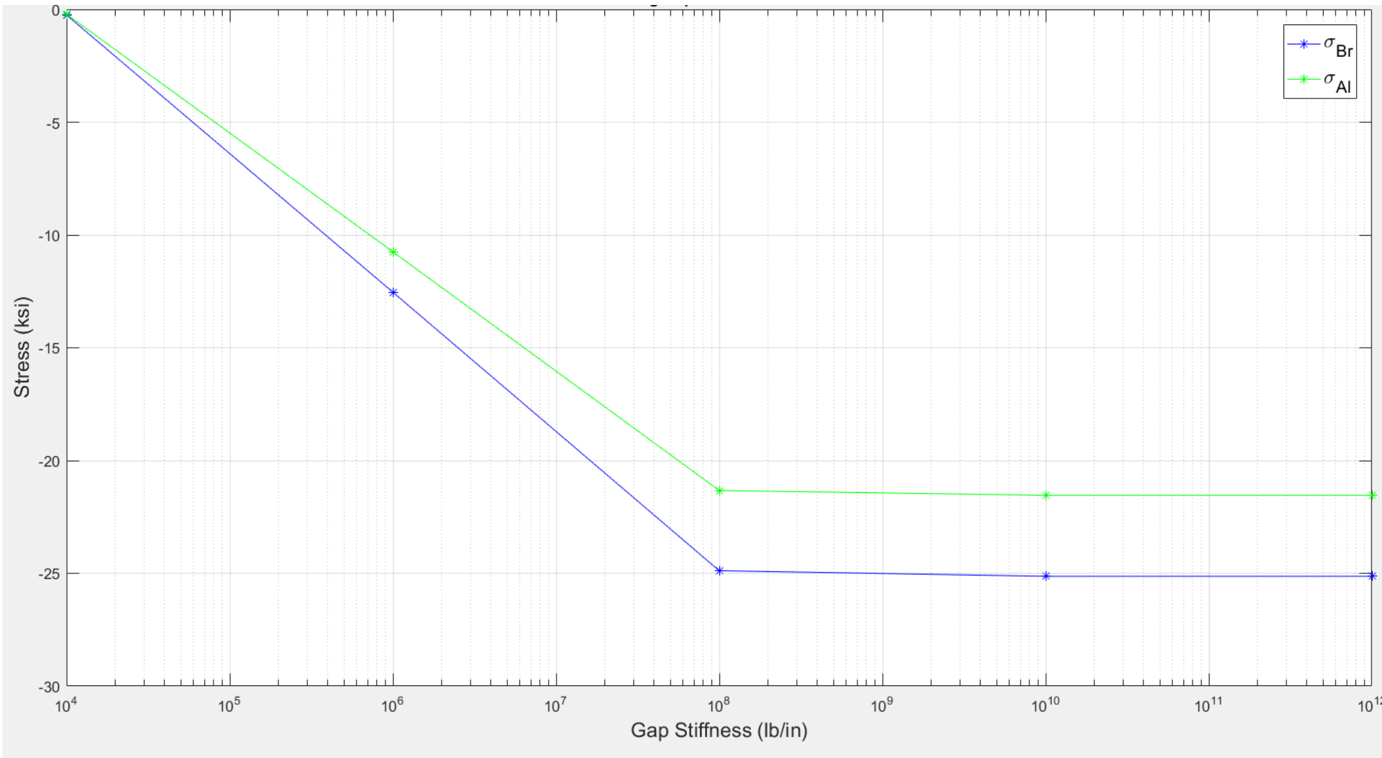
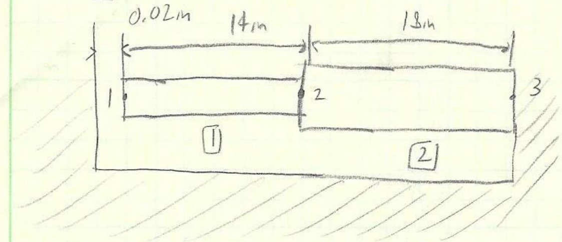


Figure 4.1.3 SemiLog Gap Stiffness vs Stress

A FEM Lumping, Property & Material Hand Calculations

pg. 1

Given



Ele 1; Bronze , $E = 15\cdot10^6$ psi , $\alpha = 12\cdot10^{-6}\frac{1}{F}$
$A = 2.4 in^2$

$T_i = 70°F$
$\Delta T = 200°F$

Ele 2; Aluminum , $E = 10.6\cdot10^6$ psi , $\alpha = 12.9\cdot10^{-6}\frac{1}{F}$
$A = 2.8 in^2$

Find

a) Axial load on the rods
b) Normal stress of each rod
c) deformation of Bronze portion of the bar.

Solution

$$f_{o_1} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} -E_{Br}\,\alpha_{Br}\,\Delta T\,A_{Br} \\ E_{Br}\,\alpha_{Br}\,\Delta T\,A_{Br} \end{Bmatrix} = \begin{Bmatrix} -15\cdot10^6\frac{lbf}{in^2}\cdot12\cdot10^{-6}\frac{1}{F}\cdot200F\cdot2.4\,in^2 \\ 15\cdot10^6\frac{lbf}{in^2}\cdot12\cdot10^{-6}\frac{1}{F}\cdot200F\cdot2.4\,in^2 \end{Bmatrix}$$

$$= \begin{Bmatrix} -86,400 \\ 86,400 \end{Bmatrix} (lbf)$$

$$f_{o_2} = \begin{Bmatrix} f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix} = E_{Al}\,\alpha_{Al}\,\Delta T A_{Al}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = 10.6\cdot10^6\frac{lbf}{in^2}\cdot12.9\cdot10^{-6}\frac{1}{F}\cdot200F\cdot2.8\,in^2\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \begin{Bmatrix} -76,574.4 \\ 76,574.4 \end{Bmatrix} (lbf)$$

$$F_o = f_{o_1} + f_{o_2} = \begin{Bmatrix} -86,400 \\ 86400 + -76,574.4 \\ 0 + 76,574.4 \end{Bmatrix} = \begin{bmatrix} -86,400 \\ 9,825.6 \\ 76,574.4 \end{bmatrix} (lbf)$$

Pg.2

$$k_1' = \frac{E_{Br}A_{Br}}{L_{Br}}\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}, \quad \frac{E_{Br}A_{Br}}{L_{Br}} = \frac{15\cdot 10^6 \frac{lbf}{in^2}\cdot 2.4\,in^2}{14\,in} = 2.571\cdot 10^6\,lbf/in$$

$$= 2.571\cdot 10^6 \frac{lbf}{in}\begin{array}{c} \begin{array}{cc} u_1 & u_2 \end{array} \\ \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \end{array}$$

$$k_2' = \frac{E_{Al}A_{Al}}{L_{Al}}\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}, \quad \frac{E_{Al}A_{Al}}{L_{Al}} = \frac{10.6\cdot 10^6 \frac{lbf}{in^2}\cdot 2.8\,in^2}{18\,in} = 1.6489\cdot 10^6\,lbf/in$$

$$= 1.6489\cdot 10^6 \frac{lbf}{in}\begin{array}{c} \begin{array}{cc} u_2 & u_3 \end{array} \\ \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \end{array}$$

$$K_{all} = k_1' + k_2' = 10^6 \frac{lbf}{in}\begin{array}{c} \begin{array}{ccc} u_1 & u_2 & u_3 \end{array} \\ \begin{vmatrix} 2.5714 & -2.5714 & 0 \\ -2.5714 & 4.2203 & -1.6489 \\ 0 & -1.6489 & 1.6489 \end{vmatrix} \end{array}$$

assume no contact
$u_1 < 0.02$

$$10^6 \frac{lbf}{in}\begin{array}{c} \begin{array}{ccc} u_1 & u_2 & u_3 \end{array} \\ \begin{vmatrix} 2.5714 & -2.5714 & 0 \\ -2.5714 & 4.2203 & -1.6489 \\ \cancel{0} & \cancel{-1.6489} & \cancel{1.6489} \end{vmatrix} \end{array} \begin{vmatrix} u_1 \\ u_2 \\ \cancel{u_3} \end{vmatrix} = \begin{vmatrix} -86,400 \\ 9,825.6 \\ \cancel{76,574.4} \end{vmatrix} (lbf)$$

$$10^6 \frac{lbf}{in}\begin{vmatrix} 2.5714 & -2.5714 \\ -2.5714 & 4.2203 \end{vmatrix}\begin{vmatrix} u_1 \\ u_2 \end{vmatrix} = \begin{vmatrix} -86,400 \\ 9,825.6 \end{vmatrix} (lbf)$$

$$\begin{vmatrix} u_1 \\ u_2 \end{vmatrix} = \begin{vmatrix} -0.08 \\ -0.0464 \end{vmatrix} (in), \quad u_1 < -0.02$$, thus contacting the wall

thus move u_1 ele to the right side & $u_1 = -0.02\,in$

$$10^6 \frac{lbf}{in}\begin{vmatrix} 4.2203 \end{vmatrix}\begin{vmatrix} u_2 \end{vmatrix} = \begin{vmatrix} \underbrace{9,825.6\,lbf + 2.5714\cdot 10^6 \frac{lbf}{in}\cdot -0.02\,in}_{= -41,602.4\,lbf} \end{vmatrix}$$

$$u_2 = \frac{-41,602.4\,lbf}{4.2203\cdot 10^6 \frac{lbf}{in}} = -0.00985\,in$$

$$\boxed{u_2 = -0.00985\,in}$$

pg.3

$$D = \begin{vmatrix} -0.02\,in \\ -0.00985\,in \\ 0\,in \end{vmatrix} \qquad F'_{(e)} = K_{all} \cdot D = \begin{vmatrix} -26{,}080 \\ -41{,}602.4 \\ 16{,}254 \end{vmatrix} (lbf)$$

$$F = F_e - F_0 = \begin{vmatrix} -26{,}080 \\ 9{,}826 \\ 16{,}254 \end{vmatrix} (lbf) - \begin{vmatrix} -86{,}400 \\ -9{,}826 \\ 76{,}574.4 \end{vmatrix} (lbf) = \begin{vmatrix} 60{,}320.0 \\ 0 \\ -60{,}320.4 \end{vmatrix} (lbf)$$

$$f_{Bv} = f_1 = k'_1 d'_1 - f_{10}$$

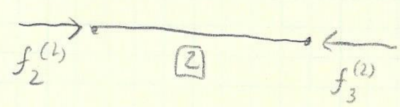
$$= 2.571 \cdot 10^6 \frac{lbf}{in} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \begin{vmatrix} -0.02\,in \\ -0.00985\,in \end{vmatrix} - \begin{vmatrix} -86{,}400 \\ 86{,}400 \end{vmatrix} (lbf)$$

$$\begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} 60{,}304.35\,lbf \\ -60{,}304.35\,lbf \end{Bmatrix}$$

$$f_{A1} = f_2 = k'_2 d'_2 - f_{2_0}$$

$$= 1.6489 \cdot 10^6 \frac{lbf}{in} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \begin{vmatrix} -0.00985\,in \\ 0\,in \end{vmatrix} - \begin{Bmatrix} -76{,}574.4 \\ 76{,}574.4 \end{Bmatrix}$$

$$= \begin{Bmatrix} f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix} = \begin{Bmatrix} 60{,}332.7 \\ -60{,}322.7 \end{Bmatrix} (lbf)$$



thus

$$\boxed{\begin{matrix} f_1 = f_2 = -60.3\ ksi \\ f_{Bv} = f_{A1} \end{matrix}}$$

$$\sigma_{Bv} = \frac{f_{Bv}}{A_{Bv}} = \frac{-60{,}320\,lbf}{2.4\,in^2} = -25{,}133\,psi, \quad \boxed{\sigma_{Bv} = -25.1\ ksi}$$

$$\sigma_{A1} = \frac{f_{A1}}{A_{A1}} = \frac{-60{,}320\,lbf}{2.8\,in^2} = -21{,}542\,psi, \quad \boxed{\sigma_{A1} = -21.5\ ksi}$$

B Hand Analysis Support

I think that has become section A, I don't think Section A and Section B were ever both used in the same report.

References

[DLe5] Daryl Logan. A First Course in the Finite Element Method. 5th Edition. Cengage Learning. 2012.