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ARO 4360

Project 7

04/09/18

California State Polytechnic University Pomona

ARO4360 – COMPOSITES HW #09

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Read the sections of your texts noted in the Syllabus. Solve the following, showing all work. Box Answers. Score your HW as directed in Syllabus.

A laminate constructed of 0.01" thick plies of unidirectional Kevlar Epoxy (Ke/Ep) oriented at [0/±30/0] experiences an external loading of N_x= 200 lb/in & M_x=20 in-lb/in. Determine the following (You do not need to restate GIVEN, FIND, SOLUTION for each problem):

[1] Determine the ABBD Matrix for the laminate.

[2] Determine the strains at the midplane of the laminate in body coordinates.

[3] Determine the principal stresses at the midplane of ply 1.

[4] Determine the margins of safety in ply 1 using the max stress criteria & state whether you expect this ply to fail as a result of this.

[5] Determine the principal stresses at the midplane of ply 4.

[6] Determine the margins of safety in ply 4 using the max stress criteria & state whether you expect this ply to fail as a result of this.

[7] Determine the stresses at the top of ply 1 in body coordinates.

[8] Determine the stresses at the bottom of ply 1 in body coordinates.

ANSWERS (40 Points Possible)

1. A_{11}= 354 kip/in, A_{12}= 49.0 kip/in, B_{13}= -333 lb, D_{11}= 56 in-lb, etc…
2. ε_x= 704 μin/in, ε_y= -614 μin/in, γ_{xy}= 0.00213 in/in, κ_x= 0.377 in^{-1}, κ_y= -0.1661 in^{-1}, κ_{xy}= 0.0574 in^{-1}.
3. σ_1= -54.4 ksi, σ_2= 159 psi, τ_{12}= 380 psi.
4. MS_1 = 0.545, MS_2 = 30.5, MS_{12} = 15.84, No.
5. σ_1= 69.6 ksi, σ_2= -763 psi, τ_{12}= 897 psi.
6. MS_1 = 1.872, MS_2 = 25.2, MS_{12} = 6.14, No.
7. σ_x= -75.0 ksi, σ_y= 312 psi, τ_{xy}= 294 psi.
8. σ_x= -33.7 ksi, σ_y= 5 psi, τ_{xy}= 466 psi.

Source Code

```
%Use homework 9, problems 1-8
%Its the same thing as Project 6, expect without Hw#8 and Hw%10

%Input type, 0 for strains input,1 for Running loads,2 for hygro-thermal
loads
fprintf('Hw#9\n')
%Hw#9: 1 to 8

%Inputs
%Material: Kevlar/Ep
E1 = 11.0*10^6;%psi
E2 = 0.8*10^6;%psi
G12 = 0.3*10^6;%psi
v12 = 0.34;
    Mat_prop = [E1,E2,G12,v12];

%ultimate strengths psi
Xt = 200*10^3; Xc = 84*10^3; Yt = 5*10^3; Yc = 20*10^3; S = 6.4*10^3;
    Mat_strength = [Xt,Xc,Yt,Yc,S];
%Layers
    %Each layer has an angle, thus the number of angles in Angle array
    %implies the number of layers
Angles = [0,30,-30,0];%degrees;
layer_thickness = 0.01*ones(1,length(Angles)); %in, all layers have the same
thickness

%Loads or strains
%N units = lbf/in
Nx = 200; Ny = 0; Nxy = 0;
%M units = (in*lbf)/in
Mx = 20; My = 0; Mxy = 0;
Loads = [Nx,Ny,Nxy,Mx,My,Mxy]';
type = 1; %Input type: 0 for strains input,1 for Running loads,2 for hygro-
thermal loads
%ply and loc vary for problems 3 to 8
%Prob 3 and 4) ply = 1, loc = 0
%Prob 5 and 6) ply = 4, loc = 0
%Prob 7) ply = 1, loc = 1
%Prob 8) ply = 1, loc = -1

%List of plies want to get strains from
ply = [1,4,1,1]; %number of ply, 1 being the topmost ply and nth ply being
the bottom ply
loc = [0,0,1,-1];%location of where the strain is being measured: 1 at the
top of the ply
% 0 at the middle of the ply and -1 at the bottom of the
% ply
ply_list = [ply;loc];
ThermalProp = [];%None for Hw#9
ARO4360_P7_1_Perez_Raul(Mat_prop,Mat_strength,Angles,layer_thickness,Loads,ty
pe,ply_list,ThermalProp);
```

```matlab
function[] =
ARO4360_P7_1_Perez_Raul(Mat_prop,Mat_strength,Angles,layer_thickness,col_in,t
ype,ply_list,ThermalProp);
%{
    Determine the stresses and strains of a ply in body and principal
    directions
%}
%Get material propties form Mat_prop array
E1 = Mat_prop(1); E2 = Mat_prop(2); G12 = Mat_prop(3); v12 = Mat_prop(4);

layers = length(Angles); %Used an end counter
v21 = v12/E1*E2;
dem = 1-v12*v21;
Q11 = E1/dem; Q12 = v12*E2/dem;
Q22 = E2/dem; Q66 = G12;
Q = [Q11, Q12, 0;
     Q12, Q22, 0;
     0,   0,   Q66]; %psi
R = [1,0,0;
     0,1,0;
     0,0,2];
%Setting up for ABBD Matrix
Q_bars = zeros(3,3,layers);
Th = Q_bars; %Transformation Matrices
z = zeros(layers,1);
z_bar = zeros(layers,1);
A = zeros(3,3); B = A; D = A;
total_thickness = sum(layer_thickness);%in
for i=1:layers
    th = Angles(i);
    Th(:,:,i) = [(cosd(th))^2, (sind(th))^2, 2*sind(th)*cosd(th);
     (sind(th))^2, (cosd(th))^2, -2*sind(th)*cosd(th);
     -sind(th)*cosd(th), sind(th)*cosd(th), (cosd(th))^2 - (sind(th))^2];
    z(i) = -total_thickness/2 + sum(layer_thickness(1:i));
    z_bar(i) = z(i) - layer_thickness(i)/2;
    Q_bars(:,:,i) = ((Th(:,:,i))^-1)*Q*R*Th(:,:,i)*(R^-1);
    A = A + Q_bars(:,:,i)*layer_thickness(i);
    B = B + Q_bars(:,:,i)*layer_thickness(i)*z_bar(i);
    D = D + Q_bars(:,:,i)*(layer_thickness(i)*(z_bar(i))^2 +
1/12*(layer_thickness(i))^3);
end
% Display_Matrix('A',A,5,1,10^-3,'kip/in') %Used to check if outputting
correctly
% Display_Matrix('B',B,5,0,1,'(in*lbf)/in')
% Display_Matrix('D',D,5,1,1,'(in*lbf)')
ABBD = [A,B;B,D];
Display_Matrix('ABBD',ABBD,8,1,1,'')

%Finds running loads or strains and curvatures
    if type == 0 %if col_in = strains

        NandM = ABBD*col_in; %Get Running loads and Monements
        N_units = 'lbf/in';
        M_units = '(in*lbf)/in';
        fprintf('Nx = %3.0f %s\nNy = %3.0f %s\nNxy = %3.0f %s\n',
NandM(1),N_units,NandM(2),N_units,NandM(3),N_units)
```

```
        fprintf('Mx = %3.2f %s\nMy = %3.2f %s\nMxy = %3.2f %s\n',
NandM(4),M_units,NandM(5),M_units,NandM(6),M_units)
%-------------------------------------------------------------------------
    elseif type == 1 %if col_in = running loads
        ek0 = (ABBD^-1)*col_in; %Get midplane strains and curvatures
        %Display the midplane strain and curvature
        ek0(1:2) = 10^6*ek0(1:2);
        e_units12 = 'x10^-6 in/in';
        e_units3 = 'in/in';
        k_units = '(1/in)';
        fprintf('Mid-plane strains and curvatures \n')
        fprintf('ex_0 = %3.0f%s\ney_0 = %3.0f%s\ngxy_0 = %3.5f %s\n',
ek0(1),e_units12,ek0(2),e_units12,ek0(3),e_units3)
        fprintf('kx_0 = %3.3f %s\nky_0 = %3.3f %s\nkxy_0 = %3.4f %s\n',
ek0(4),k_units,ek0(5),k_units,ek0(6),k_units)
        fprintf('\n')
        ek0(1:2) = 10^-6*ek0(1:2);%nullify *10^6 from line 58
%Mat_prop,Mat_strength,Angles,layer_thickness,col_in,type,ply_list,ThermalPro
p);
        %Get Material Ultimate Strenghts
         Xt = Mat_strength(1); Xc = Mat_strength(2);
   Yt = Mat_strength(3); Yc = Mat_strength(4); S = Mat_strength(5);
        for i=1:length(ply_list)
            ply = ply_list(1,i); %row 1 of ply_list is the # of ply
            loc = ply_list(2,i);%row 2 is location of where to determine the
stresses of the plies

            if loc == 1 %top
                if ply == 1
                    loc_disp = 'top';
                    z_ply = -total_thickness/2;%in, special condtion
                else
                    z_ply = z(ply-1);
                end
            elseif loc == 0 %middle
                z_ply = z_bar(ply);
                loc_disp = 'middle';
            elseif loc == -1 %bottom
                z_ply = z(ply);
                loc_disp = 'bottom';
            else
                fprint('error in loc')
            end
            strains_ply =  ek0(1:3) + z_ply*ek0(4:6);%(in/in)
            stress_local = Q_bars(:,:,ply)*strains_ply;%psi
            stress_principal = Th(:,:,ply)*stress_local;
            units = 'psi';

            fprintf('Ply %2.0d at the %s = \n',ply,loc_disp)
            %Display global stresses
            fprintf('Global stresses: \n')
            fprintf('sigma_x = %3.0f %s\nsigma_y = %3.0f %s\ntau_xy = %3.0f
%s\n',...

stress_local(1),units,stress_local(2),units,stress_local(3),units)
            %Display Principal Stresses
            fprintf('Principal stresses: \n')
```

```
            fprintf('sigma1 = %3.0f %s\nsigma2 = %3.0f %s\ntau12 = %3.0f
%s\n',...

stress_principal(1),units,stress_principal(2),units,stress_principal(3),units
)
         %Margin of Satefty using Max Stress Criterion
         if stress_principal(1) < 0
             MS1 = Xc/abs(stress_principal(1)) - 1;
         else
             MS1 = Xt/stress_principal(1) - 1;
         end
         if stress_principal(2) < 0
             MS2 = Yc/abs(stress_principal(2)) - 1;
         else
             MS2 = Yt/stress_principal(2) - 1;
         end
         MS12 = S/stress_principal(3) - 1;
         fprintf('Margin of Safety\n')
         fprintf('MS1 = %3.3f\nMS2 = %3.3f\nMS12 = %3.3f\n', MS1,MS2,MS12)
         fprintf('\n')
        end
%--------------------------------------------------------------------------
    elseif type == 2 %hygro-thermal loads only
    %col_in = [deltaF,deltaC,alpha1,alpha2,Beta1,Beta2];
    %other = 0; not used
    deltaF = col_in(1); deltaC = col_in(2);
    alpha1 = col_in(3)*10^-6; alpha2 = col_in(4)*10^-6; Beta1 = col_in(4);
Beta2 = col_in(6);
    alpha_layers = zeros(3,1,layers);
    Beta_layers = alpha_layers; N_ht = alpha_layers; %ht - hygro-thermal
    strain_hg_layers = alpha_layers;
    M_ht = alpha_layers;
    N_ht_total = 0; M_ht_total = 0;
    for i=1:layers
        alpha_layers(:,:,i) = ((Th(:,:,i))^-1)*[alpha1;alpha2;0];
        alpha_layers(3,1,i) = 2*alpha_layers(3,1,i);
        Beta_layers(:,:,i) = ((Th(:,:,i))^-1)*[Beta1;Beta2;0];
        Beta_layers(3,1,i) = 2*Beta_layers(3,1,i);
        strain_hg_layers(:,:,i) =  alpha_layers(:,:,i)*deltaF +
Beta_layers(:,:,i)*deltaC;
        N_ht(:,:,i) =
Q_bars(:,:,i)*10^6*strain_hg_layers(:,:,i)*layer_thickness(i);
        N_ht_total = N_ht_total + N_ht(:,:,i);
        M_ht(:,:,i) = N_ht(:,:,i)*z_bar(i);
        M_ht_total = M_ht_total + M_ht(:,:,i);
    end
    N_units = 'lbf/in'; M_units = '(in*lbf)/in';
    fprintf('Hygro-Thermal Running Loads\n')
    fprintf(' Nx = %3.0f %s\n Ny = %3.0f %s\n Nxy = %3.0f %s\n',
N_ht_total(1),N_units,...
    N_ht_total(2),N_units,N_ht_total(3),N_units);
    fprintf(' Mx = %3.3f %s\n My = %3.3f %s\n Mxy = %3.3f %s\n',
M_ht_total(1),M_units,...
    M_ht_total(2),M_units,M_ht_total(3),M_units);
    ek_0 = (ABBD^-1)*[N_ht_total;M_ht_total];
    e_units = 'x10^-6 in/in'; k_units = '1/in';
    ek_0(1:3) = ek_0(1:3)*10^6;
```

```
    fprintf(' ex_ht0 = %3.0f%s\n ey_ht0 = %3.0f%s\n gxy_ht0 = %3.0f%s\n',
ek_0(1),e_units,...
        ek_0(2),e_units,ek_0(3),e_units);
    fprintf(' kx_ht0 = %3.4f %s\n ky_ht0 = %3.4f %s\n kxy_ht0 = %3.4f %s\n',
ek_0(4),k_units,...
        ek_0(5),k_units,ek_0(6),k_units);
    ek_0(1:3) = ek_0(1:3)*10^-6;
    else
        fprintf('error in type')
    end


end
%In Program Functions ---------------------------------------
function [] = Display_Matrix(name,X,Fw,Pre,mag,Note)
     X = X*mag;
     if( mod(length(X),2) ~= 0)
        middle = (length(X)+1)/2;
     else
        middle = length(X)/2;
     end
fprintf('%s = \n',name)
     for i=1:length(X)
        fprintf('[%*.*f ',Fw,Pre,X(i,1))
        for b=2:(length(X)-1)
            fprintf('%*.*f ',Fw,Pre,X(i,b))
        end
             if (i == middle)
               fprintf( '%*.*f] %s\n',Fw,Pre,X(i,end),Note)
            else
                fprintf('%*.*f]\n',Fw,Pre,X(i,end))
             end

     end
       fprintf('\n')
end
```

Outputs

(Bold words is not output from the program but show what the program is outputting for Homework 9)

>> InputsForProject7

Hw#9

Problem 1

ABBD =

[354230.3	49040.1	0.0	-0.0	-0.0	-332.6]
[49040.1	45635.8	0.0	-0.0	-0.0	-112.8]
[0.0	0.0	50067.9	-332.6	-112.8	-0.0]
[-0.0	-0.0	-332.6	56.2	2.7	0.0]
[-0.0	-0.0	-112.8	2.7	4.7	0.0]
[-332.6	-112.8	-0.0	0.0	0.0	2.9]

Problem 2

Mid-plane strains and curvatures

ex_0 = 704x10^-6 in/in

ey_0 = -614x10^-6 in/in

gxy_0 = 0.00213 in/in

kx_0 = 0.377 (1/in)

ky_0 = -0.166 (1/in)

kxy_0 = 0.0574 (1/in)

Problem 3 along with global coordinate stresses

Ply 1 at the middle =

Global stresses:

sigma_x = -54358 psi

sigma_y = 159 psi

tau_xy = 380 psi

Principal stresses:

sigma1 = -54358 psi

sigma2 = 159 psi

tau12 = 380 psi

Problem 4

Margin of Safety

MS1 = 0.545

MS2 = 30.544

MS12 = 15.843

Problem 5 along with global coordinate stresses

Ply 4 at the middle =

Global stresses:

sigma_x = 69630 psi

sigma_y = -763 psi

tau_xy = 897 psi

Principal stresses:

sigma1 = 69630 psi

sigma2 = -763 psi

tau12 = 897 psi

Problem 6

Margin of Safety

MS1 = 1.872

MS2 = 25.199

MS12 = 6.137

Problem 7 along with global coordinate stresses and Margins of Safety

Ply 1 at the top =

Global stresses:

sigma_x = -75023 psi

sigma_y = 312 psi

tau_xy = 294 psi

Principal stresses:

sigma1 = -75023 psi

sigma2 = 312 psi

tau12 = 294 psi

Margin of Safety

MS1 = 0.120

MS2 = 15.017

MS12 = 20.779

Problem 8 along with global coordinate stresses and Margins of Safety

Ply 1 at the bottom =

Global stresses:

sigma_x = -33693 psi

sigma_y = 5 psi

tau_xy = 466 psi

Principal stresses:

sigma1 = -33693 psi

sigma2 = 5 psi

tau12 = 466 psi

Margin of Safety

MS1 = 1.493

MS2 = 1028.145

MS12 = 12.731