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A New Potential Field-Based Algorithm for Path Planning

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Abstract. In this paper, the path-planning problem is considered. We introduce a new potential function for path planning that has the remarkable feature that it is free from any local minima in the free space irrespective of the number of obstacles in the configuration space. The only global minimum is the goal configuration whose region of attraction extends over the whole free space. We also propose a new method for path optimization using an expanding sphere that can be used with any potential or penalty function. Simulations using a point mobile robot and smooth obstacles are presented to demonstrate the qualities of the new potential function. Finally, practical considerations are also discussed for nonpoint robots

Key words: smooth objects, potential function, local minimum, path optimization.

1. Introduction

A number of industrial applications involve finding the shortest path from a starting point to a destination in the presence of polyhedral obstacles. Finding the shortest paths in these applications is helpful in improving productivity and reducing costs. Such applications include path planning for manipulator and mobile robots, routing of oil, gas and water piping systems, communication cables, etc.

Path planning has emerged as one of the most challenging problems in the development of autonomous robots. Consequently, a lot of research efforts have been devoted to it over the past two decades. The literature is rich with various classes of algorithms for solving this problem. However, about four main approaches to the path-planning (or path-finding) problem can be identified, which are the following:

- Roadmap methods.
- Cell decomposition methods.
- Optimal control methods.
- Potential field methods.

Furthermore, algorithms can be classified as being exact or heuristic. Exact algorithms either find a solution or prove that none exists, and they tend to have high complexity. Heuristic methods, on the other hand, attain faster solutions at the expense of reliability.

The roadmap method consists of constructing a network of one-dimensional curves that capture the connectivity of the robots free space (C_{free}) or its closure. Once a roadmap $\mathcal{R}$ has been constructed, path planning reduces to connecting the initial and the goal configurations through points in $\mathcal{R}$ that do not interfere with the obstacles. Variants of the roadmap method include *visibility graph, freeway net,* and *silhouette* [4, 17, 18].

Cell-decomposition methods involve partitioning of the robot's free space into smaller regions called cells, and searching the connectivity of these cells for a valid path. The graph arising from connecting adjacent cells is called the *'connectivity graph'*. The major types of cell decomposition are the exact and approximate cell decomposition [6, 17, 19, 24].

Optimal control methods for path planning determine a path or trajectory while minimizing certain performance indexes – usually time or distance. The path is parameterized as a function of a scalar variable and the objective function (or performance index) is minimized subject to constraints due to the robot dynamics and kinematics. In this way, the path derived is both dynamically and kinematically optimal. The path is also smooth. An added advantage to this method is that, it can directly yield the optimal controls required to move on the path. However, the algorithms are computationally intensive [2, 7, 10, 11, 14, 20, 25–27].

Other methods use a combination of various techniques such as cell decomposition, roadmap, orthogonal projections, distance estimation, collision detection, shortest-path, sensing and other computational geometric approaches [16, 29, 30, 32].

Most of the above-mentioned methods have three disadvantages. First, the allowed shapes may be too restricted to be applicable in general; secondly, they may fail to find a solution even if one exists; and thirdly, their computational time may be too high.

The roadmap and the cell-decomposition methods are global methods, i.e., these methods search for a free path by first analyzing the connectivity of the whole free space of the robot, and they are guaranteed to find a path if it exists. However, their computational time increases exponentially as the degree of freedom of the robot increases [17]. On the other hand, the potentional field method is a local approach that depends on local information of the resultant force due to an artificial potential induced by the obstacles and goal. The robot is represented as a point mass under the influence of an artificial potential U. This potential is usually defined over free-space as the sum of an attractive potential pulling the robot toward the goal configuration, and a repulsive potential pushing the robot away from the obstacles. In analogy to the electrostatic potential, the robot and the obstacles are assumed to carry positive electric charges while the goal point is assumed to carry a negative charge. The resulting potential field is used to represent the free-space.

The potential-field method is elegant and can be very efficient requiring no prior model of the obstacles when used in an on-line collision avoidance scheme [3, 15, 16, 28]. Although not as global as the graph-searching techniques, the speed of the algorithms and their easy extension to global path planning, offer an excellent alternative to the graph-searching techniques [9]. However, the potential-field method has one serious drawback. Since it is essentially a fast optimization descent scheme, it can get stuck in local minima other than the goal configuration. For few obstacles in the configuration space, this problem may not arise. But for a cluttered environment, this problem seriously limits the application of the potential-field method.

The local minima are created due to the addition of attractive potential due to the goal and repulsive potentials from the obstacles. Therefore, at certain points in the potential field, the net force on the robot becomes zero which are local minima of the potential field, and thereby the robot stops at an unintended location.

In this paper, we will consider the geometric path-planning problem for the case of a point mass moving in a space where there is accurate information about the nature, position and orientation of the static configuration–space obstacles (CS-obstacles). Furthermore, we shall assume that the obstacles are not interpenetrating, i.e., they are separated by some distance, or are at most only touching. We shall introduce a new approach to the potential-field method which is essentially a penalty-function method but has the remarkable feature that there is no local minimum in the free space. The only global minimum is the goal configuration whose domain of attraction is over the whole of the free space C_{free}. We shall also propose a new approach, based on expanding spheres, for optimizing the path that can be used with any potential or penalty function.

We begin in Section 2 by reviewing the requirements desired for a potential function, and then discuss some of the drawbacks of ealier approaches. We then introduce the new approach in Section 3. In Section 4, we discuss practical considerations for path planning of nonpoint robots, and in Section 5, we give representative simulations. Finally, we give conclusions and recommendations for future work in Section 6.

2. Requirements for an Artificial Potential Function and Problems with Earlier Approaches

In this section we look at the properties required for a continuous function to serve as a potential function. These requirements have motivated the development of the new potential function, as most of the potential functions proposed so far, have fallen short of these requirements. We will then pinpoint the shortcomings of the earlier potential functions.

Khosla and Volpe [13] have summarized the properties of a repulsive artificial potential function as follows:

(1) It should have spherical symmetry for large distances from the obstacle so that no local minimum is created when this potential is added to other potentials (e.g. an attractive well).

(2) It should mimick the obstacle surface at close distance so as to maximize the robots free-space.

(3) Its range of influence should be limited to the vicinity of the obstacle so as not to affect the robots motion away from the obstacle.

(4) It should be a continuously differentiable function of class $C^m[0,\infty)$ where $m \geqslant 2$.

An attractive potential is usually a quadratic well which has good stabilizing properties and gives a constant gain when used in a feedback control. A conic well also serves as a good attractive potential but does not have the stabilizing properties of the quadratic well.

The potential field approach was pioneered by Khatib [12] who used it in real-time collision avoidance. Khatib first used the FIRAS (force inducing artificial repulsion from a surface) function. This potential function does not satisfy properties (1) and (4) above, and hence, it is not free from the problem of local minima when added to an attractive well or other potentials.

Pavlov and Voronin [21] have proposed a number of potential functions for coding the presence of obstacles in the external space of a robot. However, none of these functions is free from the local minimum problem.

The superquadric artificial potentials proposed by Khosla and Volpe [13, 31] are very attractive and satisfy almost all the requirements of the ideal. However, the inverse dependence of the potential with the pseudo-distance 'k' from the obstacle retains a finite value for the potential at distances away from the obstacle. Moreover, the choice of the scaling parameters and the decay constant for the potential still remains a trial-and-error problem.

Again, the harmonic potentials proposed by Kim and Khosla [15] eliminate the local minima problem but do not optimize the path. Moreover, as the complexity of the obstacles increases, the numbers of panels required to represent them increases and the computational cost of the algorithm may become very high.

The potential function proposed by Huang and Ahuja [9] has the advantage that it can handle convex polyhedral obstacles but it still remains finite at distances away from the obstacles. Therefore, the generation of local minima in the potential field cannot be ruled out. Although, the use of a global planner and a local planner has given rise to a powerful algorithm.

The notion of a navigation function proposed by Khoditchek and Rimon [22, 23], is achievable for only limited classes of objects shapes and configuration space.

In the next section, we introduce the potential function.

3. The New Potential Function

Consider an analytic description of a smooth convex object (such as a sphere or ellipsoid) defined by the set S

$$S = \{x\colon\ g(x) \leqslant 0;\ g(x) \in C^2[0,\infty),\ x \in \mathbb{R}^n;\ n = 2,3\} \tag{1}$$

Then the function

$$p(x) = \mu \max\,(0, -g(x))^r \tag{2}$$

where μ is a very large positive penalty parameter and $r \geqslant 2$, is everywhere zero outside the object, and attains its maximum in the interior of S. Hence, such a function can serve as a good potential function. Furthermore, the addition of such potentials arising from many objects in a configuration space, will not create local minima in the free-space.

For a point robot moving amongst N stationary obstacles defined by $g_i(x) \leqslant 0$; $i = 1, 2, \ldots, N$, a useful navigation function can be defined by

$$U(x) = \frac{1}{2}\eta\|x - x_g\|^2 + \mu\sum_{i=1}^{N} \max(0, -g_i)^r \tag{3}$$

where x_g is the goal configuration and η is a constant. The above total potential function is continuous, differentiable and has zero contribution from the second term at locations away from the obstacles. Its remarkable property is described in the following proposition.

PROPOSITION 3.1. *The potential function $U(x)$ given in Equation (3) has a unique global minimum at the goal configuration whose domain of attraction extends all over C_{free}, for all $r \geqslant 2$.*

Proof. The gradient vector of the potential function is given by

$$\nabla U = \eta(x - x_g) + (-1)^r r\mu\sum_{i=1}^{N} \max(0, -g_i)^{r-1}\nabla g_i \tag{4}$$

Because of the large penalty parameter μ, the interior of the obstacles is a forbidden region for the robot, and hence if we exclude all points in the interior of the obstacles, the second term in the gradient is always zero, and the solution of

$$\nabla U = 0 \tag{5}$$

is uniquely $x = x_g$. Also, the hessian matrix of U is positive definite at this point, and hence the result. □

The function U can also be defined in the cylindrical (ρ, θ, z) or spherical (r, θ, ϕ) coordinates to reflect other robot geometries. The only drawback of the

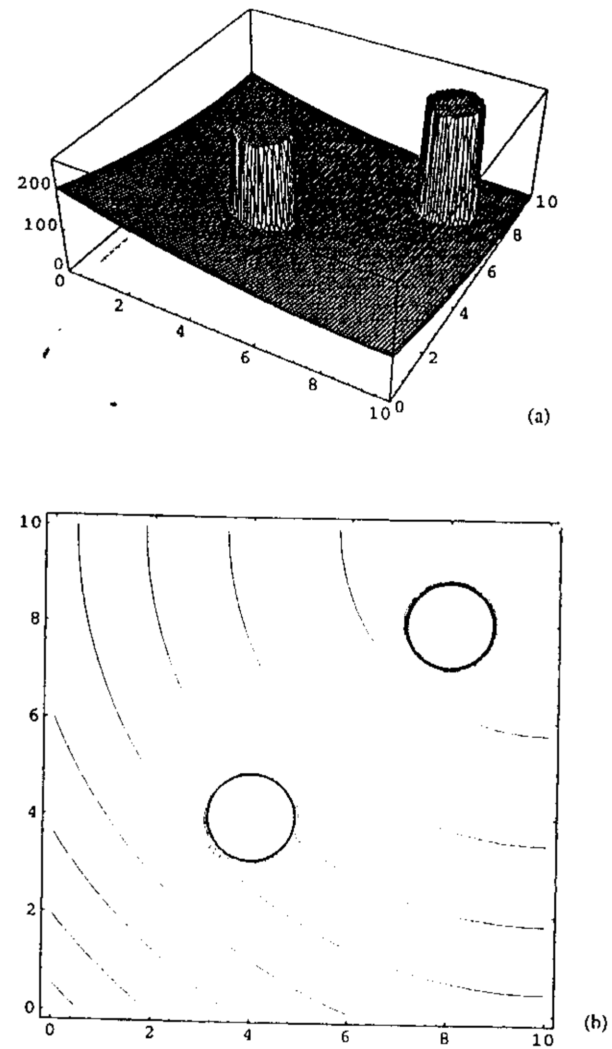

Figure 1. (a) Potential field of two circular objects. (b) Contour plot of the field.

above potential function is that it allows the robot to approach too close to the surfaces of the obstacles. However, this is not a very serious problem as observed in the visibility graph method [18]. The obstacles could be expanded by a small safety factor ϵ to avoid this problem. Furthermore, this seeming drawback can be used to advantage since certain robot tasks such as docking, parts mating, and

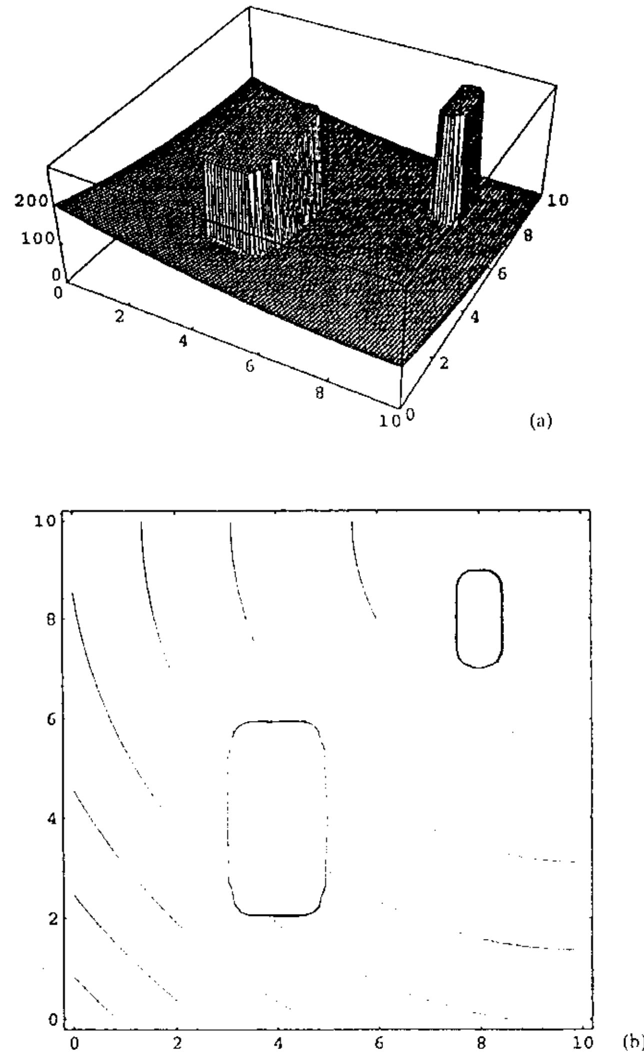

Figure 2. (a) Potential field of two rectangular objects. (b) Contour plot of the field.

more general motion tasks, all require navigation at or along the boundary of the configuration space. Figure 1 shows the distribution of the potential due to two circular objects in $\mathbb{R}^2$ and Figure 2 shows the distribution of the potential due to two rectangular objects obtained by approximation with an n-ellipse.

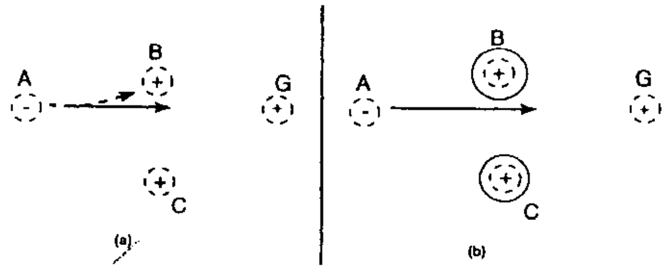

Figure 3. A system of electrostatic charges.

An analogy of the above potential field to the electrostatic potential can be drawn from Figure 3. The figure shows a system of three positive charge regions $(B, C$ and $G)$ and a region of negative charge (A) at the end. It is desired to accelerate the center of the negative charge A to G through the field. To avoid the use of excessively high potential difference between A and G, the problem could be solved by screening the regions B and C with metallic surfaces so that their contribution to the total potential in the region is reduced to zero outside the metallic screens. This is exactly the purpose served by the function $p(x)$.

4. Path Optimization

The path planning problem can be viewed as the solution of the dynamical system

$$\dot{x} = -\nabla U(x) \tag{6}$$

where ∇U now acts as the control (or input) to the system. Such a system is called a *gradient system*, and has the following nice properties [8]:

- $\dot{U}(x) \leqslant 0$ for all $x \in c_{\text{free}}$; and $\dot{U}(x) = 0$ if and only if x is an equilibrium of (6). Where $\dot{U}$ is derivative of $U(x)$ with respect to t.
- If $\bar{x}$ is an isolated equilibrium of U, then $\bar{x}$ is an asymptotically stable equilibrium of the system (6).
- If $x(t)$ is a solution of (6), then
  $\dot{U}(x(t)) = -\|\nabla U(x(t))\|^2$
- If $x(t)$ is not constant, then $U(x(t))$ is a decreasing function of t.

From these properties we can understand that x_g above is an isolated equilibrium of the system (6) and it is unique. This is the assertion of Proposition 3.1. Furthermore, it implies that the function $U(x)$ is a Lyapunov function candidate for the system. This is a very important result for controller design.

Now once the total potential function from the goal and obstacles has been constructed, path planning is usually implemented in a depth-first fashion by

integrating the system (6). This involves generating successive path segments starting from the initial configuration along the direction of the negated gradient of the total potential function $U(x)$ at each successive point until the goal is reached. The amplitude of each segment is chosen so that the segment lies in C_{free}. Hence the coordinates of the configuration x_{i+1} attained at the ith iteration is given by

$$x_{i+1} = x_i + \delta_i(-\nabla U(x_i)) \tag{7}$$

Where δ_i denotes the length of the ith segment. Usually, the choice of δ_i is not optimal.

The above method of depth-first planning is a gradient-based approach which uses only first-order information about the potential function. This makes it very inefficient. We propose a new scheme that will avail the path planner the utilization of efficient optimization algorithms in existence [1]. As such consider the following problem.

$$\begin{aligned} &\min U(x) = \frac{1}{2}\eta\|x - x_g\|^2 + \mu\sum_{i=1}^{N}\max(0, -g_i)^r \\ &\text{s.t. } \|x - x_0| \leqslant R(t) \end{aligned} \tag{8}$$

The additional constraint is an expanding sphere, where x_0 is the initial configuration, and $R(t)$ is a time function which represents the radius of the bounding sphere. By the above constraint, problem (3) is transformed into a set of problems parameterized by $R(t)$, where $R(t)$ goes from zero to d_g or the Euclidean distance from the initial point to the goal point.

The optimality conditions for problem (8) are

$$\begin{aligned} &(x - x_g) + (-1)^r r\mu\sum_{i=1}^{N}\max(0, -g_i)^{r-1}\nabla g_i - \alpha(x - x_0) = 0 \\ &(x - x_0)^2 = R^2(t) \\ &\alpha \geqslant 0 \end{aligned} \tag{9}$$

The conditions (9) are the Karush–Kuhn–Tucker conditions [1] for problem (8) and if we ignore the condition $\alpha \geqslant 0$, then (9) is a system of $n + 1$ equations in $n + 1$ unknowns parameterized by $R(t) \in [0, d_g]$, or a homotopy in $R(t)$.

Newton's method can be used to solve the above system for various values of $R(t) \in [0, d_g]$. Clearly, all the conditions of the implicit function theorem hold, which implies that the solution will constitute a path that is continuous and smooth in x, which is the desired objective.

The above has been the theoretical framework for the path-planning problem. Implementation can be done either along the same theoretical approach outlined above, where $\alpha \geqslant 0$ can be taken care of implicitly (i.e through starting with

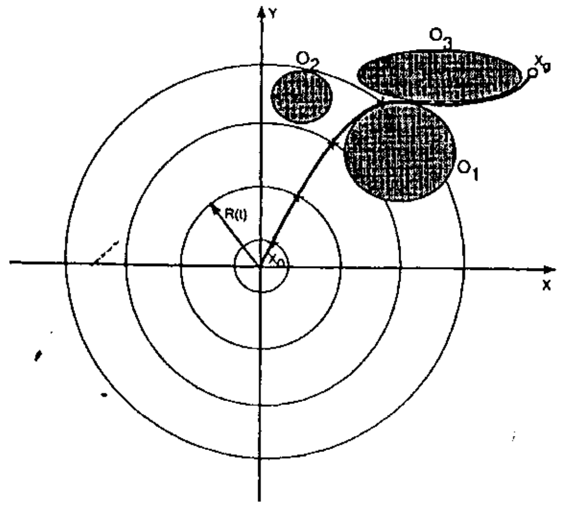

Figure 4. Depth-first planning using expanding circles.

$\alpha > 0$ and applying the minimum ratio test to make sure it is always nonnegative) or by solving problem (8) directly using any of the efficient optimization routines which are readily available nowadays.

Figure 4 shows a geometrical interpretation of the above formulation in two dimensions. In each iteration of the path search, the radius of the sphere $R(t)$ is incremented, and the minimum of the potential function is sought within the new sphere. Consequently, a new configuration is generated closer to the goal and away from the obstacles. The rate of change of $R(t)$ can be typically made equal to the maximum speed of the robot. By a judicious choice of the rate of change of $R(t)$, a smooth path can be generated from the initial configuration to the goal configuration without collision with any of the obstacles.

5. Practical Considerations

In practice, the robot links or body will have nonzero length, width and thickness; hence, the assumption of a point robot will not work in practice. However, a non-point robot can be transformed into a point robot by expanding the obstacles by the largest radius of the robot. This operation is known as *growing the obstacles by the robot* [18, 19], and it transforms the workspace obstacles into CS-obstacles. Although the operation is not exact, it provides a good practical solution to the above problem.

In Figure 5 we show the case of a circular robot amongst circular obstacles. But note that, the growth operation has led to the intersection of the grown

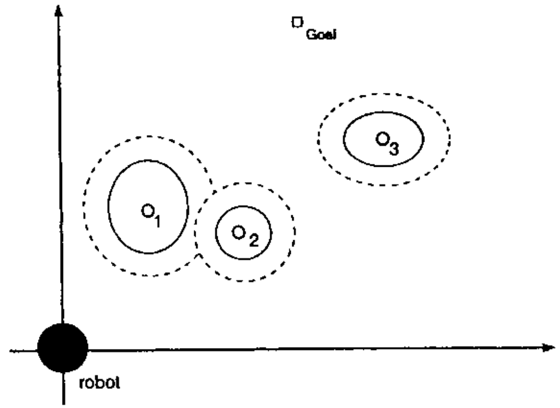

Figure 5. A circular robot moving among circular obstacles.

obstacles. Depending on the location of the goal configuration, if the robot has to pass in between the obstacles in order to reach the goal, the robot will get stuck. Yet, the potential field approach has no way of detecting such a problem a priori. A reasonable solution to the above problem is to define a new obstacle which contains the union of the intersecting obstacles. We shall call this new object the *unionizing object*. For the case of the circular obstacles, this new object can be approximated by an n-ellipse defined as follows: let the radii of the grown obstacles be r_1, r_2 and their centers be $(a_1, b_1), (a_2, b_2)$ respectively. Then, the length of the major and minor axes of the unionizing n-ellipse l_1, l_2 respectively, are given by

$$l_1 = (a + b) + \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}.$$
$$l_2 := \max(r_1, r_2)$$

The center of the n-ellipse (a, b) is similarly given by

$$(a, b) = \frac{1}{2}\Big((a_1 - a_2), (b_1 - b_2)\Big)$$

Hence the n-ellipse can be defined by

$$\left(\frac{\bar{x} - a}{l_1}\right)^{2n} + \left(\frac{b}{a}\right)^2 \left(\frac{\bar{y} - b}{l_2}\right)^{2n} = 1$$

Where $n \geqslant 1$ is an appropriate shaping parameter.

$$\bar{x} = x\cos\theta - y\sin\theta, \qquad \bar{y} = x\sin\theta + y\cos\theta$$

and

$$\theta = \cos^{-1} \frac{a_1 - a_2}{\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}}$$

6. Representative Simulations

In the simulations, we considered problems in two dimensions with circular and elliptical obstacles. Of course, the case of three dimensions is also applicable. All the simulations were implemented using the MATLAB Optimization Toolbox routine 'CONSTR' [33].

Figures 6–8 show the path of a point mobile robot starting from the reference position (0, 0), the origin (without any loss of generality), to the point (10, 10). Path 1 was obtained with three circular obstacles of radii '1' centered at (2, 2), (5, 5), (8, 8) respectively. Path 2 was obtained with two circular obstacles with radii '1' centered at (2, 0) and (2, 2). The goal in this case was (10, 2). Path 3 was obtained with a rectangular obstacle approximated by a 4-ellipse with major and minor axes lengths '2', '1' respectively, and centered at (5, 0.5). The goal was in this case (10, 2). Path 4 was obtained using ellipses centered at (3, 3) and (6, 6) with major axis length '4' along the y-axis and minor axis length '2' along the x-axis.

Clearly, Path 4 indicate the failure (i.e. jumping and crossing of the obstacles by the sequence of points) of the algorithm to handle elliptical obstacles. However, this does not mean that the new potential function proposed has failed to

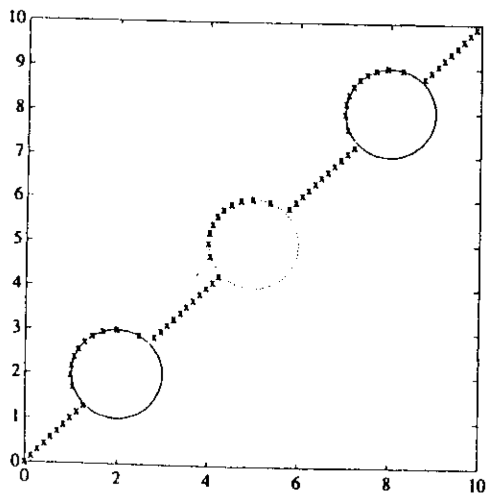

Figure 6. Path 1.

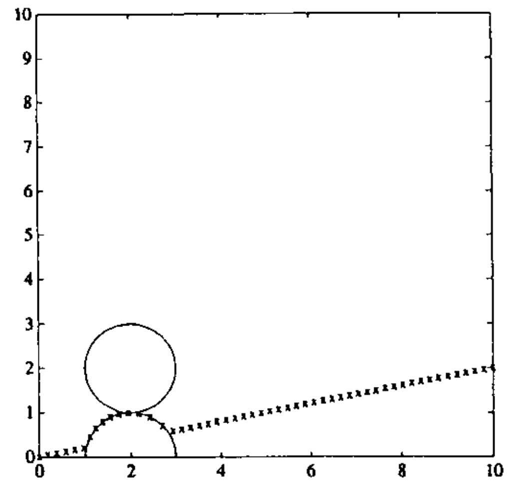

Figure 7. Path 2.

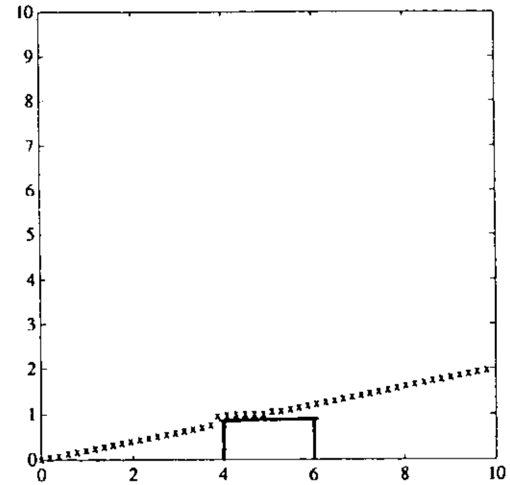

Figure 8. Path 3.

avoid the obstacles, but it is rather the formulation of the path-planning problem in Equation (8) that has failed. Nevertheless, the problem can be remedied by replacing the expanding sphere (constraint in (8)) with a very narrow expanding ellipsoid. This back-up procedure has been used to resolve all the above prob-

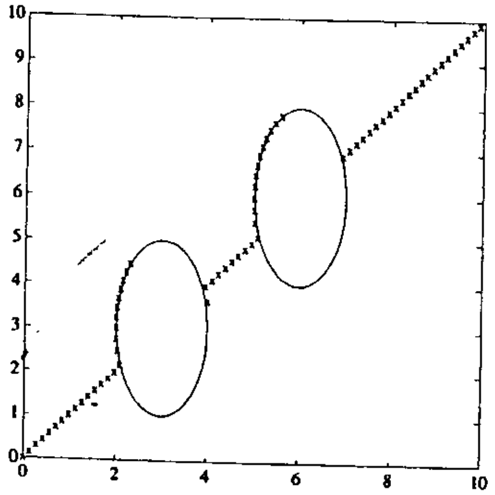

Figure 9. Path 4.

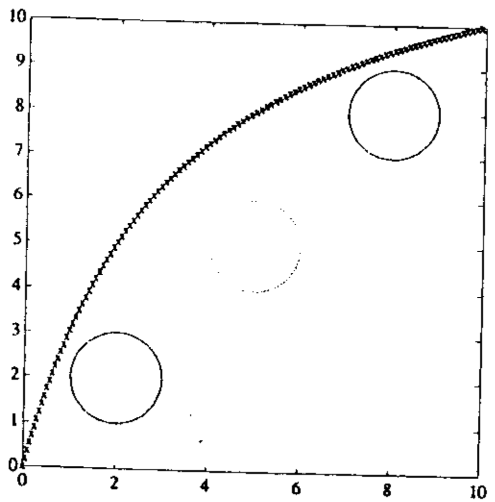

Figure 10. Path 5.

lems, and the results are shown in Figures 10–13. Paths 5, 6, 7, 8 generated using the ellipse correspond to paths 1, 2, 3, 4 generated using the circle. It has in fact turned out that this procedure yields more optimal paths and solves richer classes

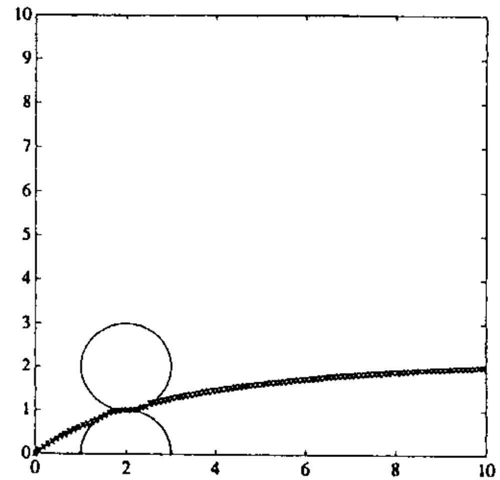

Figure 11. Path 6.

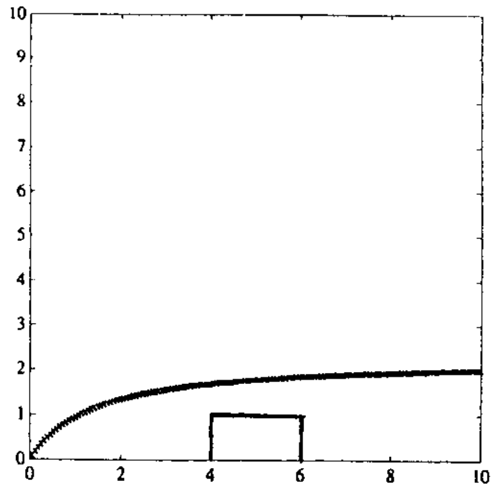

Figure 12. Path 7.

of problems. The computational times for all the above simulation problems on an IBM 486/433 DX machine are tabulated on Table I.

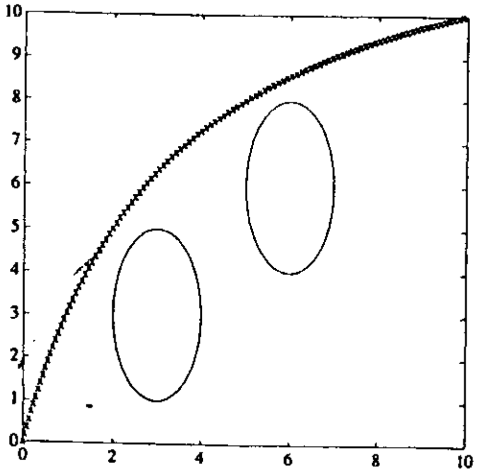

Figure 13. Path 8.

Table I. Computational times for simulation problems (Figs 6–13)

Path	Time (sec)[a]
Path 1	77.17
Path 2	72.55
Path 3	50.75
Path 4	73.77
Path 5	47.89
Path 6	74.92
Path 7	39.82
Path 8	46.13

[a]seconds on IBM 486/433 DX machine with clock speed 33 MHz.

7. Conclusion

In this paper, we have presented a methodology for local off-line path planning based on the method of potential functions for robotic application. It applies to static obstacles in the work space whose presence is coded using a new artificial potential function. The new function possesses all the properties desired for an artificial potential function, the most important of which is freedom from local minima in the free space that can be created by addition of several potentials. All that is required to generate a potential field using the new function is an analytic description of the obstacles. For smooth objects such as spheres and ellipsoids,

this is readily available. Other convex objects can be approximately described using superquadric functions [13].

We have also proposed a new approach to path optimization by parameterizing the path as a homotopy in the radius of an expanding sphere. Using simulations in two dimensions with a point mobile robot, we have demonstrated that by a combination of the new potential function for obstacle avoidance and the path-optimization scheme, a smooth continuous path can be generated from start to goal within a very short time. This is possible because the above formulation allows the planner the utilization of efficient optimization routines vis-a-vis the use of first-order information provided by the gradient. We have solved several classes of problems and we have shown representative solutions. Furthermore, we have shown that the potential function can be applied to the path planning of a rich class of robots by transforming the work-space obstacles to configuration-space (CS) obstacles using an object growth algorithm.

As areas of future research, it will be worthwhile to investigate the use of the new potential function for on-line obstacle avoidance using feedback control. We also intend to unify our approach to the path-planning problem by considering both translations and rotations of the robot in three dimensions. The use of the new potential function for motion planning in nonstationary environments and multiple (possibly interacting) robots in a work space, should also be investigated. Applications to motion planning with uncertainty, holonomic and nonholonomic constraints will also be worthwhile problems for investigation.

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Perception-Based Learning for Motion in Contact in Task Planning

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Abstract. This paper presents a new approach to error detection during motion in contact under uncertainty for robotic manufacturing tasks. In this approach, artificial neural networks are used for perception-based learning. The six force-and-torque signals from the wrist sensor of a robot arm are fed into the network. A self-organizing map is what learns the different contact states in an unsupervised way. The method is intended to work properly in complex real-world manufacturing environments, for which existent approaches based on geometric analytical models may not be feasible, or may be too difficult. It is used for different tasks involving motion in contact, particularly the peg-in-hole insertion task, and complex insertion or extraction operations in a flexible manufacturing system. Several real examples for these cases are presented.

Key words: manufacturing, motion in contact, force/torque sensors, error detection, plan monitoring, uncertainty, robotics, neural networks.

Category: (8) AI in Robotics and Manufacturing/FMS.

1. Introduction

The field of robotic assembly and task planning must play an important role in the automation and flexibility of manufacturing systems. For a given design, a sequence of subtasks has to be determined for each operation. Each particular subtask requires a lower level plan that may involve gross motion, fine motion, or grasping actions. For gross motion planning uncertainty is not critical. Since a main point is collision avoidance, there is no contact and the clearances between objects can be kept large enough by using adequate spatial representations [15]. Fine motion planning, on the other hand, deals with small clearances and contact. The existence of uncertainty may render a synthesized plan useless. The use of sensors – mainly force-and-torque sensors – is necessary to get information about

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