

LATEX PRACTICE

RAUL VILLALOBOS

Quiz Practice
Calculate probabilities and expectation values.

$$\begin{aligned}\psi(x) &= C(1 - x^2) \\ \text{for } 0 \leq x \leq 1 \\ &= 0 \text{ everywhere else}\end{aligned}$$

$$\begin{aligned}&\int |\psi(x)|^2 dx \\ &\int |C(1 - x^2)|^2 dx \\ &C^2 \int (1 - x^2)^2 dx \\ &C^2 \int (1 - 2x^2 + x^4) dx \\ &C^2 \left[x - \frac{2x^3}{3} + \frac{x^5}{5} \right]\end{aligned}$$

Find normalization

$$\begin{aligned}C^2 \left[x - \frac{2x^3}{3} + \frac{x^5}{5} \right]_0^1 &= 1 \\ C^2 \left[\left(1 - \frac{2 \times 1^3}{3} + \frac{1^5}{5} \right) - \left(0 - \frac{2 \times 0^3}{3} + \frac{0^5}{5} \right) \right] &= 1 \\ C^2 \left[\left(1 - \frac{2}{3} + \frac{1}{5} \right) - (0) \right] &= 1 \\ C^2 \left(\frac{8}{15} \right) &= 1 \\ C^2 &= \frac{15}{8} \\ C &= \sqrt{\frac{15}{8}}\end{aligned}$$

Find expected pos

$$\begin{aligned}&\int x |\psi(x)|^2 dx \\ &\int x |C(1 - x^2)|^2 dx \\ &C^2 \int x (1 - x^2)^2 dx \\ &C^2 \int x (1 - 2x^2 + x^4) dx \\ &C^2 \int (x - 2x^3 + x^5) dx \\ &C^2 \left[\frac{x^2}{2} - \frac{2x^4}{4} + \frac{x^6}{6} \right] \\ &C^2 \left[\frac{x^2}{2} - \frac{x^4}{2} + \frac{x^6}{6} \right] \\ &\frac{15}{8} \left[x^2 - x^4 + \frac{x^6}{3} \right]_0^1\end{aligned}$$

$$\begin{aligned}\frac{15}{16} \left[\left(1^2 - 1^4 + \frac{1^6}{3} \right) - \left(0^2 - 0^4 + \frac{0^6}{3} \right) \right] \\ \frac{15}{16} \left[\left(1 - 1 + \frac{1}{3} \right) - (0) \right] \\ \frac{15}{16} \left(\frac{1}{3} \right) = \frac{5}{16}\end{aligned}$$