6.2 REPRESENTING SIMPLE FACTS IN LOGIC

⑤ Bi-Conditional $p \leftrightarrow q$
p if and only if q

There are various types of logic such as logic of sentences (propositional logic), logic of objects (predicate logic), logic involving uncertainties, logic dealing with fuzziness, temporal logic etc. Here we are going to explain predicate logic. Before that, we will study propositional logic and its limitation.

PROPOSITIONAL LOGIC : Boolean Logic (It can't predict it just say True/False.

Propositional logic is a logic at the sentential level. Sentences considered in propositional logic are not arbitrary sentences but are the ones that are either true or false, but not both. These kind of sentences are called proposition. If a proposition is true, then we say it has a truth value of "true"; if a proposition is false, its truth value is "false". Some of the proposition example are:

It is used to make computer understand & helps in learning

1. Grass is green.
2. 2 + 5 = 5
3. It is raining.

The first proposition has the truth value of "true" and the second "false". But "Close the door", and "Is it hot outside ?"are not propositions. Also "x is greater than 2", where x is a variable representing a number, is not a proposition, because unless a specific value is given to x we can not say whether it is true or false, nor do we know what x represents. Lets see some of the examples of propositional logic representation:

- It is humid: Q
- If it is humid, then it is hot: $Q \rightarrow P$
- If it is hot and humid, then it is raining: $(P \wedge Q \rightarrow R)$ (AND)

Logical Conjunction

The statements which do not contain any connectives such as $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$ etc. are called **atomic statements**, whereas, those statements which contain one or more atomic statements connected by connectives are called **compound statements.** In the above sentences, Q is atomic statement whereas, $Q \rightarrow P$ and $P \wedge Q \rightarrow R$ are compound statements. The various operations performed on atomic statements P and Q is represented by truth table 6.1 given below :

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

$= (p \rightarrow q) \wedge (q \rightarrow p)$

$P \rightarrow Q \equiv \neg P \vee Q$

Truth Table 6.1

Example 1. Construct truth table for the expression given below :

(a) $(A \wedge (A \vee B))$ [Raj. Univ. 2006]

(b) $(A \vee \neg B) \wedge (\neg A \vee B)$ [Raj. Univ. 2008]

Determine whether there is any single term in these expression equivalent to ?

Solution : (a) $(A \wedge (A \vee B))$

A	B	$A \vee B$	$A \wedge (A \vee B)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

Truth Table 6.2

The expression $A \wedge (A \vee B)$ is equivalent to single term A.

(b) $(A \vee \neg B) \wedge (\neg A \vee B)$

A	B	$\neg A$	$\neg B$	$A \vee \neg B$	$\neg A \vee B$	$(A \vee \neg B) \wedge (\neg A \vee B)$
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

Truth Table 6.3

There is no single term for which expression $(A \vee \neg B) \wedge (\neg A \vee B)$ is equivalent.

final Answers are have all True U K/a tautology (universally True

eg $P \vee \neg P$

- A statement formula which is true regardless of the truth values of the statements which replace the variables in it is called universally valid formula or tautology.

eg $P \wedge \neg P$

- A statement formula which is false regardless of the truth values of the statements which replace the variables in it is called a **contradiction.** final value will all be false
- Otherwise, statement is **satisfiable** i.e. there is some interpretation for which it is true.
- The two sentences are equivalent if they have the same truth value under every interpretation.

Suppose we want to express the following sentences in propositional logic, but we are not able to do so.

1. Luis is Jack's sister.

   –need to distinguish properties from the things to which they apply.

2. All sisters are female.

   –need to express "all".

3. Some people don't have sisters.

   –need to express "some".

4. No one is their own sister.

   –need notion of equality.

Propositional logic is not powerful enough as a general knowledge representation language. ① It lacks the **expressive power to describe** an environment with many objects concisely. To cope with deficiencies of propositional logic, predicate logic and quantifiers are used.

② Can't draw inference.

③ Can't No symbol of All, Everybody, Somebody etc. (quantifiers)

① Predicate Logic is an extension of propositional Logic
② Allows the structure of Facts to be defined.
③ For eg. A = The Ball Color is Red

PREDICATE LOGIC:

Color (Ball, Red) → Arguments (object)
Predicate/Relation (Assertion)

A **predicate** is a verb phrase template that describes a property of objects, or a relationship among objects represented by the variables. Its basic unit is a predicate/argument structure called an **atomic sentence**. For example:

likes(alison, chocolate) // Alison likes chocolate

tall(fred) // Fred is tall

Arguments can be any of the constant symbol, such as 'alison', variable symbol such as x, function expression such as *motherof(fred)*. So it can be said;

likes(x, richard) // x can be any person who likes richard say; fred, jame, etc.

friends(motherof(fred), motherof(jame)) // Fred and Jame mothers are friends

These atomic sentences can be combined using logic connectives.

likes(john, fred) $\wedge$ *tall(fred)* // *John likes Fred and Fred is tall.*

tall(john) $\vee$ *nice(fred)* // *John is tall or nice.*

Sentences can also be formed using quantifiers **"forall ($\forall$)"** and **"there exists ($\exists$)"** to indicate how to treat variables.

- The expression $\forall x: P(x)$, denotes the universal quantification of the atomic formula $P(x)$. When translated into the english language, the expression is understood as: "*for all* x, $P(x)$ holds", "*for each* x, $P(x)$ holds" or "*for every* x, $P(x)$ holds". $\forall$ is called the **universal quantifier**, and $\forall x$ means all the objects x in the universe. If this is followed by $P(x)$ then the meaning is that $P(x)$ is true for every object x in the universe. For example,

$\forall x$: *king(x)* $\rightarrow$ *person(x)* // All kings are person.

$\forall x$: *car(x)* $\rightarrow$ *wheel(x)* // All cars has wheel.

$\forall x$: $\forall y$: *brother(x,y)* $\rightarrow$ *sibling (x,y)* // Brothers are siblings.

- The expression $\exists x: P(x)$, denotes the existential quantification of $P(x)$. When translated into the english language, the expression is understood as: "*there exists an* x *such that* $P(x)$" or "*there is atleast one* x *such that* $P(x)$". $\exists$ is called the **existential quantifier**, and $\exists x$ means atleast one object x in the universe. If this is followed by $P(x)$ then the meaning is that $P(x)$ is true for atleast one object x of the universe. For example:

$\exists x$: *father(Bill, x)* $\wedge$ *mother(Hillary,x)* // There is a kid whose father is Bill and
// whose mother is Hillary.

$\exists x$: *cat(x)* $\wedge$ *mean(x)* // There is a mean cat.

$\forall x$: $\exists y$: *loves(x,y)* // Everybody loves somebody.

This means for all x, if their exists somebody y, then x loves y. This is nested quantification i.e. universal and existential quantifiers are used together in single expression.

The two quantifiers are connected with each other, through **negation.** For example the sentence *"Everyone likes icecream"* also means that there exist no one who does not like ice cream.

$\forall x: likes(x, icecream)$ is equivalent to, $\neg\exists x: \neg\ likes(x, icecream)$

The predicate logic representation of some of the sentences are:

1. Luis is Jack's sister.

   $sister(Jack, Luis)$

2. All sisters are female.

   $\forall x: \forall y: sister(x, y) \rightarrow female(x)$

3. Some people don't have sisters.

   $\exists y: \neg(\exists y: sister(x, y))$

4. No one is their own sister.

   $\forall x: (\forall y: sister(x, y) \rightarrow \neg x = y)$

In order, to support inheritance in knowledge representation, objects must be organized into classes and classes must be arranged in generalization hierarchy. For this purpose, two attributes ***isa*** and ***instance*** are used. The ***isa*** attribute is used to show *class inclusion* and ***instance*** is used to show *class membership.*

- John was a man.

  $man(john)$

- John was a Russian.

  $russian(John)$

- All Russians were Roman.

  $\forall x: russian(x) \rightarrow roman(x)$

The instance predicate representation is shown below:

$instance(John, man)$

$instance(John, russian)$

$\forall x: instance(x, russian) \rightarrow instance(x, roman)$

Here binary instance predicate, is one with first argument as object and second as class to which object belongs. The *isa* predicate representation of third sentence is shown below:

$isa(russian, roman)$