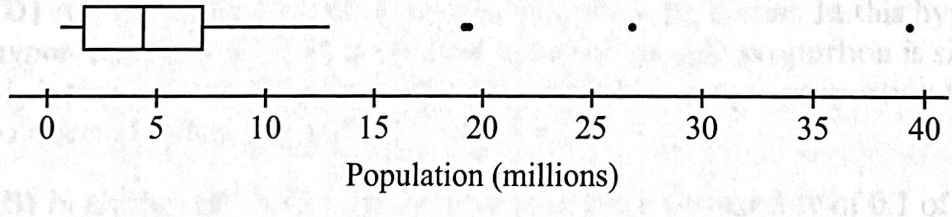
SECTION II PART A

Questions 1–5

Question One

(a) IQR = 7.0 – 1.6 = 5.4.
$Q_1 - 1.5(\text{IQR}) = 1.6 - 1.5(5.4) = -6.5.$
$Q_3 + 1.5(\text{IQR}) = 7.0 + 1.5(5.4) = 15.1.$
There are no populations less than –6.5. Any population greater than 15.1 is an outlier, so the outliers are 19.7, 19.9, 27.0, and 38.8.

(b)



(c) Missouri's population is greater than the median population, so Missouri is among the largest fifty percent of states. However, its population is less than the upper quartile population, so Missouri is not in the largest 25% of states.

Question Two

(a)

		Score on Spinner 1					
		1	2	3	4	5	6
Score on Spinner 2	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2

(b)

Amount Won ($)	0	1	2	3	4	5
Probability	1/6	7/24	1/4	1/6	1/12	1/24

(c) $E(\text{Amount won}) = 0\,(1/6) + 1\,(7/24) + 2\,(1/4) + 3\,(1/6) + 4\,(1/12) + 5\,(1/24) = 1.83.$

(d) The stallholder charges $2.50 for each play and, on average, the participant wins $1.83. So, on average, the stallholder makes 2.50 – 1.83 = $0.67 per play of the game. So, in 400 plays, the stallholder can expect to make 400 (0.67) = $268.

Question Three

(a) She could number the hospitals 1–439, and then use the computer's random number generator to select 40 different numbers in that range. The 40 hospitals with these numbers would form the sample.

(b) Randomly select 10 urban teaching hospitals, 10 urban non-teaching hospitals, 10 rural teaching hospitals, and 10 rural non-teaching hospitals. These 40 hospitals form the sample.

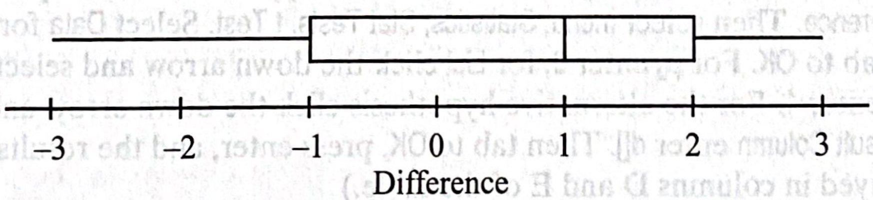
(c) It could be suggested that urban hospitals are more expensive than rural hospitals and teaching hospitals are more expensive than non-teaching hospitals.

Question Four

Student	No Music	Music	Difference (No Music – Music)
1	26	27	1
2	10	7	−3
3	19	20	1
4	25	28	3
5	8	5	−3
6	23	24	1
7	21	24	3
8	13	12	−1
9	9	11	2
10	26	26	0
11	10	12	2
12	25	24	−1

Hypotheses:
Let μ_d be the true mean difference in on-task time (music minus no music).
H_0: $\mu_d = 0$
H_a: $\mu_d > 0$

Check of conditions and naming of test:
This is a paired t-test.
We are told that the students were randomly assigned to the two orders of treatments. Since this is a small sample we consider a boxplot of the differences:



There are no outliers in the differences and the boxplot is close enough to being symmetrical for a sample this small. So we can proceed with the paired t-test.

Mechanics:

$$t=\frac{\bar{d}-\mu_d}{\left(\frac{s_d}{\sqrt{n}}\right)}=\frac{0.417-0}{\left(\frac{2.065}{\sqrt{12}}\right)}=0.699.$$

We use the t-distribution with $12-1=11$ degrees of freedom. So the p-value is $P(t_{11}>0.699)=0.250$.
(This p-value can be obtained on the TI-83, TI-84, and TI-Nspire calculators by entering tcdf(0.699,999,11). The tcdf function is accessed on the TI-83/84 by 2nd,DISTR, and on the TI-Nspire, in the *Scratchpad* or *Calculator* application, tCdf is accessed by selecting menu, Statistics, Distributions.)

Conclusion:
Since the p-value is 0.250, which is greater than 0.05, we do not reject H_0. The results do not provide convincing evidence that the true mean on-task time when music is playing is greater than when there is no music.

(The numerical results of this hypothesis test can be found directly, using a calculator, as follows. On the TI-83 and TI-84 calculators, first select STAT, Edit and enter the scores for last year into L1 and the scores for this year into L2. Go to the home screen and type L2−L1→L3; the differences are then stored in L3. Then select STAT, TESTS, T-Test. For Inpt select Data, for μ_0 enter 0, for List enter L3, and for Freq enter 1. Select $>\mu_0$ for the alternative hypothesis, select Calculate, and press Enter. The results of the test are now displayed.

On the TI-Nspire, from the home screen tab to the Add Lists & Spreadsheet icon. Press enter, and enter the two data sets into columns A and B. Name the lists list1 and list2, respectively. Move to the formula cell for column C, press enter, and then type list2−list1, and press enter. Name column C difference. Then select menu, Statistics, Stat Tests, t Test. Select Data for the data input method and tab to OK. For μ_0 enter 0, for List click the down arrow and select difference. For Frequency List enter 1. For the alternative hypothesis click the down arrow and select Ha: $\mu>\mu_0$, and for 1st Result Column enter d[]. Then tab to OK, press enter, and the results of the hypothesis test are displayed in columns D and E of the table.)

Question Five

(a) The equation of the least squares regression line is $\hat{y} = 7.418 - 0.01670x$, where x = temperature and y = pH.

(b) The predicted pH when $x = 30$ is $\hat{y} = 7.418 - 0.01670(30) = 6.917$. So the residual for the given observation is $7.08 - 6.917 = 0.163$. The pH for this observation was 0.163 higher than the predicted pH for water at 30°C.

(c) Yes. The p-value for the t-test for the slope of the population regression line is given by the computer output to be 0.007. The computer output always supplies the p-value for a two-tailed test, and the fact that this is less than 0.05 (or any commonly-used significance level) provides us with convincing evidence of a linear relationship between temperature and pH.

SECTION II PART B

Question 6

Question Six

(a) The Central Limit Theorem tells us that if n is large the sampling distribution of $\overline{X}$ is approximately normal.

(b) $\overline{X} = \dfrac{5+3+4+6+1+3+4+3+6+1}{10} = 3.6$

(c) The standard deviation of the sampling distribution of $\overline{X}$ is $\sigma/\sqrt{n}$, where σ is the population standard deviation and n is the sample size. (Here, the population is the set of all possible scores when the die is rolled.) The population standard deviation is given to be 1.708, and $n = 10$. So the standard deviation of the sampling distribution of $\overline{X}$ is $1.708/\sqrt{10} = 0.540$.

(d) $P(\overline{X} \le 3.15) = P\left(Z \le \dfrac{3.15 - 3.5}{0.540}\right) = P(Z \le -0.6481) = 0.2584$. So the number of values of $\overline{X}$ that you would expect to be less than or equal to 3.15 is $0.2584(100) = 25.84$.

(The probability in the calculation above can be found using the TI-83 and TI-84 calculators by entering normalcdf(−999,−0.6481) or normalcdf(−999,−0.6481,0,1) and on the TI-Nspire by entering normalCdf(−999,−0.6481,0,1). The normalcdf function is accessed on the TI-83/84 by 2nd, DISTR, and the normalCdf function is accessed on the TI-Nspire, in the *Scratchpad* or *Calculator* application, by selecting menu, Statistics, Distributions. Alternatively, the answer can be found directly by entering normalcdf(−999,3.15,3.5,0.540).)

(e)
(i) The expected count calculated in part (d) has been written in the table in part (ii).
(ii) $100 - (25.84 + 21.52 + 16.75 + 15.42) = 20.47$

$\overline{X}$	≤ 3.15	$3.15 - 3.45$	$3.45 - 3.75$	$3.75 - 4.05$	≥ 4.05
Observed Count	20	20	28	21	11
Expected Count	25.84	20.47	21.52	16.75	15.42

(f) Since 0.229 is greater than 0.05 (or any commonly used significance level), H_0 is not rejected. The results of the investigation do not provide convincing evidence that the sampling distribution of $\overline{X}$ is not exactly normal.

(g) The sampling distribution of $\overline{X}$ is not exactly normal, so H_0 is false. The correct result of the hypothesis test would therefore be to reject H_0. We have failed to reject H_0 when H_0 is false, and so this is a Type II error.