Hall Effect in 2-Dimensional Electron Gas

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Abstract

Many modern systems rely on the Hall Effect for accurate positioning and non-contact sensing. The effect can be attributed to the Lorentz force acting on charge carriers traveling within a magnetic field. Here, we demonstrated the temperature dependence of the Hall Effect and other physical properties in a 2-dimensional electron gas (2DEG) at 300 K and 77 K. The 2DEG sample consisted of an approximately 400-atom thick layer of Gallium Arsenide which was produced using Molecular Beam Epitaxy (MBE).

1 Introduction

When a charged particle is placed in an electric and magnetic field, which are perpendicular to each other, it will experience a Lorentz force. This Lorentz Force will deflect the particle perpendicular to both the magnetic field direction and the particle's original direction of motion. Knowing this principle we can now imagine an electric field across a semiconductor and a magnetic field pointing into the conductor. As the free electrons in the conductor begin to move due to the electric field, they will be deflected by Lorentz Force due to the magnetic field. This force will produce a transverse voltage due to the deflection of electrons to one side of the semiconductor. This transverse voltage, known as the Hall voltage, is directly proportional to both the strength of the magnetic field and the current flowing through the conductor, while inversely proportional to the thickness of the conductor.

The Hall Effect, first discovered by Edwin Hall in 1879 eighteen years before electrons were discovered, was monumental in understanding electromagnetism and its applications in modern science and technology. The Hall Effect has applications in various fields, including semiconductor physics, material science, and engineering. Its utility ranges from precise measurements of magnetic fields and currents to developing devices such as Hall sensors, magnetic field probes, and magnetometers.

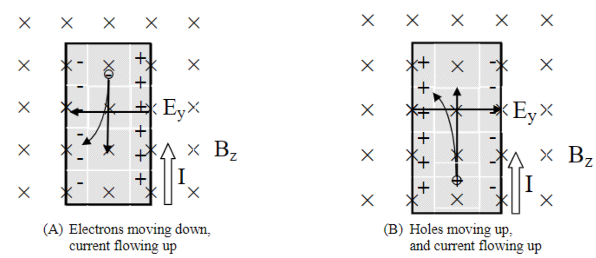

Figure 1: Hall Effect for electron and holes

1.1 In this paper

In our experiment, we used a semiconductor containing two-dimensional electron gas (abbreviated as 2DEG). This semiconductor was placed in between an electromagnet and attached to a current source so an electric field could be generated across the sample. The voltage is measured at room temperature (300 K) and when submerged in liquid nitrogen (77 K). The main purpose of this experiment is to understand standard DC resistivity and the Hall Effect. We calculated the sheet resistance of the semiconductor and the Hall coefficient. We determined the mean free path, the average distance between collisions, and relaxation time, the average time between collisions. Finally, we computed the maximum velocity of an electron in the electron gas, the Fermi velocity, and the maximum energy difference between states, the Fermi energy, both at absolute zero.

1.2 Review of Previous Work

Before the discovery of the Hall Effect, Edwin Hall was researching electromagnetism through the texts of Erik Edlund, and James Clerk Maxwell. Ellund stated that the way a magnetic force acts upon a current in a conductor wouldn't change if the conductor is fixed or free to move. However, Maxwell argued that current moves per a magnetic force when current exists in a fixed conductor but when it is free to move, the magnetic force does not affect its motion; rather, the conductor is affected by the magnetic force rather than the current.

Hall saw that these ideologies contradicted each other and sought to run an experiment himself. Hall used an electromagnet with two poles between it to mount a gold leaf on a glass plate as part of a current. A galvanometer's poles were positioned around the gold leaf, and the current was passed through them until the equipotential was reached. The magnetic force was then applied, and the galvanometer was checked for any notable changes. This experiment led Hall to the conclusion that a magnetic force did, in fact, act upon a current. A Lorentz force will cause current traveling perpendicular to a magnetic field to be deflected from its original path and toward the direction perpendicular to the vector of the magnetic field.

2 Theory

2.1 Electron Gas

Free electrons in a conductor can be thought of as an electron gas on a macroscopic level, where the negatively charged gas moves in relation to the positively charged lattice of atoms that forms the conductor, which is static [1]. The system is electrically neutral and although there is motion in the gas, the total momentum of the system is 0. Devoid of any electrical field the electrons' motion is random and at any moment the average of all their velocities is zero. Even at absolute zero, the free electrons in the gas will still have an intrinsic velocity. The maximum velocity of one of these electrons is the Fermi velocity. The Fermi velocity is a result of the Pauli Exclusion Principle, which states that no more than one electron can occupy any given state. This leads to electrons filling all of the lowest available energy states, leaving a packed group of filled energy states. The highest energy of the group is known as the Fermi Level (or Fermi Energy at absolute zero) [2].

2.2 2-Dimensional Electron Gas

Restricting the movement of these free electrons to a plane results in a system known as a 2-dimensional electron gas. The sample utilized in this experiment was made via molecular beam epitaxy and was composed of layers of doped and undoped GaAlAs and GaAs that were grown on a GaAs substrate that was 1 mm thick. [1]. The resulting configuration of the layers results in a potential well as shown in Figure 2. This limits the movement of electrons to the plane at the boundary of layers 1 and 2, thus these electrons can be modeled by a 2DEG.

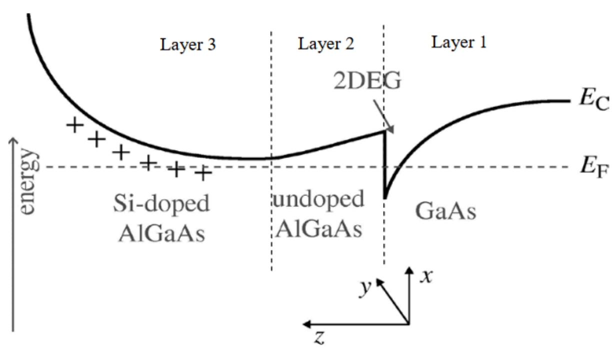

Figure 2: Energy diagram for the conduction band of sample

2.3 Sheet Resistance

Ohm's Law (Eq. 1) tells us that voltage (V) is equal to the current (I) multiplied by the resistance (R).

$$V = IR \tag{1}$$

In a 3-dimensional conductor, we see that $R = \rho\left(\frac{L}{w\delta}\right)$, where ρ is the resistivity of the material, L is the length in the axis of current flow, and w and δ are the width and thickness of the material respectively. If we set $L = w$, we obtain the equation for sheet resistance in 3 dimensions $R_\square = \frac{\rho}{\delta}$, which is the resistance of a square of a material of uniform thickness. However, in 2 dimensions we obtain the following

$$R_\square = \frac{R}{\gamma} \quad (2)$$

where R is the measured resistance across a sample and $\gamma = \frac{L}{w}$ or the number of squares that would fit in a sample as depicted in Figure 3.

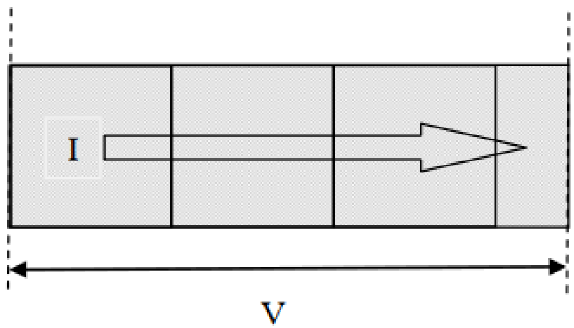

Figure 3: $\gamma = 3.44$, showing the number of squares that would fit into a rectangular area

2.4 Hall Effect

As charge carriers (in our case electrons) move with a velocity v through an electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$, they experience the Lorentz force given by Eq. 3.

$$\boldsymbol{F} = e(\boldsymbol{E} + \boldsymbol{B} \times \boldsymbol{v}) \quad (3)$$

In a 2DEG, where the magnetic field is perpendicular to the surface of the plane, as in Figure 4, electrons will build up on one side of the conductor in the y direction, leaving the other side electron deficient.

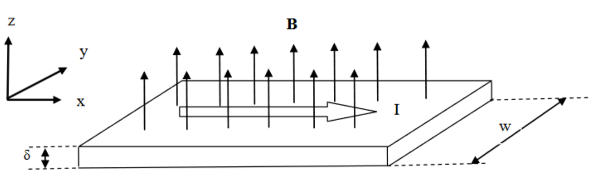

Figure 4: Conductor with uniform magnetic applied perpendicular to its surface

This accumulation will continue until an equilibrium is established, at which point the magnetic field's effect is neutralized by the electric field created by the excess electrons on one side of the wire and the electron shortage on the other. The resulting potential difference, V_H, is Hall voltage and, in 2 dimensions, can be described by

$$\frac{V_H}{I} = R_H B_Z \quad (4)$$

R_H, known as the Hall coefficient can be defined by

$$R_H = \frac{1}{ne} \quad (5)$$

where n is the carrier density of the material and e, the charge of an electron. The carrier density is the number of charge carriers within a unit volume.

2.5 Properties of 2DEG

Carrier mobility, μ, determines the speed at which a charge carrier can move within a material when an electric field is applied to it. It can be calculated by the following equation

$$\mu = \frac{R_H}{R_\square} \quad (6)$$

The relaxation time, τ, is the average amount of time between collisions for charge carriers and is described by Eq. 7, where m^* is the effective mass of an electron within our material which is equal to $0.067 \times m$ (the mass of the electron).

$$\tau = \frac{\mu m^*}{e} \quad (7)$$

The Fermi velocity, v_F, is the maximum velocity of a charge carrier at 0 K, and can be found with

$$v_F = \frac{2\pi\hbar\sqrt{n}}{m^*} \quad (8)$$

The corresponding Fermi Energy would be

$$E_F = \frac{m^* v_F^2}{2} \quad (9)$$

Lastly, the mean free path, l, also abbreviated mpf, which defines the average distance between collisions is

$$l = v_F \tau \quad (10)$$

3 Experimental Setup

3.1 General Setup

The apparatus for our experiment is shown in Figure 5. The 2DEG semiconductor sample is placed in a glass dewar. The sample is placed at the bottom of the dewar and wires running along a metallic strip connect the sample to the four-probe spine. The glass dewar is sandwiched by two electromagnets. To ensure that the magnetic field is uniform at the sample the electromagnets are placed relatively close together. The magnets by themselves have a magnetic field of 0.073 kG and a maximum magnetic field strength of around 1.539 kG. Measurements of the magnetic field are taken using a Gauss meter (the blue probe in Figure 5), making sure that the plane of the probe is perpendicular to the magnetic field for an accurate reading. The sample should also be perpendicular to the magnetic field. When taking measurements at 77 K, we poured liquid nitrogen into the dewar so that the sample is fully submerged. The dewar was filled to the top to provide enough time to collect data, before having to refill the container.

3.2 Sample Composition

The GaAs semiconducor sample is composed of several layers stacked on top of each other. These layers are a GaAs substrate (1 mm), a GaAs undoped layer (1 μm), a GaAlAs undoped layer (20 nm), a GaAlAs doped layer (60 nm), and a GaAs cap layer (10 nm). Between the GaAs undoped layer and the GaAlAs undoped layer is a few atomic layers of the 2DEG.

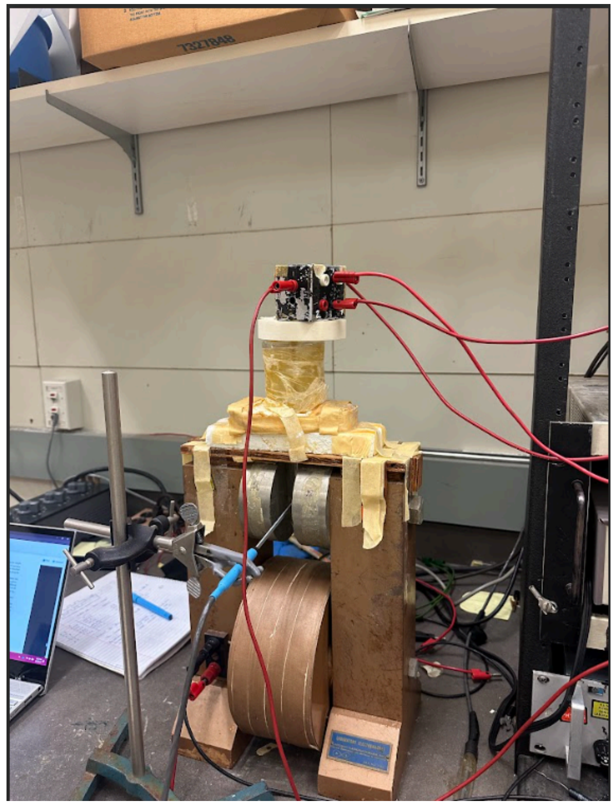

Figure 5: Experimental apparatus. Metal cylinders are the electromagnets with the glass dewar in between. Blue probe is a Gauss meter

3.3 Four Probe Method

Typical measurements of resistance are often done by measuring both the current through the resistor and the voltage across the resistor. This a two-terminal measurement as we measure the voltage drop across two points, normally before and after the resistor. It assumed that the contact resistance, the resistance created from the pins on the circuit, and the resistance of the wires are negligible in comparison to the resistance of the resistor. In some cases, including our own, contact resistance and lead resistance can't be ignored. This can be solved with the four-probe method; used frequently across solid-state physics to measure resistivity (ρ) or sheet resistance ($R_{\square}$) of materials.

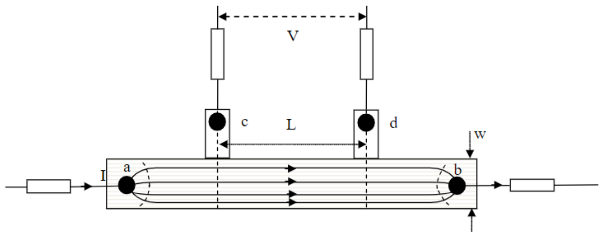

Figure 6: Four-Probe Measurement

In order to properly use the four-probe method the material must be delineated into a particular geometric shape. The shape is a long spine with fingers protruding out all with a uniform thickness, see Figure 6 for a better understanding. On the spine, there will be two electrical contacts, one at the front and one at the back. There will also be electrical contacts on each finger. A known current I will flow through along the contacts of the spine; and the potential drop V across the contacts on the fingers will be measured. The fingers must be relatively long to not disturb the flow of current.

4 Measurements and Results

As a result of problems with our sample (shown in Figure 7), we were unable to obtain useful data for any measurements made on pins 3, 4, 6, and 7. As a result, measurements of sheet resistance and Hall voltage were made between pins 2 and 8 and between pins 2 and 3. We belive that the sample appears to have developed cracks and exhibits a thermoelectric effect across many pin pairs.

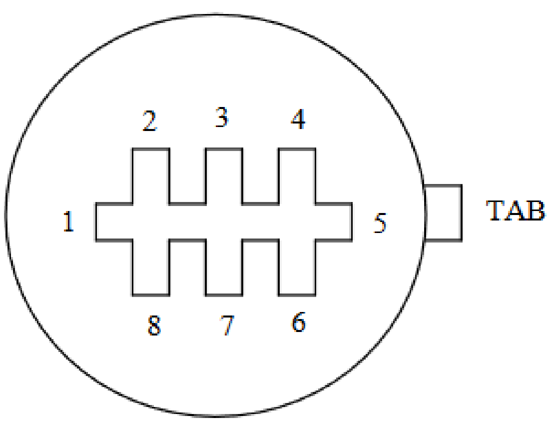

Figure 7: Schematic of sample with pin numbering

4.1 Sheet Resistance

Using the 4-probe method to measure the voltage drop across pins 2 and 3 while varying the current between pins 1 and 5 we observed the following in a 70 G magnetic field perpendicular to the 2DEG plane surface:

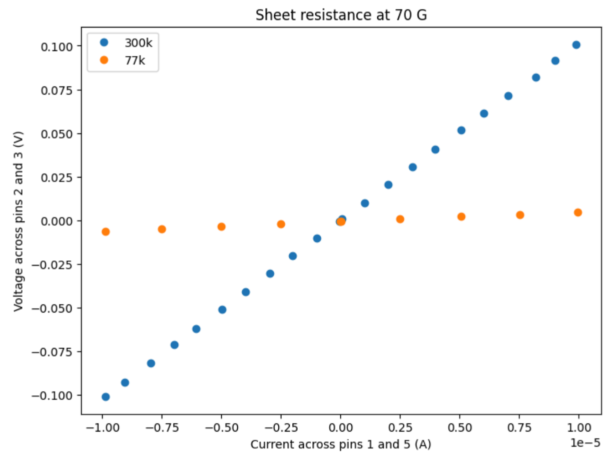

Figure 8: Sheet resistance measurements at 70 G

When increasing the magnetic field to 1542 G we collected data shown in Figure 9.

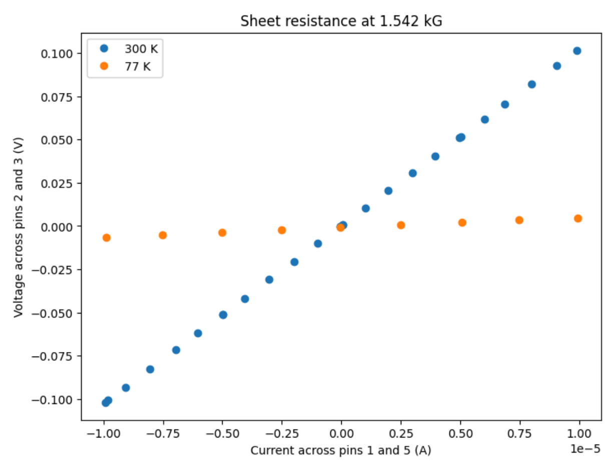

Figure 9: Sheet resistance measurements at 1542 G

In both cases, we varied the current on pins 1 and 5 between −10 uA and 10 uA.

Our sample indicated negligible signs of magnetoresistance, as indicated by Figure 10 and Figure 11. Due to the proximity of the lines of best fit and their similarity in their slopes, we know that there was only one type of charge carrier in our sample. A compass may have been used to identify the magnetic field's orientation and establish whether the carrier type was electrons or holes. Unfortunately, since we don't have a compass in the lab, we rely on earlier studies that showed the charge carriers in GaAs are, in fact, electrons.

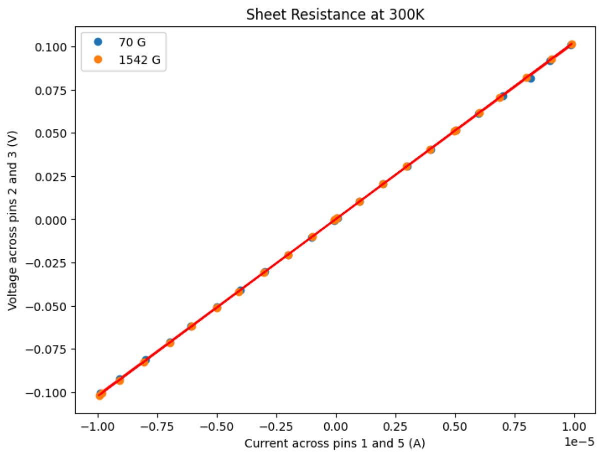

Figure 10: Plot of sheet resistance measurements at 300 K

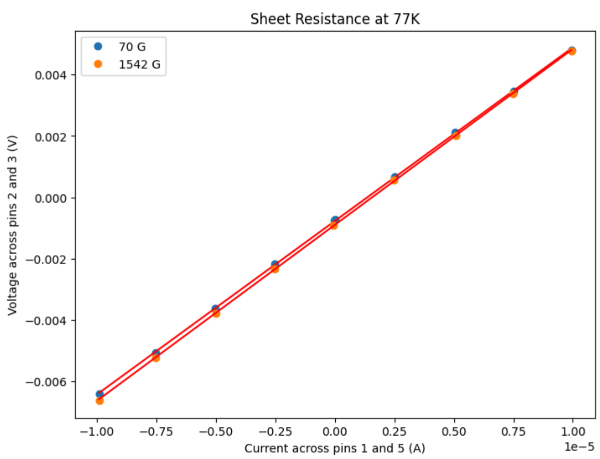

Figure 11: Plot of sheet resistance measurements at 77 K

As shown in Figure 12 we had a strong linear relationship at both 300 K and 77 K.

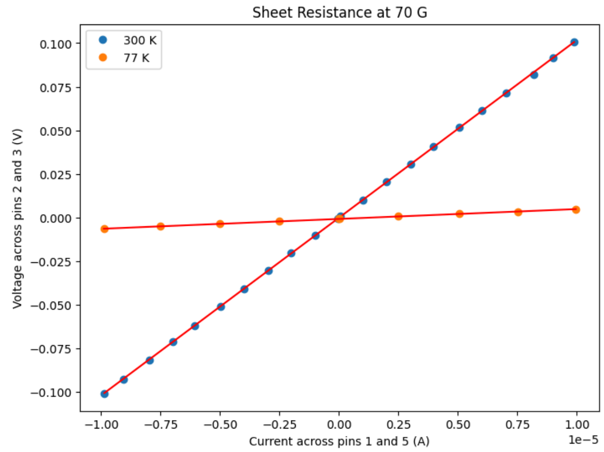

Figure 12: Plot of sheet resistance measurements at 300 K and 77 K at 70 G

4.2 Hall Coefficient

To measure the Hall Coefficient, R_H, the current between pins 1 and 5 was kept constant at about 10 μA, which is the limit of how much current we can pass through the sample without damaging it. The magnetic field strength perpendicular to the 2DEG plane was varied between 70 G and 1542 G to obtain the data in Figure 13.

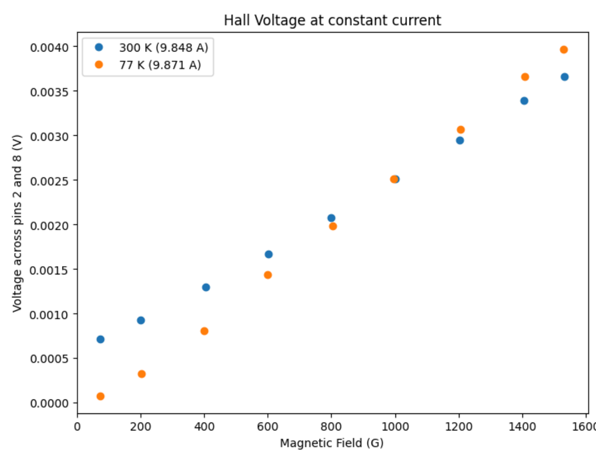

Figure 13: Hall voltage measurements at ~10uA

Figure 14 shows the linear relationship between the magnetic field strength and the measured Hall voltage at both 300 K and 77 K.

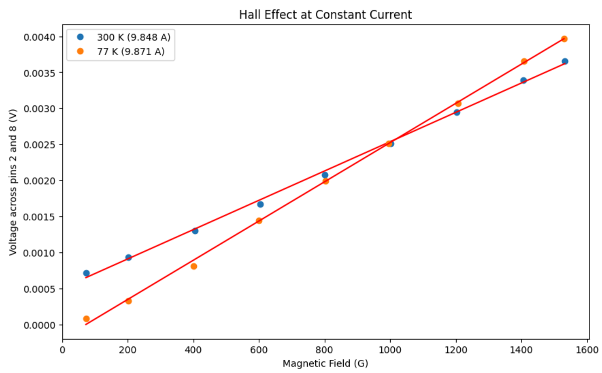

Figure 14: Plot of hall voltage at 300 K and 77 K at constant current

γ from Eq. 2 was determined by counting the number of pixels between pins 2 and 3 and pins 2 and 8 in the image of the sample depicted in Figure 15. Using the provided scale on the lower right of the image it was determined that $L = 496$ μm and $w = 101$ μm, giving us a $\gamma = 4.91 \pm 0.02$.

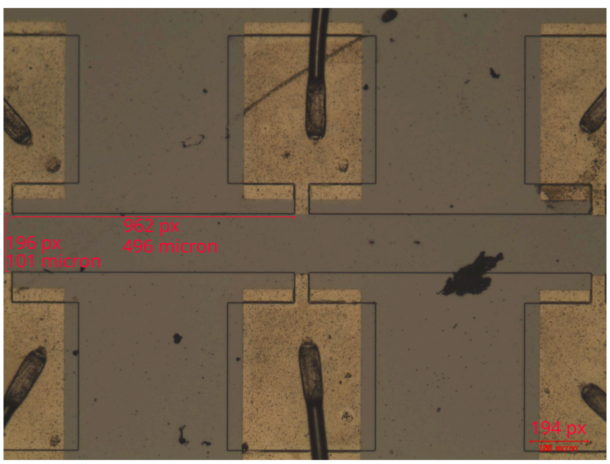

Figure 15: Sample and contact pads

4.3 Results

The following table contains the calculated values we found for the sample:

	300 K	**77 K**
Sheet Resistance $R_\square$ ($\frac{\Omega}{\square}$)	2070±10	120±10
Hall Coefficient R_H ($\frac{\Omega}{T}$)	2070±20	2720±20
Mobility μ ($\frac{m^2}{Vs}$)	1.00±0.01	23±2
Carrier Density n ($\frac{10^{15}}{m^2}$)	3.02±0.03	2.29±0.02
Relaxation Time τ ($10^{-12}s$)	0.381±.004	9.0±0.8
Fermi Velocity v_F ($10^5\frac{m}{s}$)	5.97±0.03	5.20±0.2
Fermi Energy E_F ($10^{-20}J$)	1.09±.01	0.824±0.007
Mean Free Path l ($10^{-6}m$)	0.227±.003	4.7±0.5

The increase of mobility as temperature decreased was an expected outcome of the experiment, and the carrier density decreased with temperature decreasing. Since the Fermi velocity was proportional to the carrier density, as shown in Eq. 8, the result of Fermi velocity and Fermi energy decreasing as the temperature decreased also was expected.

5 Conclusion

Throughout our experiment, we explored the properties of the 2-dimensional electron gases, such as carrier mobility, carrier density, Fermi velocity, and Fermi Energy. We used the materials sheet resistance and Hall effect voltage to calculate these values. We saw a linear correlation between current and voltage for the sheet resistance at different temperatures and different magnetic field strengths. We noticed a significant decrease in the sheet resistance due to temperature change. On the other hand, we saw almost no change in sheet resistance when deferring the magnetic field strength, indicating there was little to no magnetoresistance present in the material. We also noticed a linear relationship between the Hall voltage and the current; as well as in the Hall voltage and the magnetic field. Potential sources of error in our experiment could have stemmed from the inability to orient our sample by rotating along the z-axis. Due to this we could not ensure that the sample was perpendicular to the magnetic field in this axis. Future areas of study could explore the Quantum Hall Effect in 2DEG.

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