1. 1D Motion

Note: The little arrow on top of a letter means vector.

- A particle's position in one dimension is given by the ~~vector~~ $\vec{x}$ vector.
- Displacement is the change in position over some interval:

$$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$$

f=final i=initial $\Delta\vec{x}$ = displacement

Note: In one dimension, a vector's direction is given by positive or negative sign.

- The average velocity of a particle $\vec{v}_{avg} = \frac{\Delta\vec{x}}{\Delta t}$
- As $\lim_{\Delta t \to 0} \vec{v}_{avg} = \frac{dx}{dt}$ = instantaneous speed of a particle
- Average acceleration of a particle is

$$\vec{a}_{avg} = \frac{\Delta v}{\Delta t}, \quad \vec{a} = \lim_{\Delta t \to 0} \vec{a}_{avg} = \frac{dv}{dt}$$

> Ex. problem: A jet plane lands at 63 m/s, what is its acceleration (assume constant) if it stops in two seconds?
>
> $$\vec{a}_{avg} = \frac{\Delta v}{\Delta t} = \frac{0\,m/s - 63\,m/s}{2\,s} = -31.5\,m/s$$

Certain equations follow when a particle undergoes constant acceleration:

1) $v_f = v_i + at$ OMG This looks like average velocity ;)
2) $\Delta x = \frac{1}{2}(v_i + v_f)t$
3) $\Delta x = v_i t + \frac{1}{2}at^2$
4) $v_f^2 = v_i^2 + 2a \cdot \Delta x$

Try to prove these yourself but if stuck check next page!

You will need to memorize these!!!

Proofs for Kinematics Equations (Acceleration Constant)

1) $a = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t - 0}$

$V_f - V_i = at \qquad V_f = V_i + at$

2) For constant acceleration, velocity $\propto t$ thus average velocity is just average of initial and final velocities.

Note: variable with line on top indicates average.

$\bar{V} = \frac{1}{2}(V_i + V_f)$

$x_f - x_i = \bar{V} t = \frac{1}{2}(V_i + V_f) t$

3) Just sub equation one into equation two.

$\Delta x = \frac{1}{2}(V_i + (V_i + at))t$

$\Delta x = V_i t + \frac{1}{2} a t^2$

4) Sub t from Eq. 1 into Eq. 2

$x_f = x_i + \frac{1}{2}(V_i + V_f)\left(\frac{V_f - V_i}{a}\right)$

$\Delta x = \frac{V_f^2 - V_i^2}{2a}$

$V_f^2 = V_i^2 + 2a\Delta x$

Ex problem: A boy drops a ball from 10 meters Gravity acts as $g = -10 m/s^2$ How long does the ball take to hit the ground?

$V_i = 0$, $\Delta x = -10$ meters
$g = 10 m/s^2$

$\Delta x = V_i t + \frac{1}{2} g t^2$

$-10 = 0 \cdot t - \frac{1}{2} \cdot 10 \cdot t^2$

$-10 = -5t^2$

$t^2 = 2 \quad t = \sqrt{2}$ seconds