1. 1D Motion

Note:
The little arrows on top of a letter means vector.

- A particle's position in one dimension is given by the ~~vector~~ $\vec{x}$ (vector).
- Displacement is the change in position over some interval:

$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$ f=final i=initial $\Delta\vec{x}$ = displacement

Note: In one dimension, a vector's direction is given by positive or negative sign.

- The average velocity of a particle $\vec{v}_{avg} = \dfrac{\Delta\vec{x}}{\Delta t}$
- As $\lim_{\Delta t \to 0} \vec{v}_{avg} = \dfrac{dx}{dt}$ = instantaneous speed of a particle
- Average acceleration of a particle is

$$\vec{a}_{avg} = \frac{\Delta v}{\Delta t}, \quad \vec{a} = \lim_{\Delta t \to 0} \vec{a}_{avg} = \frac{dv}{dt}$$

$$\boxed{\begin{array}{l}\text{Ex. problem: A jet plane lands at 63 m/s, what is its acceleration (assume constant)} \\ \text{if it stops in two seconds?} \\ \vec{a}_{avg} = \dfrac{\Delta v}{\Delta t} = \dfrac{0\,m/s - 63\,m/s}{2\,s} = -31.5\,m/s\end{array}}$$

Certain equations follow when a particle undergoes constant acceleration:

1) $V_f = V_i + at$

2) $\Delta x = \underbrace{\tfrac{1}{2}(V_i + V_f)}_{\text{OMG This looks like avergage velocity ;)}} t$

3) $\Delta x = V_i t + \frac{1}{2}at^2$

4) $V_f^2 = V_i^2 + 2a \cdot \Delta x$

Try to prove these yourself but if stuck check next page!

You will need to memorize these!!!

Proofs for Kinematics Equations (Acceleration Constant)

1) $a = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t - 0}$

$V_f - V_i = at$ $\quad V_f = V_i + at$

2) For constant acceleration, velocity $\propto t$ thus average velocity is just average of initial and final velocities.

Note: variable with line on top indicates average.

$\bar{V} = \frac{1}{2}(V_i + V_f)$

$x_f - x_i = \bar{V} t = \frac{1}{2}(V_i + V_f) t$

3) Just sub equation one into equation two.

$\Delta x = \frac{1}{2}(V_i + (V_i + at)) t$

$\Delta x = V_i t + \frac{1}{2} a t^2$

4) Sub t from Eq. 1 into Eq. 2

$x_f = x_i + \frac{1}{2}(V_i + V_f)\left(\frac{V_f - V_i}{a}\right)$

$\Delta x = \frac{V_f^2 - V_i^2}{2a}$

$V_f^2 = V_i^2 + 2a\Delta x$

Ex problem: A boy drops a ball from 10 meters. Gravity acts as $g = -10 m/s^2$. How long does the ball take to hit the ground?

$V_i = 0$, $\Delta x = -10$ meters
$g = 10 m/s^2$

$\Delta x = V_i t + \frac{1}{2} g t^2$

$-10 = 0 \cdot t - \frac{1}{2} \cdot 10 \cdot t^2$

$-10 = -5t^2$

$t^2 = 2 \quad t = \sqrt{2}$ seconds

1. Dimensional Analysis

1. A particle is thrown into the air from ground level height $h = 0$ with upward velocity v. Which of the following gives the height of the particle at time t from launch?
   a. $h(t) = v \cdot t - \frac{1}{2}gt$
   b. $h(t) = v \cdot t^2 - \frac{1}{2}gt$
   c. $h(t) = v \cdot t - \frac{1}{2}gt^2$
   d. $h(t) = v \cdot t^2 - \frac{1}{2}gt^2$
2. Newton's second law of motion states $F = ma$, what are the dimensions of F (force) in basic dimensions?
3. Challenge question! The universal law of gravitation states $F_{gravity} = G\frac{m_1 m_2}{R^2}$, where $F_{gravity}$ is a force, and m_1 and m_2 are masses, and R is a distance. What are the units of the gravitational constant G?
4. Super extra bonus challenge question! (From a past F=ma exam but it's really easy and you should be able to do it)

Inspired by a problem from the 2012 International Physics Olympiad, Estonia.

A very large number of small particles forms a spherical cloud. Initially they are at rest, have uniform mass density per unit volume ρ_0, and occupy a region of radius r_0. The cloud collapses due to gravitation; the particles do not interact with each other in any other way.

How much time passes until the cloud collapses fully? (The constant 0.5427 is actually $\sqrt{\frac{3\pi}{32}}$.)

(A) $\frac{0.5427}{{r_0}^2\sqrt{G\rho_0}}$

(B) $\frac{0.5427}{r_0\sqrt{G\rho_0}}$

(C) $\frac{0.5427}{\sqrt{r_0}\sqrt{G\rho_0}}$

(D) $\frac{0.5427}{\sqrt{G\rho_0}}$

(E) $\frac{0.5427}{\sqrt{G\rho_0}}r_0$

(Use the backside if you need to for this one)

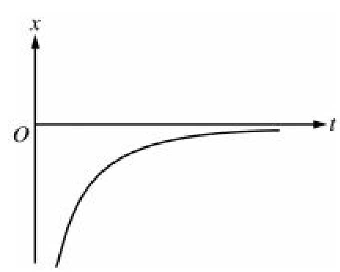
2. Dimensional Analysis

1. A boy throws a ball upward from the ground ($h = 0m$) with velocity $v = 10m/s$ and acceleration due to gravity $g = -10m/s^2$
    a. Calculate the time it takes for the ball to get to the highest point of its motion.

    b. Calculate the maximum height the ball reaches.

    c. Calculate the velocity of the ball once it hits the ground again.

2. A boy stands on the edge of a building. He throws a ball upward with velocity $v = 10m/s$ and the ball hits the ground after 10 seconds. How high is the cliff?
3. Some easy money AP test questions. Assume g = 9.8 m/s^2 for these question.



The position x as a function of time t for an object moving in a straight line is shown in the graph above. Which of the following best describes the object's speed and direction of motion during the time interval shown?

Speed	Direction of Motion
(A) Decreasing	Positive
(B) Increasing	Positive
(C) Constant	Positive
(D) Decreasing	Negative
(E) Increasing	Negative

Which of the following statements must be true for a falling object that has been dropped from rest near the surface of Earth?

(A) The derivative of the distance the object falls with respect to time equals $9.8\ \text{m/s}^2$.
(B) The object falls a vertical distance of 9.8 m during the first second only.
(C) The object falls a vertical distance of 9.8 m during each second.
(D) The speed of the object as it falls is a constant $9.8\ \text{m/s}$.
(E) The speed of the object increases by $9.8\ \text{m/s}$ during each second.

The position x of an object is given as a function of time t by the equation $x = 8 + 4t - 6t^3$, where x is in meters and t is in seconds. What is the maximum positive velocity attained by this object?

(A) $4\ \text{m/s}$
(B) $8\ \text{m/s}$
(C) $18\ \text{m/s}$
(D) $36\ \text{m/s}$
(E) There is no maximum positive velocity because the object never moves in the positive direction.

4. Challenge question (it's p hard lol dw if you can't do it [first question on an old f=ma test])

An observer stands on the side of the front of a stationary train. When the train starts moving with constant acceleration, it takes 5 seconds for the first car to pass the observer. How long will it take for the 10th car to pass?

(A) 1.07s

(B) 0.98s

(C) 0.91s

(D) 0.86s

(E) 0.81s