

To determine the number of clauses in terms of the number of vertices (n) and the number of edges (m) of graph G , we need to consider two types of clauses generated by the CNF expression for the 3-colouring problem:

Vertex-colouring constraints: Each vertex has a clause specifying that it must be assigned exactly one colour and another clause that ensures it cannot be assigned all three colours simultaneously. This results in $2 * n$ clauses.

Adjacent vertices having different colours: For each edge in the graph, we have clauses that ensure the adjacent vertices have different colours. Since there are m edges in the graph, this results in $3 * m$ clauses.

An expression for the number of clauses of ϕ_G , after adding these two types of clauses together in terms of n (the number of vertices of G) and m (the number of edges of G), the total number of clauses (C) in terms of n and m is:

$$C = 2 * n + 3 * m$$

For graph G , the number of vertices (n) is 4, and the number of edges (m) is 5. Substituting these values into the formula:

$$C = 2 * 4 + 3 * 5 = 8 + 15 = 23$$

So, the expression for the number of clauses in terms of n and m is **$2n + 3m$** , which in this case is 23.

This matches the number of clauses in the output given by the awk script from problem 2.