Challenge 12: Bonus — Commuting Pauli Phase Program

In quantum simulation and fault-tolerant compilation, one often needs to implement products of small-angle exponentials of Pauli strings. In this task, you are given a dense list of commuting Pauli strings with fixed $(\pi/8)$-quantized angles, and you must compile the resulting unitary into Clifford+T with as few T gates as possible.

Input

You are given a JSON file `challenge12.json` with the following fields:

- **n**: The number of qubits.
- **angle_unit**: The string `"pi/8"`.
- **terms**: A list of objects of the form:
  - `pauli`: A length-n string over $\{I, X, Y, Z\}$.
  - `k`: An odd integer (in this instance, $k \in \{1, 7\}$).

  Interpret each term $(\texttt{pauli}_j, k_j)$ as an n-qubit Pauli operator P_j and define the target unitary:

$$U := \prod_{j=1}^{m} \exp\left(-i\frac{\pi}{8} k_j P_j\right)$$

Important: In this instance, all P_j commute pairwise, so the product is order-independent and is an exact unitary (this is not a Trotterization problem).

Mapping from string to operator: If `pauli[i]` is the i-th character, it denotes the single-qubit Pauli acting on qubit i, tensored across all qubits.
Example (n=4): `"XIZY"` means $X \otimes I \otimes Z \otimes Y$.

Submission Format

Save your solution as a plaintext OpenQASM but don't upload it to the website. Include it along with your writeup. Your circuit must use **only** the gate set:

$$\{H,\ T,\ T^\dagger,\ \text{CNOT}\}$$

Submissions containing other gates will be rejected.

Hints

- Since all terms commute, you can treat them as a single structured object rather than unrelated gates.
- Recall: Clifford gates conjugate Pauli operators to Pauli operators (e.g., $HXH = Z, HZH = X$, etc.).
- Looking for a transformation that makes many terms "simpler" at once can pay off.