

Theorem: suppose $f : [a, b] \rightarrow \mathbb{R}$ and suppose f has either a local maximum or a local minimum at $x \in (a, b)$. if f is differentiable at x then

$$f'(x) = 0$$

proof: suppose f has a local maximum at $x \in (a, b)$. For small (enough) h , $f(x + h) \leq f(x)$.

if $h > 0$ then

$$\frac{f(x + h) - f(x)}{h} \leq 0$$

similarly, if $h < 0$, then

$$\frac{f(x + h) - f(x)}{h} \geq 0$$

By elementary properties of the limit, it follows that $f'(x) = 0$