Assignment - 6
Mathematical Methods
Practice Problems

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Q1. Check whether $x=0$ is an ordinary/singular point of the differential equation

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1-x^2) = 0$$

and find power series solution of the ODE.

Ans: $y = ax\left[1+\frac{x^2}{2.5}+\frac{x^4}{2.4.5.9}+\cdots\right] + bx^{1/2}\left[1+\frac{x^2}{2.3}+\frac{x^4}{2.4.3.7}+\cdots\right]$

Q2. Transform the equation $\frac{d^2y}{dx^2} - y = 0$ by the substitution $x = \frac{1}{z}$ and show that the power series method / Frobenious method fails to find an ascending power series solution in z about $z=0$ for the transformed equation. Why does the method fail?

Ans: $z^4\frac{d^2y}{dz^2} + 2z^3\frac{dy}{dz} - y = 0$

$z=0$ is not a regular singular point.

Q3. Prove that (i) $P_n(0) = 0$ for n odd

and (ii) $P_n(0) = \dfrac{(-1)^{n/2}\, n!}{2^n\left\{\left(\frac{n}{2}\right)!\right\}^2}$ for n even

Q4. Use the formula $P_n(x) = \dfrac{1}{2^n n!}\dfrac{d^n}{dx^n}(x^2-1)^n$

to prove

(i) $\int_{-1}^{+1} P_n(x)\,dx = 0 \qquad n \neq 0$

(ii) $\int_{-1}^{+1} P_0(x)\,dx = 2$

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