Assignment 5
Mathematical Methods

Koeli Ghoshal

1. Show that

$P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}(3x^2 - 1)$,
$P_3(x) = \frac{1}{2}(5x^3 - 3x)$ $P_4(x) = (35x^4 - 30x^2 + 3)/8$

[Hint: Start from $\sum_{n=0}^{\infty} h^n P_n(x) = \{1 - h(2x - h)\}^{-1/2}$
Expand and compare like powers of h]

2. Prove that $P = x^4 + 2x^3 + 2x^2 - x - 3$ can be expressed in terms of Legendre's polynomials as
$$P = \frac{8}{35} P_4(x) + \frac{4}{5} P_3(x) + \frac{40}{21} P_2(x) + \frac{1}{5} P_1(x) - \frac{224}{105} P_0(x)$$

3. Use the recurrence formula $(2n+1) P_n = P'_{n+1} - P'_{n-1}$ to prove that
$$P'_{n+1} + P'_n = P_0 + 3P_1 + 5P_2 + \cdots + (2n+1) P_n$$

4. Prove that $\int_{-1}^{+1} (1 - x^2) P_m' P_n' \, dx = 0$ where m and n are distinct positive integers.

5. Use the recurrence formula $(2n+1) x P_n = (n+1) P_{n+1} + n P_{n-1}$ prove that $\int_{-1}^{+1} x^2 P_{n+1} P_{n-1} \, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$.

6. Show that $\frac{1 - h^2}{(1 - 2xh + h^2)^{3/2}} = \sum_{n=0}^{\infty} (2n+1) P_n(x) h^n$

7. Show that $P_n(1) = 1$

* * * The End * * *