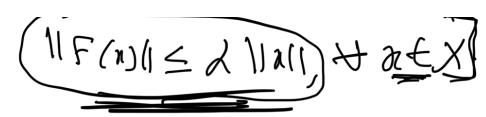
06/09/2020 det: A linear map F: X-J> is Said to be bounded if there exists m > 0 feech that When x 2 y are h.l.g MFM/12X, HREX Supporte Eine Connect At, de la we court à large chorth to that E _ U(o,r) Then Fin who bonder as E Also we know That Fig bounded on U(Ox) iff 7 × 250 3



Theorem: let X and Y be n.l.s and F: X —) Y be a linear mar. They F is bounded if F F mary bounded lety in X to bounded lety in X to bounded lety in Y.

Proof: let F be bounded.

Then there exists on >0 feech that

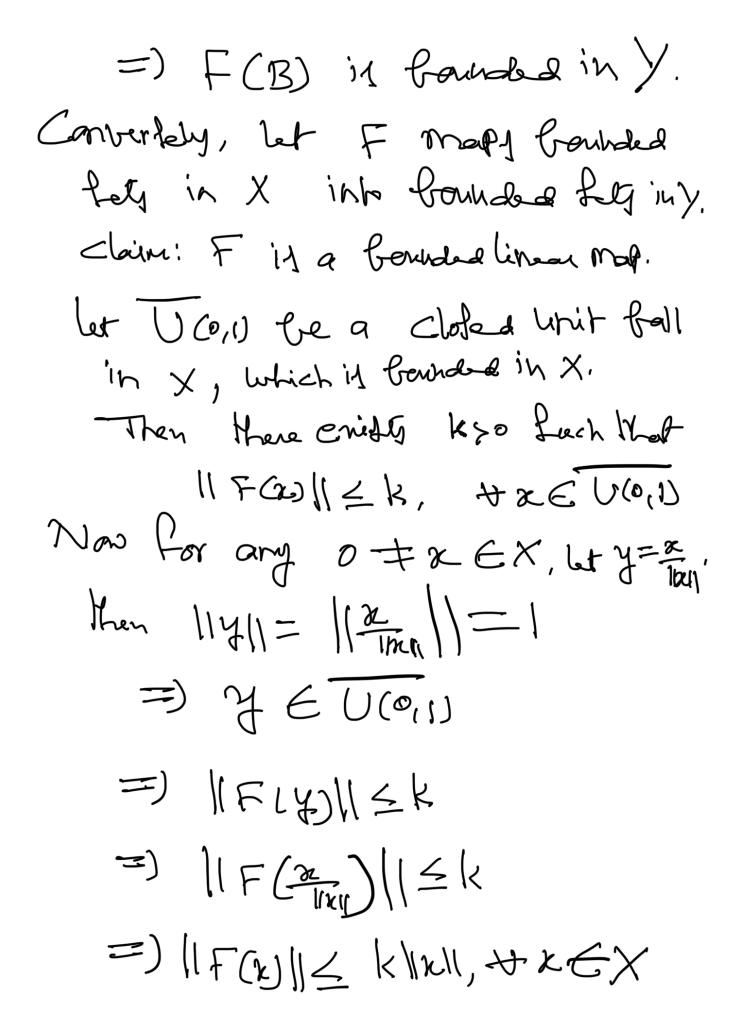
IF Gy (1 & m || 211, + 2 E X.

let B= U(o,r) be a bounded fet in X

Then REB=] ||NI(<Y.

Then from O, we have IF Ca) | \le mo, tres

=) 11F/2011 5 Mr 20



=) Fil a Coulded linear map L: V(O,1)=Bigar founde Let Thu F(B) if bounded iny => + Kgo > ||F(v)| \leq k
HREB] of F: X and Y be n.R.J.

9/ F: X -> Y is continue only linear map, then it is uniformly Continuony. Proof: 3/ F is Continuous at the origin. :. Given E70, 7 1) >0) 112114d) => 11FM 11 < E Heha for any UEX, replacing

& by &- 4 in O, we fet

112-4(12) => 11Fa-41/2E i.e., IIFaJ-Fay/(LE Line & in independent of UEX, it follows that F is uniformly continuous on X. Combining above cell the reput we have the following theorem: Theoren: let X and Y be n.l. Jand F: X-Jy be a linear map. They the following are equivalent: (i) Fis Continuous at the origin F is continuous at every exex. (iii) F is whifamy continuous on x.

(14) There enisty 250 Luch Mat 11 FGW11 = 211211, 4 26x. (V) {F(2)/1/211=1,2EX | is a bounded det in y. (Vi) For every boundard for ESX, the fet F(E)= {FGU/2EE} is bounded in y. Theorem: Let X and Y be h. l. I and F:X-) y le a linear mont. let

F:X—Jy be a linear month let

Z(F) be a hull space of X.

Then F is Continuory if f

Z(F) is closed in X and

F: Zif —) Y define by

F(n+Z(F)) = F(n) is Continuony. Proof: let F be continuent on X. Then Z(F) = FLOY is closed in X, line folis a Cloted fet in y. Polet $\alpha_n \in Z(F) \rightarrow \alpha_n - 12 \text{ in } X$. The F(20 20 1 also of Fil Continuous, and \Rightarrow $F(x_{4}) \longrightarrow F(x_{5})$ =) F(x)=0=) 2EZ(F). : Z(F) is a lolea

:. X in a h.l.s.
Z(F) Then map F: X -> y be Z(F) delined by F(2+Z(F)) = F(21) + N-EX il a linear map. "F:X-y is continuous, 7 d>0 3 11F(x)|15d11x11, HREX. Now let a EX, z EZ(f). They [[F (2+Z(F))] = ||F(2+Z(F))] = || FGL+2]] ∠ | 12+211 Line above inequality is true for any ZEZ(F), it follows that

11 F (2+2(F)) 1 < 2 inf [10x+211/267(F)] = X 11/2+ Z(F)11/ $=) \stackrel{\sim}{F}: \frac{\chi}{2(F)} \rightarrow$ Continuony. Convertey, let Z(F) is close and F: X) y is Continuoy. claim: F: X - JY is continuous. Confider for any XGX, 11F6y) - 11F(n+Z(F)) < X 112+ 7(F) 11 =) Fis Continuous

