Test on 17.11.2021
Indian Institute of Technology Kharagpur
Department of Mathematics
Subject: Fluid Mechanics (MA40011/MA51003)
No. of Students: 93, Time: 1hr 45 mins., Full Marks: 40

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**Attention:** Answer all Questions. Each question carry 5 marks. Submit the single PDF file of your answerscript on MS Teams (in the assignment created as Test 2). Name the file with your roll number. Mention your name, roll number and course name on the first page of the answerscript.

1. a. The velocity field at a point in fluid is given by $\mathbf{q} = (x/t, y, 0)$. Determine the pathlines.
   b. Determine whether the motion specified by $\mathbf{q} = \alpha(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}), \alpha > 0$ is a possible motion for an incompressible fluid. Find the streamlines.
2. Let $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 2\frac{1}{3}, 0 \leq y \leq 4\frac{8}{9}\}$. Then, for all $(x, y) \in D$, investigate the nature of the fluid flow given by

$$u(x, y) = \frac{ax - by}{x^2 + y^2}, v(x, y) = \frac{ay + bx}{x^2 + y^2} \text{ and } w(x, y) = 0.$$

   Find the velocity potential.

3. A quantity of liquid occupies a length $2l$ of a straight tube of uniform small bore (perforation) under the action of a force to a point in the tube varying as the distance from that point. Determine the motion and the pressure.

4. A circular cylinder is fixed across a stream of velocity $U$ with circulation $k$ round the cylinder. Show that the maximum velocity in the liquid is $2U + \frac{k}{2\pi a}$, where $a$ is the radius of the cylinder.

5. a. The stress tensor at a point $P$ is

$$\sigma_{ij} = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{bmatrix}.$$

   Determine the stress vector on the plane at $P$ whose unit normal is $\mathbf{n} = (2/3, -2/3, 1/3)$.
   b. Determine the principal stresses if the stress tensor at a point is given by

$$\sigma_{ij} = \begin{bmatrix} 5 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

6. A circular cylinder is placed in a uniform stream, find the forces acting on the cylinder.

7. Consider a $2-$dimensional laminar flow of an incompressible fluid with constant viscosity between two parallel plates at a distance $y_0$ apart. We assume that both the plates are horizontal so that $y$ is measured vertically. Let the either of the two surfaces (plates) are moving at a constant velocity and there is also an external pressure gradient present. Then, determine the velocity $\mathbf{q}$ of the fluid.

8. Consider the flow of an incompressible viscous fluid past a thin flat plate which is placed in the direction of a uniform velocity $U_\infty$. Let the leading edge of the plate be at the origin $x = 0$. Discuss the boundary layer flow along the flat plate. Obtain the Blasius solution.

********** End of question paper. Good luck! **********