Assignment 1

Fluid Mechanics (MA51003, MA40011)

Deadline:18.09.21 | Autumn 2021-22 | Max mark: 20

Attempt all questions.

1. Check whether or not the given two-dimensional incompressible flow satisfy the equation of continuity:
   (a) $v_r(r,\theta) = c_1\left(\frac{1}{r^2} - 1\right)\cos\theta$, $v_\theta(r,\theta) = c_1\left(\frac{1}{r^2} + 1\right)\sin\theta$, where c_1 is an arbitrary non-zero constant. $(1\frac{1}{2})$
   (b) $v_r = \frac{c_1}{r^2} + c_2\cos\theta$, $v_\theta = -c_2\cos\theta$ and $v_\phi = 0$, where c_1 and c_2 are arbitrary constants and $r > 0$. $(1\frac{1}{2})$

2. In a three dimensional incompressible flow the velocity components in x and z directions are $u = ax + by^2 + cz^3$ and $w = -ax^2 + by + cxz$. Determine the missing component of velocity direction such that the continuity equation is satisfied. **(3)**

3. (a) From the law of conservation of mass, show that whether the flow fluid represented by $u = 2x - 3yz + \frac{1}{z^2}$ and $v = \frac{1}{x} + e^z$ is a possible velocity field for two-dimensional incompressible fluid flow. $(1\frac{1}{2})$
   (b) Show that the following velocity field is a possible case of irrotaional flow of an incompressible flow $u = \frac{1}{x}, v = e^y + z, w = y + logz$. $(1\frac{1}{2})$

4. If the velocity of an incompressible fluid at the point (x, y, z) is given by $u = \frac{x}{r^3}, v = \frac{y}{r^3}, w = \frac{z}{r^3}$, where $r^2 = x^2 + y^2 + z^2$. Prove that the liquid motion is possible and find out the velocity potential. Also determine the streamlines.**(3)**

5. A two dimensional flow field is given by $\psi = xy$.
   (a) Show that the flow is irrotational. $(\frac{1}{2})$
   (b) Find the velocity potential. $(\frac{1}{2})$
   (c) Verify that ψ and ϕ satisfy the Laplace equation. **(1)**
   (d) Find the streamlines and potential lines. **(1)**

6. Find the stream function of the two dimension motion due to equal sources and an equal sink situated midway between them. **(3)**

7. The velocity field at a point in fluid is given by $q = (xt, \frac{y}{t}, t)$. Obtain the path lines and stream lines. **(2)**

Best wishes!