# Assignment 2 on Fluid Mechanics.

1 (a) Determine the constants L, m and n such that the velocity

$$\vec{q} = \frac{(x+lr)\hat{i} + (y+mr)\hat{j} + (z+nr)\hat{k}}{r(x+r)}, \text{ where}$$

$\vec{r} = \sqrt{x^2+y^2+z^2}$ and $(x,y,z) \neq (0,0,0)$ must satisfy the equation of Continuity for a flow.

(b) Show that the velocity potential

$$\phi(x,y,z) = \frac{a}{2}\left(x^2+y^2-2z^2\right), \; a>0$$

satisfies Laplace's equation. Also determine the stream lines.

(c) For an incompressible flow $u = -\alpha y$, $v = \alpha x$, $w = 0$, show that the surfaces intersecting the stream lines orthogonally exist and are the planes through z-axis, althou the velocity potential does not exist. Discuss the nature of the flow.

2(a). Show that the velocity field

$$u(x,y) = \frac{A(x^2-y^2)}{(x^2+y^2)^2}, \; v(x,y) = \frac{2Axy}{(x^2+y^2)^2}, \; w=0$$

satisfies the equation of motion for inviscid incompressible flow. Determine the associated pressure of the flow field.

(b) Find the vorticity of the fluid motion for below veloci components:

(i) $u = A(x+y)$, $v = -A(x+y)$

(ii) $u = 2Axz$, $v = A(c^2+x^2-z^2)$

(iii) $u = Ay^2 + By + C$, $v = 0$, $A, B, C$ are Constants.

(c) Prove that if the speed is same everywhere, then streamlines are straight lines.

3(a) A steady inviscid incompressible fluid flow has a velocity field $u = \alpha x$, $v = -\alpha y$ and $w = 0$, where $\alpha > 0$ is a consta[nt]. Derive an expression for the pressure field $p(x,y,z)$ if the pressure $p\big|_{(0,0,0)} = p_0$ and $\vec{F} = -\beta \hat{z}$, where $\beta > 0$.

(b) For a steady motion of inviscid incompressible fluid of uniform density under conservative forces, show that the vorticity $\vec{\omega}$ and the velocity $\vec{q}$ satisfies

$$(\vec{q}\cdot\vec{\nabla})\vec{\omega} = (\vec{\omega}\cdot\vec{\nabla})\vec{q}.$$

(c) If the motion of an ideal fluid, for which density is a function of pressure $p$ only, is steady and the external forces are conservative, then $\exists$ a family of Surfaces which Contain the streamline and vortex lines.

End

Submit by 17.10.20 by 5PM.