FUNCTIONL ANLYSIS TEST-2
AUTUMN 2020
DEPT OF MATHEMATICS,
IIT KHARAGPUR

Answer All the questions

1. Let $X = C'[0,1]$ be a normed linear space with norm $\|x\|_1 = \|x\|_\infty + \|x'\|_\infty$ and $Y = C[0,1]$ be a normed linear space with norm $\|.\|_\infty$. Let $F : X \to Y$ be a map defined by $Fx = x'$, $\forall x \in X$. Is F continuous ? Justify.

2. Let $F : X \to Y$ be a linear operator between two normed linear spaces X and Y. If F is bounded below, then what can you say about the continuity of inverse of F and its norm?

3. Let $X = C[a,b]$ be a normed linear space with the norm $\|x\|_2 = \left(\int_a^b |x(t)|^2 dt \right)^{\frac{1}{2}}$, $x \in C[a,b]$. Let $k(.,.) \in C([a,b] \times [a,b])$. For $x \in C[a,b]$, let

$$Fx(s) = \int_a^b k(s,t)x(t)dt.$$

Discuss the boundedness of F and its norm.