Test on 01.10.21
Indian Institute of Technology Kharagpur
Department of Mathematics
Subject: Fluid Mechanics (MA40011/MA51003)
No. of Students: 93, Time: 75 mins., Full Marks: 30

Attention: Answer all Questions. First two questions carry 3 marks and the rest of the questions carry 4 marks. Submit the single PDF file of your answerscript on MS Teams (in the assignment created as Test 1). Name the file with your roll number. Mention your name, roll number and course name on the first page of the answerscript.

1. If the velocity potential of an irrotational flow is given by $\phi = \phi_1 + \phi_2$, where $\phi_1(x, y) = (x^2 - y^2)$ and $\phi_2(r, \theta) = r^{\frac{1}{2}} \cos\left(\frac{\theta}{2}\right)$, then verify whether ϕ satisfies the Laplace equation.

2. If the velocity potential of an irrotational flow is given by $\phi(x, y, z) = \frac{a}{2}(x^2+y^2-2z^2)$, then determine the streamlineas associated with the flow.

3. Show that the velocity field $u(x, y) = \frac{A(x^2-y^2)}{(x^2+y^2)^2}$, $v(x, y) = \frac{2Axy}{(x^2+y^2)^2}$ and $w = 0$ ($A > 0$ is constant) satisfies the equation of motion for an inviscid incompressible flow. Determine the pressure associated with this velocity field.

4. Verify whether the velocity field associated with the flow $\mathbf{q} = \frac{-y\mathbf{i}+x\mathbf{j}}{x^2+y^2}$ is irrotational. Calculate the circulation round the curve $(1, 0), (2, 0), (2, 1)$ and $(1, 1)$.

5. Verify whether the potenital function $\phi(x, y) = \frac{1}{2} \log \frac{(x+a)^2+y^2}{(x-a)^2+y^2}$ corresponds to a possible fluid motion. Determine the streamlines.

6. In the region bounded by a fixed quadrantal arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the streamline leaving either end at an angle α with radius r is $r^2 \sin(\alpha + \theta) = a^2 \sin(\alpha - \theta)$.

7. Derive the equation of motion of an inviscid fluid.

8. Let us consider the streaming and circulation about a fixed circular cylinder. Obtain the velocity potential ϕ, stream function ψ, stagnation points and the lift on the cylinder.

********** End of question paper. Good luck! **********