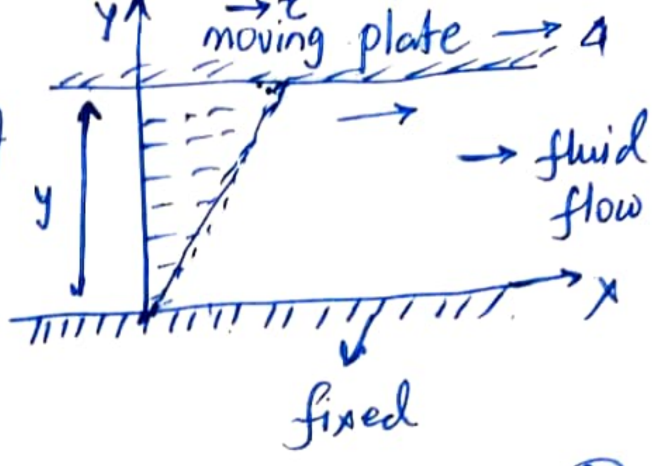
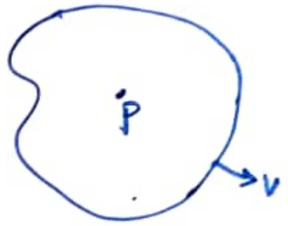
u is the velocity of the velocity



13.08.21 ①

$P(\vec{x}) = (x_1, x_2, x_3)$, $p = f(x_1, x_2, x_3, t)$

$$p = f(x_1, x_2, x_3)$$



$p : \mathbb{R}^3 \longrightarrow \mathbb{R}$

$$\tau = \mu \frac{du}{dy} \rightarrow \text{Newton's law of viscosity.}$$

i). If $\tau = 0$ then $\mu = 0$, this represents an ideal fluid / perfect fluid. [ideal fluid: (Frictionless, homogeneous and incompressible.) The ideal fluid is incapable of sustaining any tangential stress or action in the form of a shear but the normal force (pressure) acts between the adjoining layers of fluid.

Real fluids: (viscous and Compressible) The real fluid is the one in which both tangential and normal force exists.

ii) If $\frac{du}{dy} = 0$, then $\mu = \infty$, then the above equ^n. represents

(iii) A fluid for which the constant of propotionality does not change, (i.e., μ is constant) with the rate of deformation (shear stress), then it is said to be Newtonian fluid. ②

(iv) If μ varies, then it is a Non-Newtonian fluid.

Ex1: A plate at a distance 0.2 cm from a fixed plate moves at a rate 2m/sec, and requires a force 40 dyn/cm^2 to maintain this speed. Find μ ~~bet~~ of the fluid between the plates.

Sol^n. The viscosity co-eff. $\mu = \dfrac{\tau}{\frac{du}{dy}}$ — ①. To calculate

$\frac{du}{dy}$ = the velocity gardient $= \dfrac{2m}{0.2cm} = \dfrac{2 \times 100\,cm}{0.2\,cm} = 10^3$

$\tau = F = 40\ dyne/cm^2$.

From ①, $\mu = \dfrac{40\ dyne/cm^2}{10^3} = 4 \times 10^{-2}\ dyne/cm^2$.

§ Steady & Unsteady Flow: $\frac{\partial p}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial \rho}{\partial t} = 0 \rightarrow$ steady flow.

$\neq 0 \rightarrow$ Unsteady flow.

§ Laminar flow and Tubulent flow:

§ Eulerian Description: In this method we select any point fixed in space occupied by a certain fluid and study the changes which take place in velocity, pressure and density as the fluid passes through this point. Let $\vec{q} = (u, v, w)$ be the velocity at the point $P(x, y, z)$ at time t. Then, we have

$$u = F_1(x, y, z, t), \quad v = F_2(x, y, z, t) \text{ and } w = F_3(x, y, z, t) \quad \text{— (II)}$$

$$\vec{q} = (u, v, w)$$

Remark 1: For velocity / accⁿ. we are no longer writing / using $\frac{dx}{dt}, \frac{d^2x}{dt^2}$ as the point under consideration being fixed x, y, z and t and they are independent variable.

§ Lagrangian → Eulerian : Suppose $\phi(x_0, y_0, z_0, t)$ be some fluid property associated with the flow. Then by Lagrangian Des

$$x = f_1(x_0, y_0, z_0, t), \quad y = f_2(x_0, y_0, z_0, t), \quad z = f_3(x_0, y_0, z_0, t) \quad \text{— (III)}$$

From (III).

$$x_0 = \hat{f}_1(x, y, z, t), \quad y_0 = \hat{f}_2(x, y, z, t), \quad z_0 = \hat{f}_3(x, y, z, t) \quad \text{— (IV)}$$

Therefore

$$\phi = \phi(x_0, y_0, z_0, t)$$
$$= \phi\left[\hat{f}_1(x, y, z, t), \hat{f}_2(x, y, z, t), \hat{f}_3(x, y, z, t), t\right]$$

→ Eulerian Description of ϕ

§ Eulerian → Lagrangian: Suppose $\varphi(x,y,z,t)$ is any physical quantity associated with the flow, i.e., $\varphi = \varphi(x,y,z,t)$. Now, we have,

$$u = F_1(x,y,z,t), \quad v = F_2(x,y,z,t), \quad w = F_3(x,y,z,t)$$

$$\Rightarrow \frac{dx}{dt} = F_1(x,y,z,t), \quad \frac{dy}{dt} = F_2(x,y,z,t), \quad \frac{dz}{dt} = F_3(x,y,z,t) \quad \text{— (v)}$$

Let us $x(t_0) = x_0$, $y(t_0) = y_0$, $z(t_0) = z_0$, then (v) we will have.

$$x = f_1(x_0,y_0,z_0,t), \quad y = f_2(x_0,y_0,z_0,t),$$

$$z = f_3(x_0,y_0,z_0,t) \quad \text{— (vi)}.$$

Therefore from (vi),

$$\varphi = \varphi(x,y,z,t)$$

$$= \varphi\left[f_1(x_0,y_0,z_0,t),\ f_2(x_0,y_0,z_0,t),\ f_3(x_0,y_0,z_0,t)\right]$$

→ Lagrangian Description.

Ex 1: The velocity components for a 2D flow in Eulerian System is given by.

$$u = 2x + 2y + 3t, \quad v = x + y + t/2.$$

Then find the displacement of a fluid particle in Lagrangian Configuration.

(6)

Sol^n. Given $u = 2x + 2y + 3t$ and $v = x + y + t/2$ —(1).

The displacements x and y (in Lagrangian sys) can be determined by using the velocity components, i.e., u and v.

$$\frac{dx}{dt} = u = 2x + 2y + 3t \quad \text{and} \quad \frac{dy}{dt} = v = x + y + \frac{t}{2}$$

—(2)

Take $\frac{d}{dt} \equiv D$, then

$$(D-2)x - 2y = 3t \qquad \text{—(3)}$$
$$-x + (D-1)y = \frac{t}{2} \qquad \text{—(4)} \times (D-2)$$

$$\Rightarrow \quad (D^2 - 3D)y = \frac{1}{2} + 2t \qquad \text{—(5)}$$

Auxillary eqn. for (5): $m^2 - 3m = 0 \Rightarrow m = 0, 3$. Hence the complementary function is, $CF = c_1 y_1 + c_2 y_2$

$$= c_1 e^{0t} + c_2 e^{3t}$$
$$= (c_1 + c_2 e^{3t})$$

Now the PI of (5) $= \frac{1}{D^2 - 3D}\left(\frac{1}{2} + 2t\right)$

$$= -\frac{1}{3D} \cdot \frac{1}{\left(1 - \frac{D}{3}\right)}\left(\frac{1}{2} + 2t\right)$$

$$= -\frac{1}{3D}\left(1 - \frac{D}{3}\right)^{-1}\left(\frac{1}{2} + 2t\right)$$

$$= -\frac{1}{3}\cdot\frac{1}{D}\left(1+\frac{D}{3}+\frac{D^2}{9}+\frac{D^3}{27}+\cdots\right)\left(\frac{1}{2}+2t\right)$$

$$= -\frac{1}{3}\cdot\frac{1}{D}\left(1+\frac{D}{3}\right)\left(\frac{1}{2}+2t\right)$$

$$= -\frac{1}{3D}\left(2t+\frac{7}{6}\right) = -\frac{1}{3}\left[2\cdot\frac{t^2}{2}+\frac{7}{6}t\right]$$

$$= -\frac{t^2}{3} - \frac{7t}{18}$$

The required displacement along y-axis is

$$y = C.F. + PI$$

$$= c_1 + c_2 e^{3t} - \frac{t^2}{3} - \frac{7}{18}t \quad — (6)$$

Similarly, for $x = -c_1 + 2c_2 e^{3t} + \frac{t^2}{3} - \frac{7t}{9} - \frac{7}{18}$ — (7)

Let $x(t_0) = x_0$ and $y(t_0) = y_0$; where $t = t_0 = 0$ be the initial time.

$$y_0 = c_1 + c_2, \quad x = -c_1 + 2c_2 - \frac{7}{18} \quad — (8)$$

$$\Rightarrow c_1 = \frac{2y_0 - x_0}{3} - \frac{7}{54}, \quad c_2 = \frac{x_0 + y_0}{3} + \frac{7}{54} \quad (9)$$

$\therefore x =$

The required displacement in Lag. System is

$$x = -\frac{2y_0 - x_0}{3} + 2\left(\frac{x_0 + y_0}{3}\cancel{e^{3t}} + \frac{7}{54}\right)e^{3t}$$

$$+ \frac{t^2}{3} - \frac{7t}{9} - \frac{7}{18}$$

$y =$ ✓

<u>velocity of a fluid particle</u>: Let the fluid particle be at P at any time t and be at Q ($\vec{r} + \delta\vec{r}$) at ti
$\vec{OP} = \vec{r}$ and $\vec{OQ} = \vec{r} + \delta\vec{r}$. $\vec{q} = \lim_{\delta t \to 0} \frac{\delta \vec{r}}{\delta t} = \frac{d\vec{r}}{dt}$.

§ Material, local and convective derivatives:

2, 6, 10, 12, 27, 29, 30,