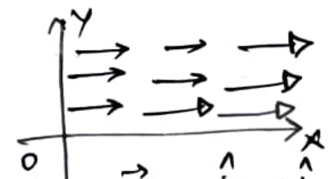
1. Steady Flows: Steady flow occurs when at various points of the flow field the conditions and the properties associated with the flow remain constant/unaltered w.r.t. time "t". Mathematically, if A represents some property (say, velocity, density, temp., pressure etc.) then

$$\frac{\partial A}{\partial t} = 0. \Rightarrow A = \phi(x, y, z) = c$$

Ex: water being pumped through a pipe at a constant speed.

On other hand, $\frac{\partial A}{\partial t} \neq 0 \Rightarrow$ A is associated with unsteady flow.

2. Uniform and non-uniform Flow: If at every point the velocity vector is identical in magnitude and direction at any given instant of time t, then it is termed as uniform flow.



$\vec{q} = 5\hat{i} + 2\hat{j}$

3. 1D, 2D and 3D Flows: The motion of a fluid at any pt. in space can be specified w.r.t. a coordinate system. One dimensional flow neglects the variations or changes in velocity, pressure etc. transverse to the main flow direction. The flow characteristics vary only in the direction of the flow.

$\vec{q} : \mathbb{R}^3 \to \mathbb{R}^3$ $\quad \vec{q} = u\hat{i} + \underset{\substack{\downarrow \\ 0}}{v\hat{j}} + \underset{\substack{\downarrow \\ 0}}{w\hat{k}}$, $\quad u : \mathbb{R}^3 \to \mathbb{R}$

$v : \mathbb{R}^3 \to \mathbb{R}$

$w : \mathbb{R}^3 \to \mathbb{R}$

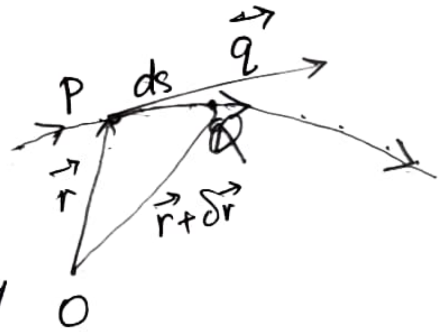
$$\vec{q} = u\hat{i} = (u, 0, 0)$$

Axi-symmetric Flow: A flow field is said to be axi-symmetric when the velocity components (u, v, w) w.r.t. cylinderical co-ordinat (r, θ, ϕ) are all independent of ϕ (azimuthal angle).

(2)

Lines of flow: A line of flow is a line whose direction coincide with the direction of the resultant velocity of the fluid.

Stream lines: A stream line is a continuous line of flow drawn in the fluid so that the tangent at every point of it at any instant of time coincides with the direction of motion of the fluid.



consider a (fluid) line element ds of the Streamline passing through $P(\vec{r})$ at time "t". Let $\vec{q}$ be the fluid velocity at P. Since the direction of the tangent and direction of velocity vector $\vec{q}$ are $\parallel$, then.

$$\vec{ds} \times \vec{q} = \vec{0}$$

$$\Rightarrow \quad (dx\hat{i} + dy\hat{j} + dz\hat{k}) \times (u\hat{i} + v\hat{j} + w\hat{k}) = \vec{0}$$

$$\Rightarrow \quad (w\,dy - v\,dz)\hat{i} + (u\,dz - w\,dx)\hat{j} + (v\,dx - u\,dy)\hat{k} = \vec{0}$$

$$\Rightarrow \quad w\,dy - v\,dz = 0, \quad u\,dz - w\,dx = 0, \quad v\,dx - u\,dy = 0$$

$$\Rightarrow \quad \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}.$$

This is known as the diff. equn. for a streamline.

Ex 1: Let $\vec{q}$ be the velocity of a 3D flow given by

$$\vec{q} = 2x\hat{i} - y\hat{j} - z\hat{k}.$$

Determine the equ^{n}s. of the streamlines passing through $(1,1,1)$.

Sol^n: The diff. equ^n. for streamlines are $\dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w}$

$$\Rightarrow \quad \frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{-z} \qquad \text{— (1)}.$$

$x = c_1 y^2$ | $y = c_2 z$.

$\Rightarrow \quad xy^2 = 1$ and $y = z$. (?)

These are the required equ^{n}s. of the stream lines.

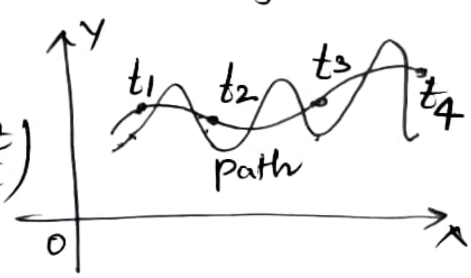
$xy^2 = c_1 = 1$ | $y = c_2 z$.

$y = z$

Pathlines: The curve described in space by moving fluid element is known as its pathline or tranjectory. In short, a path line is a line traced by a particle in the fluid.

The pathline is given by

$$\vec{q} = \frac{d\vec{r}}{dt} \Rightarrow (u, v, w) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$$



$$\Rightarrow \quad \frac{dx}{dt} = u(x,y,z,t), \quad \frac{dy}{dt} = v(x,y,z,t), \quad \frac{dz}{dt} = w(x,y,z,t).$$

Ex 2: The velocity vector $\vec{q}$ in a 2D flow is given by

$$\vec{q} = \left(\frac{x}{t}, y, 0\right).$$ Then find path lines.

Sol^n: The diff. equ^{n}s. for pathlines are:

$$\frac{dx}{dt} = \frac{x}{t}, \quad \frac{dy}{dt} = y, \quad \frac{dz}{dt} = 0.$$

④

$\Rightarrow x = c_1 t, \quad y = c_2 e^{t}, \quad z = \text{constant} = c_3.$

et $(\cancel{x}, 1, 1)$ be the particle's position at $t=1$. Then,

$c_1 = 1, \; c_2 = e^{-1}, \; c_3 = 1.$ Therefore the required

pathline is

$$x = t, \quad y = e^{(t-1)}, \quad z = 1.$$

Streamlines $\rightarrow \dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w} = \cancel{c/f(t)}$ $10^{10^{10}} \rightarrow c$

Pathlines $\rightarrow \quad \dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w} = \cancel{dt}$ $\neq 0$