FUNCTIONL ANLYSIS TEST-1
AUTUMN 2020
DEPT OF MATHEMATICS,
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Answer All the questions

1. Let X be a finite dimensional linear space with an ordered basis $\{u_1, u_2, \ldots, u_m\}$ and let $\|.\|_p$ be any norm for $1 \leq p < \infty$ on K^m. Prove that (i) there exists an injective linear map from X to K^m, and (ii) show that X is also a normed linear space.

2. Consider the norms $\|.\|_1$ and $\|.\|_\infty$ on $C[a, b]$. Are these two norms equivalent? Justify?

3. Prove that every closed and bounded subset of a finite dimensional normed linear space is compact.

4. Let $X_p = C_{00}$ be a normed linear space with norm $\|.\|_p$, $1 \leq p < \infty$. Is X_p is a Banach space? Justify

5. Let p and q be positive real numbers satisfying $\frac{1}{p} + \frac{1}{q} = 1$. The for any x, $y \in C[a, b]$, show that
$$\int_a^b |x(t)y(t)|dt \leq \left(\int_a^b |x(t)|^p dt\right)^{\frac{1}{p}} \left(\int_a^b |y(t)|^q dt\right)^{\frac{1}{q}}$$