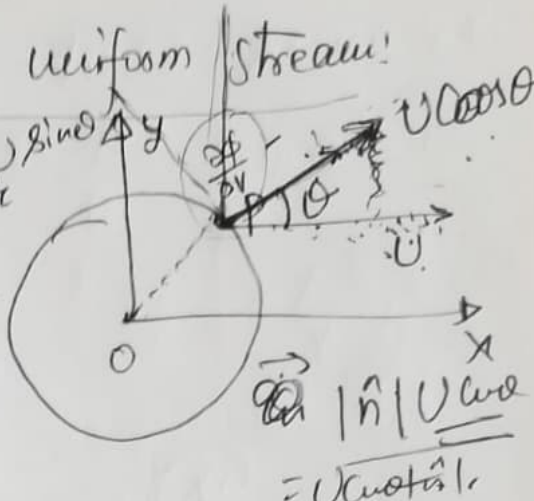
§ Motion of circlular cylinder. in an uniform stream:



" To determine the motion of a circular cylinder in an infinite mass of fluid at rest at infinity with velocity U $\parallel$ to x-axis.

Consider the fluid flow is irrotational which started at rest at infinity. Let $\vec{q}$ be the velocity vector, Let o be the center of the ~~circular~~ base of the circular cylinder which is taken as origin of the co-ordinates axes.

There exists a ϕ s.t. $\vec{q} = -\nabla\phi$

$\nabla^2\phi = 0$ $\Rightarrow \vec{\nabla}\cdot\vec{q} = 0 \Rightarrow \nabla^2\phi = 0$ — (I)

$\phi(x,y) = ax + by$

(r,θ)

$\Rightarrow \phi \neq 0$

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0.$$

$$\Rightarrow \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0 \qquad \text{— (II)}$$

$\phi(r,\theta) = R(r)\,\Phi(\theta)$

The solution of (II) has the forms,

$r^n \cos n\theta$ and $r^n \sin n\theta$, $n \in \mathbb{Z}$

Hence the sum of any number of terms of the form

$A_n r^n \cos n\theta$ or, $B_n r^n \sin n\theta$, i.e.,

$\frac{d^2y}{dx^2}+y=f(x)$

$y_1 \& y_2$

y_1, y_2

$$\phi(r,\theta) = A_n r^n \cos n\theta \text{ or, } B_n r^n \sin n\theta \quad \text{— (vi)}$$

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(i) Normal velocity at any pt. of the surface of the cylinder = velocity of the liquid at that point in that direction.

$$\Rightarrow \quad -\frac{\partial \phi}{\partial r} = U\cos\theta. \text{ when } r=a \quad \text{— (vii)}$$

(ii) Since the liquid is at rest at infinity, the velocity must be zero as $r \to \infty$ i.e.,

$$-\frac{\partial \phi}{\partial r} \to 0, \text{ and } -\frac{1}{r}\frac{\partial \phi}{\partial \theta} \to 0 \text{ as } r \to \infty \quad \text{— (viii)}$$

From (vi) ; $\frac{\partial \phi}{\partial r} = n r^{n-1} A_n \cos n\theta$ or, $B_n n r^{n-1} \sin n\theta$ — (ix)

$$\left[\begin{array}{l} \frac{\partial \phi}{\partial \theta} = -A_n r^n \sin n\theta\,(n) \text{ or, } B_n n r^{n-1} \cos n\theta\, n \\ \Rightarrow \frac{1}{r}\cdot\frac{\partial \phi}{\partial \theta} = -A_n n\cdot r^{n-1}\sin n\theta \text{ or, } n\cdot B_n\cdot r^{n-1}\cos n\theta \quad \text{— } (*) \end{array}\right.$$

As $\frac{\partial \phi}{\partial r} \to 0$ as $r \to \infty$.

For $n = -1, -2, \ldots$

| ~~$A \to 1 < 0$~~ $\phi(r,\theta) = Ar\cos\theta + Br\sin\theta$

$n-1 \leq -1$ $\frac{\partial \phi}{\partial r} = A\cos\theta + B\sin\theta$

Step: $\phi(r,\theta) = \sum_n A_n r^n \cos n\theta$ ✓

$\frac{\partial \phi}{\partial r} \to 0$ as $r \to \infty$ ✓

$\phi(r,\theta) = A r \cos\theta + \cancel{B r}$

$\frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{A r \sin\theta}{r} + B r \cos\theta$

$\frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{A \sin\theta}{r} + B\cos\theta$
$\to 0$ as $r \to \infty$

$\phi(r,\theta) = \left(A r \cos\theta + \frac{B}{r}\cos\theta\right)$ — (x1)

$\cos\theta = 0$
$\Rightarrow \theta = (2n-1)\frac{\pi}{2}$
$n = 1, 2 \ldots$

We Know $-\left.\frac{\partial \phi}{\partial r}\right|_{r=a} = U\cos\theta$

$\Rightarrow -\left[A\cos\theta - \frac{B}{r^2}\cos\theta\right]\Big|_{r=a} = U\cos\theta$

$\Rightarrow -A + \frac{B}{a^2} = U \Rightarrow -A + \frac{B}{a^2} = U$

From (x1), $\frac{\partial \phi}{\partial r} = A\cos\theta - \frac{B}{r^2}\cos\theta$

$\Rightarrow -\frac{\partial \phi}{\partial r} = -A\cos\theta + \frac{B}{r^2}\cos\theta$

Since $\boxed{\frac{\partial \phi}{\partial r} \to 0 \text{ as } r \to \infty}$ $\Rightarrow A\cos\theta = 0$ ✓

$\Rightarrow A = 0$ ✓ $\Rightarrow B = Ua^2$

From (x1), $\phi(r,\theta) = \frac{B}{r}\cos\theta$ ✓ $= \frac{Ua^2}{r}\cos\theta$

$$\phi(r,\theta) = \frac{Ua^2}{r}\cos\theta \ .$$

From CR equ^n of ϕ and ψ

$$\frac{\partial\psi}{\partial r} = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{1}{r}\cdot\frac{Ua^2}{r}\,-\sin\theta$$

$$\Rightarrow \frac{\partial\psi}{\partial r} = \frac{Ua^2}{r^2}\sin\theta$$

$$\Rightarrow \psi(r,\theta) = -\frac{Ua^2}{r}\sin\theta + f(\theta)$$

The fluid is at rest at infinity, i.e., $\psi(r,\theta) \to 0$ as $r \to \infty$.

$$\psi(r,\theta) = -\frac{Ua^2}{r}\sin\theta \ .$$

The required complex potential of the flow,

$$w(z) = \phi + i\psi = \frac{Ua^2}{r}\cos\theta - i\frac{Ua^2}{r}\sin\theta$$

$$= \frac{Ua^2}{r}(\cos\theta - i\sin\theta) = \frac{Ua^2}{r}e^{-i\theta}$$

$$= \frac{Ua^2}{re^{i\theta}} = \frac{Ua^2}{z}, \quad z = re^{i\theta}$$

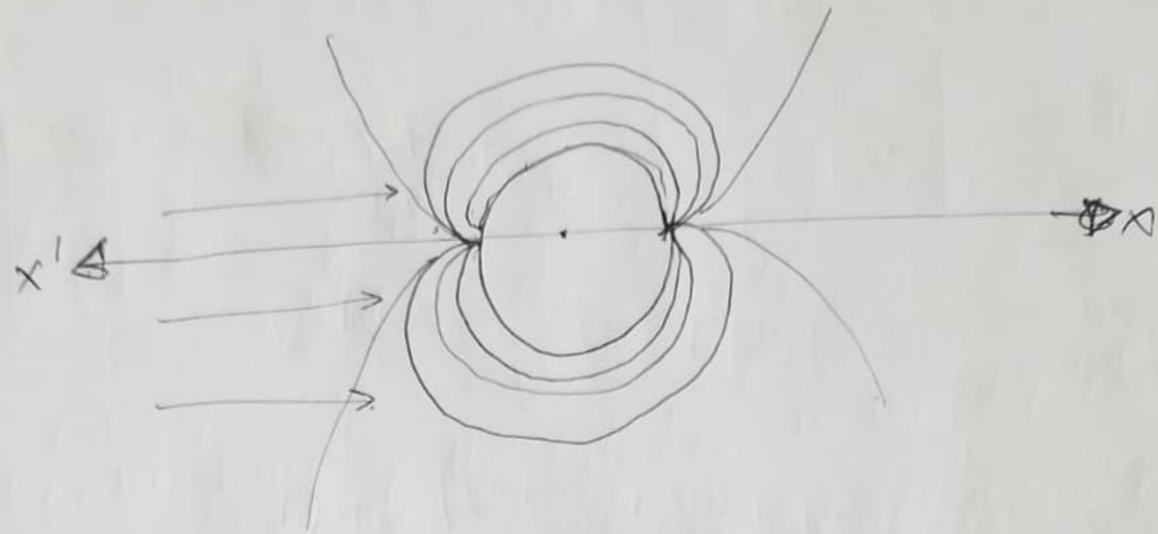
The st. lines are given by

$\psi = c$

$\Rightarrow -\frac{Ua^2}{r}\sin\theta = c \ .$

$\Rightarrow kr\sin\theta = r^2 \Rightarrow x^2+y^2-ky=0 \ . \Rightarrow x^2+\left(y-\frac{k}{2}\right)^2 = \frac{k^2}{4}$

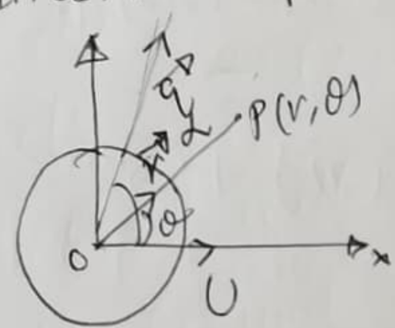
Which are circles touching x-axis at origin.



Ex 1: A circular cylinder of radius "a" is moving with velocity U along the x-axis and if its complex potential is given by

$$w(z) = \phi + i\psi = \frac{a^2 U}{z - Ut}$$

Then find the magnitude and the direction of the velocity.



Sol^n:

$$w(z) = \frac{a^2 U}{z - Ut} \quad \text{— (1)}$$

$$\frac{dw}{dz} = -\frac{a^2 U}{(z - Ut)^2}$$

$$\Rightarrow \underline{\underline{-u + iv}} = -\frac{a^2 U}{(z - Ut)^2}$$

$z - Ut = re^{i\theta}$

$$-u + iv = -\frac{Ua^2}{r^2 e^{2i\theta}} = -\frac{Ua^2}{r^2}(\cos 2\theta - i\sin 2\theta)$$

$$\Rightarrow \quad u = \frac{Ua^2}{r^2}\cos 2\theta \;, \quad v = \frac{Ua^2}{r^2}\sin 2\theta .$$

Magnitude of $\vec{q} = \sqrt{u^2+v^2} = \frac{Ua^2}{r^2}$ ✓

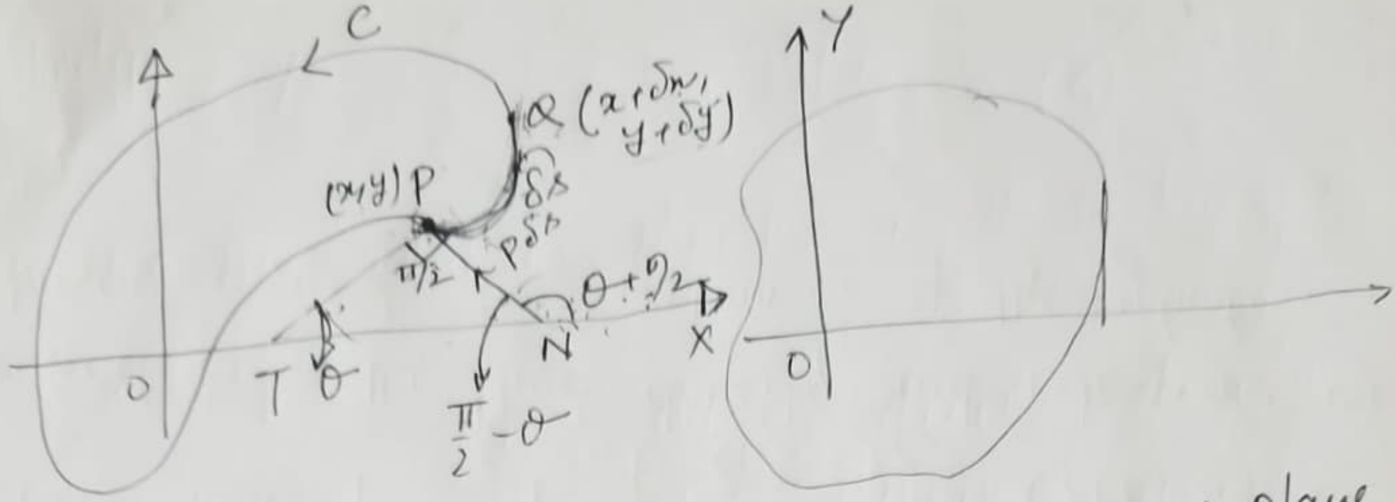
Direction of $\vec{q}$: $\tan\alpha = \frac{v}{u} = \tan 2\theta \Rightarrow \underline{\alpha = 2\theta}$

§ The Blasius Theorem: In a steady 2D irrotational flow of an incompressible fluid under no external forces is given by the complex potential $w = f(z)$. If the pressure thrusts on the fixed cylinder of any shape are represented by a force (X, Y). and a couple of moment M about the origin of co-ordinate axes then $|F| = \sqrt{X^2+Y^2}$

$$X - iY = \frac{1}{2} i\rho \int_C \left(\frac{dw}{dz}\right)^2 dz \quad \text{and} ✓$$

$$M = \text{Real part of } \left\{ -\frac{1}{2} i\rho \int_C z \left(\frac{dw}{dz}\right)^2 dz \right\} ✓$$

Where ρ is the fluid density and integrals are taken around the contour C of the cylinder.



Let C be the contour of the base of the cylinder in the XY-plane. Let P and Q be any two arb. pts. of the curve C, where $P=(x,y)$ and $Q=(x+\delta x, y+\delta y)$ s.t. $\widehat{PQ}=\delta s$.

Let $\angle PNX = \frac{\pi}{2}+\theta$, $\angle PTN = \theta$, $\angle PNT = \frac{\pi}{2}-\theta$ and $\angle TPN = \angle NPT = \frac{\pi}{2}$. Here ~~clearly~~, PT is a tangent to C making an angle θ with x-axis. Then

$$\cos\theta = \frac{dx}{ds} \text{ and } \sin\theta = \frac{dy}{ds} \quad \text{—(1)}$$

Also, the normal PN makes an angle $\theta+\frac{\pi}{2}$ with +ve x-axis. Let p be the pressure per unit area, the force on the length of section δs is $p\delta s$.

$$(2)\left\{\begin{aligned} X &= \int_C p\cos\left(\theta+\frac{\pi}{2}\right) ds = -\int_C p\sin\theta\, ds = -\int_C p\,dy \\ Y &= \int_C p\sin\left(\theta+\frac{\pi}{2}\right) ds = \int_C p\cos\theta\, ds = \int_C p\,dx \end{aligned}\right.$$

[Moment of the force: It is a measure of its tendency to cause a body to rotate about a specific pt. or axis]

Moment $= |\text{Force}| \times |\text{distance}| = |\vec{F}| \times |\vec{d}|$ $\quad (x, y)$ $\quad P(x, y)$

The moment acting on a small element δs is

$$\delta M = (p\,ds \sin\theta)\,y + (p\,ds \cos\theta)\,x$$

$$\Rightarrow \int_C dM = \int_C [y p \underline{ds \sin\theta} + p x \underline{\cos\theta\, ds}]$$

$$\Rightarrow M = \int p\,[y\,dy + x\,dx] \quad \text{— (3)}$$

We will calculate/derive $X = ?,\ Y = ?,\ \underline{M = ?}$

Now, we will apply Bernoulli's eqn.

$$\frac{1}{2}|\vec{q}|^2 + \frac{p}{\rho} = B,$$

where $\vec{q} = (u, v)$ is the velocity, p is the pressure, ρ is the density and B is constant. Also $\underline{q^2 = |\vec{q}|^2 = u^2 + v^2}$

$$\Rightarrow p = \underbrace{\rho B}_{\text{Constant}} - \frac{1}{2}\rho q^2 = \underbrace{N}_{\text{Constant}} - \frac{1}{2}\rho(u^2 + v^2) \quad \text{— (4)}$$

From (2) and (4),

$$X = -\int_C \left(N - \frac{1}{2}\rho(u^2 + v^2)\right) dy = \frac{\rho}{2}\int_C (u^2 + v^2)\,dy, \quad \text{— (5)}$$

$\because C$ is a closed contour, $\int_C dy = \int_C dx = \underline{0}$

Similarly, $Y = -\frac{\rho}{2}\int_C (u^2+v^2)\,dx$ — (6)

Now, we know that the streamline is the contour of the cylinder, Therefore,

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dx+idy}{u+iv} = \frac{dx-idy}{u-iv}$$

$$\Rightarrow \frac{dx-idy}{dx+idy} = \frac{u-iv}{u+iv} = \frac{(u-iv)^2}{u^2+v^2}$$

$$\Rightarrow (u-iv)^2\,dx+idy = (u^2+v^2)(dx-idy) \quad (7)$$

Now,

$$X - iY = \frac{\rho}{2}\left[\int_C (u^2+v^2)\,dy + i\int_C (u^2+v^2)\,dx\right]$$

$$= \frac{\rho}{2}\int_C (u^2+v^2)(idx+dy)$$

$$= \frac{i\rho}{2}\int_C (u^2+v^2)(dx-idy)$$

$$= \frac{i\rho}{2}\int_C (u-iv)^2(dx+idy) \text{ by } (7)$$

$$= \frac{i\rho}{2}\int_C (-u+iv)^2(dx+idy)$$

Since $\frac{dw}{dz} = -u + iv$,

$$x - iy = \frac{i\rho}{2} \int_C \left(\frac{dw}{dz}\right)^2 dz, \qquad z = x + iy, \quad dz = dx + idy$$

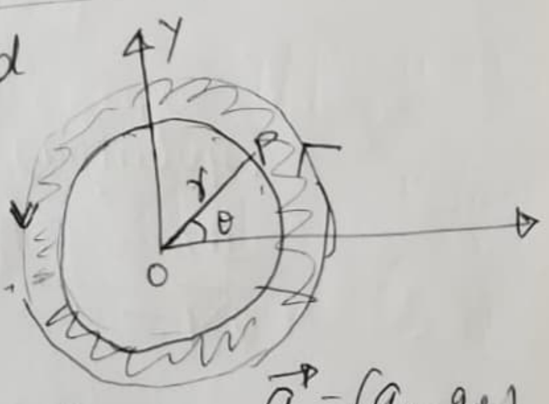
$$M = -\frac{\rho}{2} \int_C (u^2 + v^2)(x\,dx + y\,dy)$$

$$= \text{Real part of} \left[-\frac{\rho}{2} \int_C (u^2+v^2)(x+iy)(dx - idy) \right]$$

$$M = \text{Real part of} \left[-\frac{\rho}{2} \int_C z \left(\frac{dw}{dz}\right)^2 dz \right], \text{ by}$$

(previous calculation)

§. Circulation around / about a circular cylinder.



Since the fluid occupies a connected region, the motion around the contour of the cylinder is possible for the irrotational fluid. For a suitably chosen velocity potential,

$$\vec{q} = (q_r, q_\theta) = -\nabla\phi = -\left(\frac{\partial\phi}{\partial r}, \frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)$$

$$K = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} \cdot 2\pi r.$$

$$\Rightarrow \frac{\partial\phi}{\partial\theta} = -\frac{K}{2\pi}.$$

$$\frac{\partial \phi}{\partial r} = 0. \qquad \int_{C} q\,dr.$$

Again, $\frac{\partial \phi}{\partial \theta} = -\frac{K}{2\pi} \Rightarrow \phi(r,\theta) = -\frac{K\theta}{2\pi} + F(r)$

$$= -\frac{K\theta}{2\pi}$$

Moreover ϕ and ψ are complex conjugates,

$$\frac{\partial \psi}{\partial r} = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

$$\Rightarrow \frac{\partial \psi}{\partial r} = \frac{K}{2\pi r}.$$

$$\Rightarrow \psi(r,\theta) = \frac{K}{2\pi}\log r$$

Therefore the required complex potential is

$$\omega = \phi + i\psi = -\frac{K\theta}{2\pi} + i\frac{K}{2\pi}\log r.$$

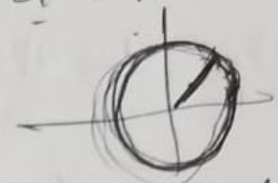
$$= \frac{iK}{2\pi}(\log r + i\theta)$$

$$= \frac{iK}{2\pi}(\log r + \log e^{i\theta}) = \frac{iK}{2\pi}\log re^{i\theta}$$

$$w(z) = \frac{iK}{2\pi}\log z.$$

$K = \oint_C \vec{q} \cdot d\vec{r}$ $\qquad \vec{r} = dx + i\,dy$

$d\vec{r}$



$r = 1/2$

$= \oint_C (q_r, q_\theta) \cdot (dr, d\phi)$ $\qquad \vec{q} = -\nabla\phi$

$= \oint_C (q_r\,dr + q_\theta\,d\theta)$ $\qquad = -\left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}\right)\phi$

$\Rightarrow q_r = -\frac{\partial \phi}{\partial r} \to 0, \quad q_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}$

$= \underbrace{\oint_C q_r\,dr}_{0} + \oint_C q_\theta\,d\theta$ ✓

$\phi =$

$= \oint_C q_\theta\,d\theta = \oint_C -\frac{1}{r}\frac{\partial \phi}{\partial \theta}\,d\theta \cong -\frac{1}{\cancel{r}}\frac{\partial \phi}{\partial \theta} \times 2\pi r$

$= -2\pi \frac{\partial \phi}{\partial \theta}$

$\int_0^1 f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} (x_i - x_{i-1})\, \underline{f(x_i)}$

24.09.21

§ Motion around a circular cylinder.

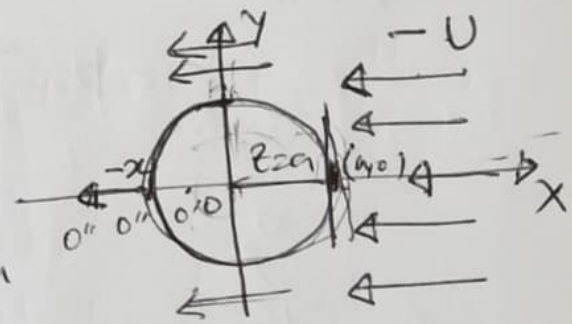
$$w(z) = \frac{Ua^2}{z} \quad \Bigg| \quad \phi = \frac{Ua^2}{r}\cos\theta, \quad \psi = -\frac{Ua^2}{r}\sin\theta$$

Circulation around a circular cylinder

$$w(z) = \frac{i\kappa}{2\pi}\log z \qquad \kappa = \int \vec{q}\cdot dr$$

§ Liquid streaming past a fixed circular cylinder (Cohen & Kundu – Fluid Mechanics)

Let a circular cylinder be at rest and let a fluid flow past the cylinder with velocity U in the $-$ve direction of x-axis. We can write the complex potential by imposing a velocity $-U$ ∥ to x-axis on both the cylinder and the fluid.



Therefore we must add to the velocity potential a term $-U(-x) = Ux$ to account for the additional velocity created by the motion of the cylinder, i.e,

$$\phi(r,\theta) = \frac{Ua^2}{r}\cos\theta + Ux = \frac{Ua^2}{r}\cos\theta + Ur\cos\theta$$

$$= U\cos\theta\left(\frac{a^2}{r} + r\right)$$

$$\psi(r,\theta) = -\frac{Ua^2}{r}\sin\theta + Uy = -\frac{Ua^2}{r}\sin\theta + Ur\sin\theta$$

$$= U\left(r - \frac{a^2}{r}\right)\sin\theta.$$

The required complex potential,

$$\omega(r,\theta) = \phi(r,\theta) + i\psi(r,\theta)$$

$$= U\left(\frac{a^2}{r} + r\right)\cos\theta + iU\left(r - \frac{a^2}{r}\right)\sin\theta$$

$$= Ur(\cos\theta + i\sin\theta) + U\frac{a^2}{r}(\cos\theta - i\sin\theta)$$

$$= Ure^{i\theta} + \frac{Ua^2}{r}e^{-i\theta} = \underbrace{Ure^{i\theta}}_{z} + \frac{Ua^2}{re^{i\theta}}$$

$$= Uz + \frac{a^2U}{z}, \quad z = re^{i\theta} = U\left(z + \frac{a^2}{z}\right).$$

Note 1:

We know, $|\vec{q}| = \left|\frac{dw}{dz}\right| = \left|U - \frac{a^2U}{z^2}\right|$

$$\Rightarrow q = \left|U - \frac{Ua^2}{a^2e^{2i\theta}}\right|, \quad z = ae^{i\theta}$$

$$= |U - Ue^{-2i\theta}|, \quad a \neq 0$$

$$= |U|\,|1 - e^{-2i\theta}|$$

$$= |U| \, |1 - \cos 2\theta + i \sin 2\theta|$$

$$= |U| \sqrt{(1-\cos 2\theta)^2 + \sin^2 2\theta}$$

$$= |U| \sqrt{2} \sqrt{1-\cos 2\theta} = |U| \sqrt{2} \sqrt{2} \sqrt{\sin^2\theta}$$

$$q = 2|U| \, |\sin\theta| \checkmark$$

Stagnation pt. ~~q (or occur)~~ (or critical pts) occurs

where $q = 0$ when $\theta = 0, \pi$

Similarly, $q = q_{max} = 2|U|$, when $\theta = \frac{\pi}{2}$.

$= 2U; \; U > 0$

$0 < \theta < \frac{\pi}{2}, \; q < q_{max}$	q_{max} at $\theta = \frac{\pi}{2}$
$\frac{\pi}{2} < \theta < \pi, \; q < q_{max}$	$q = 0$ at $\theta = 0, \pi$

§. Streaming and the circulation about a fixed circular cylinder.

We know, the circulation created due to the motion of the fluid if denoted by κ, then complex potential ω_1 is given by

$$\omega_1(z) = \frac{i\kappa}{2\pi} \log z \quad \text{— (i)}$$

From previous calculation, we know the complex poten ω_2 for streaming past a fixed circular cylinder of radius "a" with velocity U in the -ve x-axis is given by

$$\omega_2(z) = Uz + \frac{Ua^2}{z} \quad \text{— (ii)}.$$

Therefore, the required complex potential of the given flow,

$$w(z) = \omega_1(z) + \omega_2(z) = \frac{i\kappa}{2\pi} \log z + Uz + \frac{Ua^2}{z}$$

$$= U\left(z + \frac{a^2}{z}\right) + \frac{i\kappa}{2\pi} \log z$$

$z = re^{i\theta}$

$$\Rightarrow \phi(r,\theta) = Ur\cos\theta + \frac{Ua^2}{r}\cos\theta - \frac{\kappa\theta}{2\pi} ✓$$

$$\psi(r,\theta) = Ur\sin\theta - \frac{Ua^2}{r}\sin\theta + \frac{\kappa}{2\pi}\log r. ✓$$

Since the velocity will be only tangential at the boundary of cylinder, $-\frac{\partial \phi}{\partial r} = 0$ and hence the magnitude of the velocity $\vec{q}$ is given by

$$q = |\vec{q}| = \left| -\frac{1}{r}\frac{\partial \phi}{\partial \theta} \right|$$

$$= \left| \frac{1}{r}\left\{ + Ur\sin\theta + \frac{Ua^2}{r}\sin\theta + \frac{K}{2\pi} \right\} \right|_{r=a}$$

$$= \left| U\sin\theta + U\sin\theta + \frac{K}{2\pi a} \right|$$

$$q = \left| 2U\sin\theta + \frac{K}{2\pi a} \right|.$$

Case I: If there is no circulation, $K=0$. Then

$$q = |2U\sin\theta| = 2|U|\sin\theta$$

$\theta = 0, \pi$ are pts. of zero velocity on the cylinder.

Case II: If the circulation is present, $K \neq 0$ Then the velocity $\vec{q}$ will be zero when.

$$2U \sin\theta + \frac{K}{2\pi a} = 0$$

$$\Rightarrow \sin\theta = -\frac{K}{4\pi U a}.$$

But $|\sin\theta| \leq 1 \Rightarrow \left|\frac{K}{4\pi U a}\right| \leq 1.$

$$\Rightarrow |K| \leq 4\pi U a = 2\pi a\, q_{max}$$

$$\Rightarrow -4\pi U a \leq K \leq 4\pi U a$$

Sub-case I: When $|K| \leq 4\pi U a$ or, $-4\pi U a \leq K \leq 4\pi U a$

$$\Rightarrow -2\pi a(2U) \leq K \leq 2\pi a(2U)$$

$$\Rightarrow -2\pi a\, q_{max} \leq K \leq 2\pi a\, q_{max}$$

$\vec{q}.dr$

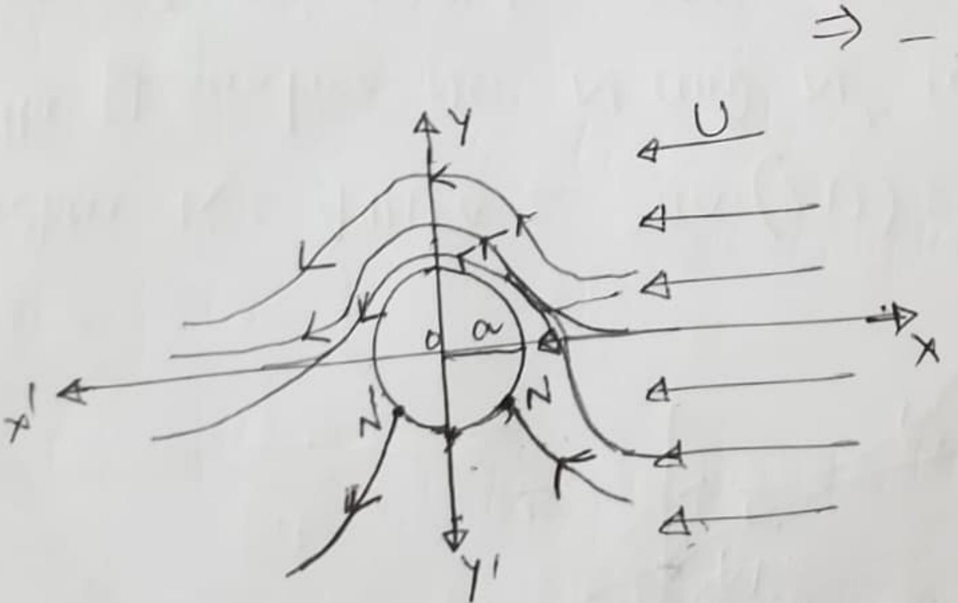
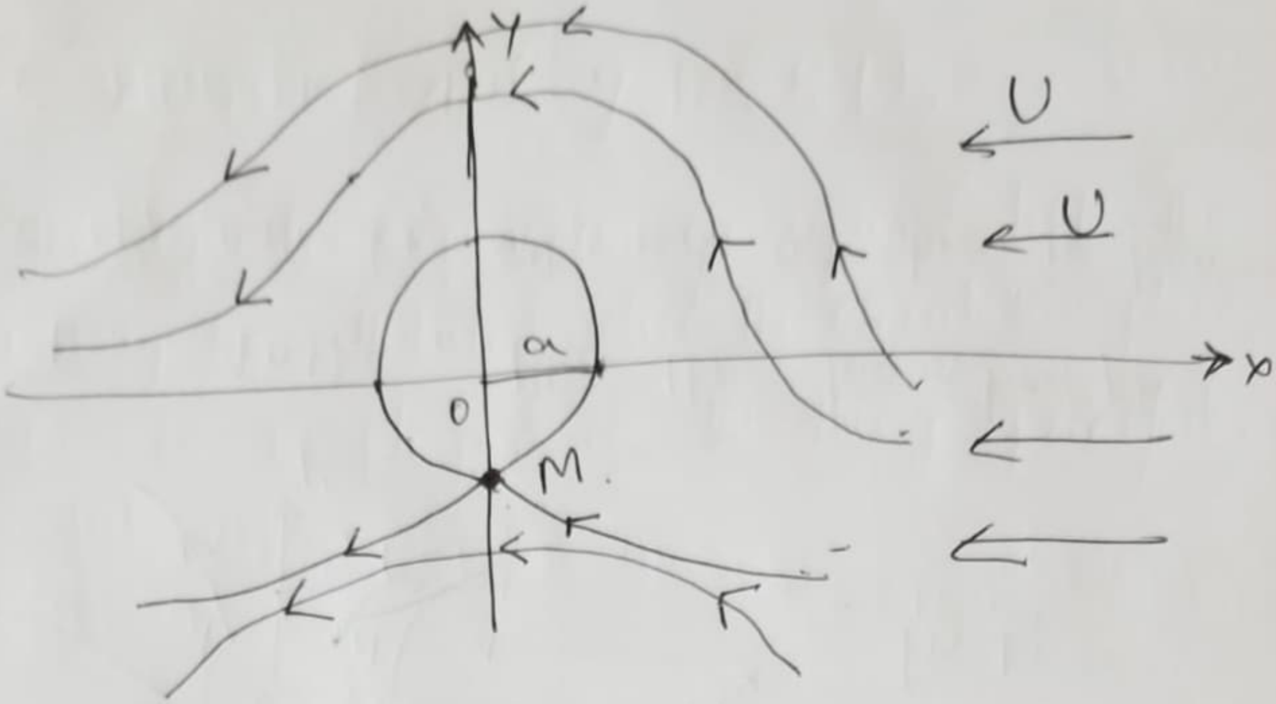


Fig 1:

When $|K| \leq 4\pi U a$, then the Stagnation pts. N and N' will be attained as show in figure.

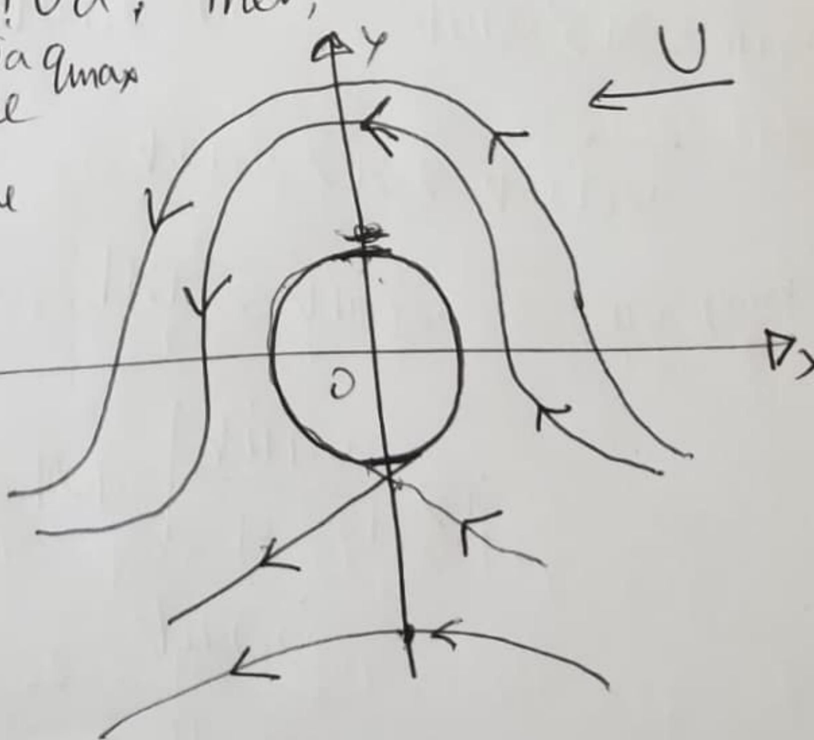
Subcase ii: When $|K| = 4\pi U a$, then.



When $K = 4\pi U a = 2\pi a \cdot (2U) = 2\pi a\, q_{max}$. then the stagnation pts. N and N' will coincide i.e., N and N' will be say, point M.

Subcase iii: When $|K| > 4\pi U a$, then,
$= 2\pi a\, q_{max}$

When $|K| > 4\pi U a$ then there is no stagnation pts. on the cylinder but there is such pt. below the cylinder on the y-axis.



$\frac{K}{U}$

From Bernoulli's equⁿ.

$$\frac{p}{\rho} = F(t) - \frac{q^2}{2} \quad \text{—} \quad (*)$$

Let π be the pressure at infinity, i.e. $p = \pi$ when $q = U$, then from $(*)$,

$$\frac{\pi}{\rho} = F(t) - \frac{U^2}{2} \Rightarrow F(t) = \frac{\pi}{\rho} + \frac{U^2}{2} \quad \text{—} \quad (**)$$

From $(*)$ & $(**)$,

$$\frac{p}{\rho} = \frac{\pi}{\rho} + \frac{U^2}{2} - \frac{1}{2}\left(2U\sin\theta + \frac{K}{2\pi a}\right)^2$$

$$\Rightarrow p = \pi + \frac{\rho U^2}{2} - \frac{\rho}{2}\left(2U\sin\theta + \frac{K}{2\pi a}\right)^2$$

Let X and Y be the thrusts on the cylinder created /exerted by the fluid. Then,

$$X = -\int_0^{2\pi} p\cos\theta\, a\, d\theta = -a\int_0^{2\pi} p\cos\theta\, d\theta$$

$$Y = -\int_0^{2\pi} p\sin\theta\, a\, d\theta = -a\int_0^{2\pi} p\sin\theta\, d\theta.$$

$$X = -a\int_0^{2\pi}\left[\pi + \frac{\rho U^2}{2} - \frac{\rho}{2}\left(2U\sin\theta + \frac{K}{2\pi a}\right)^2\right]\cos\theta\, d\theta$$

$$Y = -a\int_0^{2\pi}\left[\pi + \frac{\rho U^2}{2} - \frac{\rho}{2}\left(2U\sin\theta + \frac{K}{2\pi a}\right)^2\right]\sin\theta\, d\theta$$

$\Rightarrow \quad X = 0 \quad$ and $\quad Y = \rho K U.$

Shows the cylinder experiences an upward lift. This ~~is~~ effect may be attributed due to the circulation phenomena. This fact is employed in the theory of aerofoils.