Indian Institute of Technology Kharagpur
Department of Mathematics
MA41007 - Functional Analysis
Test - 1, AUTUMN 2021

Instructions: Answers all the questions. No queries will be entertained during examination.

1. **Find the closure of C_{00} in $(\ell^1, \|.\|_1)$, $(\ell^2, \|.\|_2)$ and in $(\ell^\infty, \|.\|_\infty)$**

2. **Let $X := C^1[a,b]$. For $x \in X$, let $\|x\|' := \max\{|x(a)|, \|x\|_\infty\}$ and $\|x\|_{1,\infty} := \max\{\|x\|_\infty, \|x'\|_\infty\}$. Are these two norms $\|.\|'$ and $\|.\|_{1,\infty}$ equivalent norms on X? Justify. Is the norm $\|.\|_{1,\infty}$ equivalent to the norm $\|.\|_\infty$ on X. Justify**

3. **Let Y be a finite dimensional proper subspace of a normed linear space $(X, \|.\|)$. Then show that there is some $x \in X$ with $\|x\| = 1$ and $dist(x, Y) = 1$.**

4. **Let the subset $\{x \in X : \|x\| \leq 1\}$ of a normed linear space X is compact. Is X finite dimensional? Justify.**

5. **Is c, the space of all convergent sequences, a closed subspace of the space ℓ^∞ ? Justify**

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