Indian Institute of Technology Kharagpur
Department of Mathematics
MA41007 - Functional Analysis
Test - 3, AUTUMN 2021

NAME:

ROLL NO:

Instructions: Answers all the questions. No queries will be entertained during examination.

1. Let X and Y be Banach spaces and $A : X \to Y$ be a injective bounded linear map such that $R(A)$ is closed in Y. Then $F^{-1} : R(A) \to X$ is a $-----------$map.

2. Let X be a Banach space with two noms $\|.\|_1$ and $\|.\|_2$ such that $\|x\|_1 \leq \alpha\|x\|_2$, for some $\alpha > 0$, for all $x \in X$. Then $\|.\|_1$ and $\|.\|_2$ are equivalent(TRUE/ FALSE) $-------$.

3. Let X be a Banach space and Y be a normed linear space. Let $A : X \to Y$ be a bounded linear map and there exists $\gamma > 0$ such that $\gamma\|x\| \leq \|Ax\|, \forall x \in X$. Then $A^{-1} : R(A) \to X$ is continuous(TRUE/ FALSE): $----------$ and $R(A)$ is $----------$.

4. Let X and Y be normed linear space and $A : X \to Y$ be a open linear map. Then A is a $-------$linear map.

5. Let $X = C[a, b]$ with norm $\|.\|_\infty$ and $Y = C[a, b]$ with norm $\|x\|_1 = \int_a^b |x(t)|dt, x \in X$. Let $A : X \to Y$ be identity linear map. Then A is continuous linear map(TRUE/FASLE): $---------$ and $A^{-1} : Y \to X$ is continuous (TRUE/FASLE): $---------$.

6. Let $X = C[a, b]$ be an inner product space with the inner product $< f, g >= \int_a^b f(t)\overline{g(t)}dt$ and Y be the space of polynomials defined on $[a, b]$ then $Y^\perp = ----------$

7. Let X be a complex inner product space. Then there hold "For all $x, y \in X$, $\|x + y\|^2 = \|x\|^2 + \|y\|^2$ if and only if $x \perp y$". Is this statement true?(Yes/No): $--------$.

8. Let $(X, < ., . >_X)$ and $(Y, < ., . >_Y)$ be inner product spaces and $F : X \to Y$ be a linear map such that $\|F(x)\| = \|x\|$, $\forall x \in X$. Then $< Fx_1, Fx_2 >_Y = -------$.

9. An infinite dimensional separable Hilbert space H is isometrically isomorphic to $-------$.

10. Let $X = R^2$ be a normed linear space over the field K with the Euclidean norm $\|.\|$ and let $Y = \{(x, y) \in R^2 \ / \ x = \gamma y, \text{for some } \gamma \in K\}$. Let $f : Y \to K$ be the functional defined by $f(x, y) = x$, $\forall (x, y) \in Y$. Then the Hahn-Banach extension $g : X \to K$ of f is given by $g(x, y) = ---------$, $\forall (x, y) \in X$.

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