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Mutrod 2: Method of Successive Substitution

 $u(x) = f(x) + \gamma (k(x,t)u(t)dt \longrightarrow (1)$

Here RHS of u(x) is substituted for u(t) inside the integral prepeatedly.

Step-1 u(x) = f(x)+ > (K(x,t)) {f(t) + > (K(t,t_i)) u(t_i) dt_i} dt_i $u(\alpha) = f(\alpha) + \sum_{i=1}^{N} K(\alpha_i, t) \left\{ f(t) + \sum_{i=1}^{N} K(t, t_i) u(t_i) dt_i \right\} dt$

= f(x) + x (K(x,t) f(t) dt + > 2/1 K(x,t) K(t,ti) u(ti)

step-2 for ain u(ti) shall be superced by its expression as given in the RHS of (D. This gives, & (x,t) K(t,t,) {f(ti) u(a) = f(a) + > \int_{a} K(a,t) t(t) dt + \int_{a} [K(a,t) K(t,t,) {f(ti)}







 $\frac{1}{1} \underbrace{(x, t)}_{k} \underbrace{(x,$ Reminder after (n+1) terms denoted as Rn+1

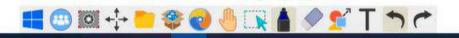
IXIM(b-a) (1; M= Sup {K(a,t): a(a,t),

then Rn+1 -> 0.

In that case the solution is given by 01/18/2022 02:59 PM 3/3 $u(x) = f(x) + \sum \{K(x,t) f(t) dt + \sum \{K(x,t) K(t,t) f(t)\}$ $+ \sum_{\alpha \alpha \alpha} \{K(x,t) K(t,t) K(t,t) \} + \sum_{\alpha \alpha} \{K(x,t) K(t,t) K(t,t) \} + \sum_{\alpha} \{K(x,t) K(t,t) K(t,t) \} + \sum_{\alpha} \{K(x,t) K(t,t) K(t,t) K(t,t) \} + \sum_{\alpha} \{K(x,t) K(t,t) K(t,t) K(t,t) \} + \sum_{\alpha} \{K(x,t) K(t,t) K(t,t) K(t,t) K(t,t) K(t,t) \} + \sum_{\alpha} \{K(x,t) K(t,t) K(t,t$

by method of enecessive embslitution. Note: f(x)=1, $r=\frac{1}{2}$, $K(x,t)=\sec x$.

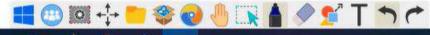




The solution to the IE is given by, 01/18/2022 03:07 PM 4/4 W(x)= f(x)+> (K(x,t) f(t) dt + >2/1 K(x,t) K(t,ti) f(t) f(x)=1, K(x,t)=sec=x, >= 1; a=0, + 1 / sec n dt + (1) / (1/4 exc n sec t dt, dt + (1) 5 My ory wear sect sect, dtzdt df = 1+ \frac{1}{2}. sec \frac{\pi}{4}. \frac{\pi}{4}. \frac{\pi}{4}. \frac{\pi}{4}. \frac{\pi}{4}. \frac{\pi}{4}. \frac{\pi}{4}. + (主)3 2022、正. = 1 + \frac{7}{4}(\frac{1}{2})(\frac{1}{2})\frac{1}{2} \frac{1}{2} \frac{1} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \f

En-2 solve u(2) = = = x + 1 (12t u(x) dt. by method of successive substitution. いいこまでももられた、まれかれもはりんれ、だれ、まれんれん + (b) 3 [] 22t. x2t, x2t, +2t, +2t, +2t, +2t, dt, dt- $= \frac{1}{8} x^{2} + \frac{1}{2} \cdot \frac{7}{8} x^{2} \int_{0}^{1} t^{3} dt + (\frac{1}{2})^{2} \cdot \frac{7}{8} x^{2} \left(\int_{0}^{1} t^{3} dt \right) \left(\int_{0}^{1} t^{3} dt_{1} \right) \left(\int_{0}^{1}$ 二 まれ [1+ 立・ 4 + (立)2(七)2+ (立)3(七)3+---- = また1-







Theorem Consider the Fredholm Intigral Equation 1916 $u(a) = f(a) + \gamma \int_{a}^{b} F(x,t) u(t) dt \longrightarrow (1)$.

If a) K(x,t) is real and continuous in R: {(x,t): a < x,t < i} such that |K(x,t)| \le M \tau (x,t) \in R.

- e) f(2) \fo in [a, b] & is great & continuous in [a, b]

c) > is a constant such that 1>1 < \frac{1}{M(l-a)}

Then method of successive substitution yields one and only one continuous solution in [a,l] for the IE (1).





Peroof. Substitute for u(t) its expression given in P. H. S. of (1). $t(x) = t(x) + x \int k(x,t) f(t) dt + x^2 \int k(x,t) k(t,t) u(t) dt dt$ $t(x) = t(x) + x \int k(x,t) f(t) dt + x^2 \int k(x,t) k(t,t) f(t) dt dt$ $= f(x) + x \int k(x,t) f(t) dt + x^2 \int k(x,t) k(t,t) f(t) dt dt$ $= f(x) + x \int k(x,t) f(t) dt + x^2 \int k(x,t) k(t,t) f(t) dt dt$ $= f(x) + x \int k(x,t) f(t) dt + x^2 \int k(x,t) f(t,t) f(t) dt dt$ $= f(x) + x \int k(x,t) f(t) dt + x^2 \int k(x,t) f(t,t) f(t) dt dt$ $\begin{array}{c} (x_{1}) = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) + \frac$

Let the last term containing re, i.e the rempleder after n terms be denoted by Rn. If u(x) is arounded to be continuous in [a,b],
then u(x) is bounded there. So, $|u(x)| \leq N$, say $\forall x \in [a,b]$ Then $|R_n| \geq |x^n| \int_{a}^{b} \cdots \int_{a}^{b} K(x,t) K(t,t_1) \cdots K(t_{n-2},t_{n-1}) u(t_{n-1})$ -! $|\lambda| \leq \frac{1}{M(b-a)}$.: $|\lambda|^{2} (b-a)^{2} M^{2} N \longrightarrow 0$ as



This shows that u(n) is given by $\frac{01/18/2022 \ 03:55 \ PM \ 9/10}{U(n)} = f(n) + \sum_{n=1}^{\infty} \frac{1}{N} \left(\frac{1}{N} \left(\frac{1}{N} \right) + \frac{1}{N} \left(\frac{1}{N} \right) \right) + \sum_{n=1}^{\infty} \frac{1}{N} \left(\frac{1}{N} \left(\frac{1}{N} \right) + \frac{1}{N} \left(\frac{1}{N} \right) \right) + \sum_{n=1}^{\infty} \frac{1}{N} \left(\frac{1}{N} \right) + \sum_{n=1}^{\infty} \frac{$

(*) represents an infinite series in which every term is continuous in [a, 6], since f(a) is cont, in [a,b] & K(a,t) is cont. in R={(a,t)!}

This review represents a continuous function in [a,b],

forovided it converges uniformly in [a,b].