Indian Institute of Technology Kharagpur
Department of Mathematics
Integral Equations and Variational Methods
Assignment - 2

Dead line: 03-04-2022 at 11.59 pm

A. Solve the following boundary value problems (BVPs) using Green's function.

1. $xy'' + y' = x;\ y(1) = y(e) = 0.$

2. $y'' - y = 2\sinh 1;\ y(0) = y(1) = 0.$

B. Reduce the following BVP to an equivalent integral equation using Green's function.

1. $y'' + \lambda y = 2x + 1;\ y(0) = y'(1), y'(0) = y(1).$

C. Find the extremals of the following functionals:

1. $I[y] = \int_1^2 \left(y'^2 + 2yy' + y^2\right) dx;\ y(1) = 1, y(2) = 0.$

2. I $[y] = \int_0^1 yy'^2 dx;\ y(0) = 1, y(1) = \sqrt[3]{4}.$

3. I $[y] = \int_0^1 \left(y'^2 - y^2 - y\right) e^{2x} dx;\ y(0) = 0, y(1) = e^{-1}$

4. $I[y] = \int_0^1 (e^y + xy')\, dx,\ y(0) = 0, y(1) = \alpha$

5. $I[y] = \int_0^\pi \left(y'^2 - y^2\right) dx; y(0) = 1, y(\pi) = -1$

6. I$[y] = \int_0^1 \left(y'^2 + 4y^2\right) dx;\ y(0) = e^2, y(1) = 1$

7. $I[y] = \int_{-1}^0 \left(240y - y'''^2\right) dx;\quad \begin{matrix} y(-1) = 1, y(0) = 0, \\ y'(-1) = -4.5, y'(0) = 0, \\ y''(-1) = 16, y''(0) = 0. \end{matrix}$

8. $I[y,z] = \int_0^{\pi} \left(2yz - 2y^2 + y'^2 - z'^2\right) dx;$ $\begin{array}{l} y(0) = 0, y(\pi) = 1, \\ z(0) = 0, z(\pi) = -1. \end{array}$

9. $I[y,z] = \int_0^{\pi/4} \left(2z - 4y^2 + y'^2 - z'^2\right) dx;$ $\begin{array}{l} y(0) = 0, y\left(\frac{\pi}{4}\right) = 1, \\ z(0) = 0, z\left(\frac{\pi}{4}\right) = 1. \end{array}$

10. $I[y(x)] = \int_0^1 \left(y'^2 + y''^2\right) dx;$ $\begin{array}{l} y(0) = 0, y(1) = \sinh 1, \\ y'(0) = 1, y'(1) = \cosh 1. \end{array}$

11.
$$I[y(x), z(x)] = \int_0^1 \left(y'^2 + z'^2 - 4xz' - 4z\right) dx,$$
$$y(0) = 0, z(0) = 0, y(1) = 1, z(1) = 1$$

subject to the condition
$$\int_0^1 \left(y'^2 - xy' - z'^2\right) dx = 2$$

12.
$$J[y(x)] = \int_0^1 y'^2(x)dx;\ y(0) = 0, y(1) = \frac{1}{4}$$

subject to the condition
$$\int_0^1 \left[y(x) - y'^2(x)\right] dx = \frac{1}{12}$$

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13. Find the shortest distance between the points $A(1,0,-1)$ and $B(0,-1,1)$ lying on the surface $x + y + z = 0$, by framing the problem as an isoperimetric problem.

14. Find the shortest distance between the point $A(1,0)$ and the ellipse $4x^2 + 9y^2 = 36$.

15. Find the shortest distance between the circle $x^2 + y^2 = 1$ and the straight tine $x + y = 4$.

16. Find the shortest distance from the point $M(0,0,3)$ to the surface $z = x^2 + y^2$.

17. Find the shortest distance between the surface $\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} = 1$ and $x^2 + y^2 + z^2 = 4$.
Ans. 1.