Started on	Tuesday, 5 April 2022, 11:37 AM
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Question 1

Complete

Marked out of 8.00

A submarine has two critical parts, each of which is needed for the submarine to work. The submarine runs continuously as long as the two required parts are working. The two parts have mutually independent exponential lifetimes before they fail. The expected lifetime of parts 1 and 2 are 10 weeks and 20 weeks, respectively. When a part fails, the submarine is shut down and an order is made for a new part of that type. When the submarine is shut down (to order a replacement part), the remaining working part is not subject to failure. The time to replace part 1 is exponentially distributed with mean 1 week; and the time to replace part 2 is uniformly distributed between 1 week and 3 weeks.

- What is the long-run proportion of time that the submarine is working? 5/6
- Assuming that all parts are initially working, what is the expected time until one of the two part fails? (in weeks) 20/3
- What is the probability that part 1 is the first part to fail? 2/3
- Suppose that new parts of type 1 each cost dollar 50; and new parts of type 2 each cost dollar 100. What is the long-run average cost of replacement parts per week? 80

Question 2

Complete

Marked out of 6.00

Suppose a parent has no offsprings with probability 1/4 and has two offspring with probability 3/4. If a population of such individuals begins with a single parent and evolves as a branching process determine u_i, the probability that the population is extinct by i th generation, i=2,3. Also find probability of ultimate extinction Π_0

Then

$u_3=$ 0.3160

$u_2=$ 0.2968

$\Pi_0=$ 0.3333

Question 3

Complete

Marked out of 2.00

Suppose that X has probability density function (pdf)

$f(x) = 2x, 0<x<1.$

Use probability intergal transform method to develope a technique to generate a realization of X. If we have a uniform variate as U=0.04, then using the developed technique the variate for X is

Answer: 0.2

Question 4

Complete

Marked out of 2.00

A stock is presently selling at a price of Rs 50 per share. After one time period, its selling price will (in present value rupees) be either Rs 150 or Rs 25. An option to purchase y units of the stock at time 1 for the (present value) price 125 y costs c y. What should c be in order for there to be no sure win?

Answer: 5

Question 5

Complete

Marked out of 2.00

Let $\{W_t, \ t\geq 0\}$ be a Wiener process(,i.e., Standard Brownian Motion process). Then for $0< s< t$,

$2W_s+W_t$ follows

Select one:

- a. N(0, 8s+t)
- b. N(0, s+t)
- c. N(0, 4s+t)
- d. N(0, 3s+t)

Question 6

Complete

Marked out of 2.00

Let $\{Xn, n \geq 0\}$ be a discrete-time Markov chain with state space {1,2,3} and tpm P=

0.1	0.3	0.6
0.0	0.4	0.6
0.5	0.2	0.3

Find $P(X_9 = 2| X_1 = 2, X_5 = 1, X_7 = 3)$

Answer: 0.29

Question 7

Complete

Marked out of 2.00

Let $\{ X_n; n \geq 0\}$ be a Markov chain with tpm P. Let $Y_n = X_{3n}$ for n =0,1,2,3,..... . The tpm of the Markov chain $\{ Y_n; n \geq 0\}$ is

Select one:

- a. P^3
- b. P^2
- c. P
- d. None of these

Question 8

Complete

Marked out of 2.00

Let {Xn, n ≥ 0} be a discrete-time Markov chain with state space {1,2,3 } and tpm P=

$$\begin{matrix} 1/2 & 1/2 & 0 \\ 0 & 2/3 & 1/3 \\ 1/4 & 0 & 3/4 \end{matrix}$$

The stationary distribution of the chain is (π_1, π_2, π_3)

Select one:

- a. 2/11, 5/11, 4/11
- b. 2/9, 1/3, 4/9
- c. 1/3, 4/9, 2/9
- d. 1/7, 2/7. 4/7

Question 9

Complete

Marked out of 5.00

Jesse is a new born baby who is always in one of the three states: eat, play and sleep.
He eats on average for 30 minutes at a time; plays on average for 1 hour; and sleeps for about 3 hours. After eating, there is a 50-50 chance he will sleep or play. After playing, there is a 50-50 chance he will eat or sleep. And after sleeping, he always plays. Jesse's life is governed by a continuous-time Markov chain. Find

(i) the holding time parameters for three states (in hours)

(ii) the transition matrix of the embedded Markov chain

(iii) the generator matrix Q

(iv) the stationary distribution of three states (eat, play, sleep).

(v) what proportion of the day does Jessey sleep?

The p(1,2), p(3,2) of the transition matrix of the embedded Markov chain	1/2,1
The stationary distribution of three states (eat, play, sleep).	(1/14, 4/14, 9/14)
The holding time parameters for three states (in hours) q1,q2,q3	2,1,1/3
The q(1,3), q(3,2) of the generator matrix Q	1,1/3
How many hours of the day does Jessey sleep?	15.43

Question 10

Complete

Marked out of 4.00

Customers arrive at a busy food truck according to the Poisson process with parameter λ. If there are *n* people already in the line, the customer will join the line with probability 1/(n+1) i.e. now the arrival rate is $\lambda_n = \lambda/(n+1)$, n=0,1,2,3...... Assume that the chef at the truck takes, on average, α (alpha) minutes to process the order. Find the long-term (i) average number of people in the system. (ii) probability that there are at least two people in the system

Select one:

- a. $\lambda\alpha$, $\lambda\alpha\, e^{-\lambda\alpha}$
- b. λ/α , $1-(1+\lambda/\alpha)e^{-(\lambda/\alpha)}$
- c. λ/α , $1-e^{-(\lambda/\alpha)}$
- d. $\lambda\alpha$, $1-(1+\lambda\alpha)\, e^{-\lambda\alpha}$

Question 11

Complete

Marked out of 3.00

Consider a random walk of a particle on states {1, 2, 3, ..., K } with two reflective barriers at 1 and K. Let p be the probability of going from state i to i + 1 and q = (1- p) be the probability of going from state i to i -1, $2 \leq i \leq K - 1$. If the particle is at state 1, it moves to state 2 with probability p and remains at state 1 with probability q. Similarly, if the particle is at state K it moves to K - 1 with probability q and remains at K with probability p. Let X_n be the position of the particle at n. (i) Draw the transition diagram of the chain. (ii) Find the transition probability matrix P of the Markov chain {Xn; n ≥ 1}. (ii) Find the limiting behavior of the Markov chain, i.e., π_i for i = 1,2, ..., K.

For p=1/3 and K=5, the value of π_3 is

Select one:

- a. 5/31
- b. 2/15
- c. 4/31 (selected)
- d. 8/15

Question 12

Complete

Marked out of 2.00

Suppose we have the option of buying, at some time in the future, one unit of a stock at a fixed price Rs. 200, independent of its current market price. The current market price of the stock is taken to be 0, and we suppose that it changes in accordance with a Brownian motion process having a drift coefficient -4. When, if ever, should we exercise our option?, i.e. we should exercise our option when market price becomes 200.125.

(write answer upto three decimal places)

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