<u>Dashboard</u> / My courses / <u>Stochastic Processes / Stochastic Process and Simulation (MA41017/ MA60067) - Spring 2022</u> / <u>Topic 1</u>

/ Class Test-1 / Preview

Started on Thursday, 10 February 2022, 4:56 PM

State Finished

Completed on Thursday, 10 February 2022, 5:20 PM

Time taken 23 mins 52 secs

Question ${f 1}$

Complete

Marked out of 3.00

An urn initially contains a single red ball and a single green ball. A ball is drawn at random, removed, and replaced by a ball of the opposite color, and this process repeats so that there are always exactly two balls in the urn. Let Xn be the number of red balls in the urn after n draws, with X0 = 1. Specify the transition probability matrix of Markov chain $\{Xn\}$, with state-space and answer the following questions.

$$p(2,2)=$$

Question **2**

Complete

Marked out of 3.00

A cab driver moves between the airport (A) and three hotels H1, H2, and H3 according to the following rule: if he is at the airport then he will move to one of the three hotels with equal probability. If he is at any one of the three hotels, then he returns to the airport with a probability of 1/4 and goes to one of the other two hotels with equal probability. Let Xn, n ≥ 1 be the position cab driver at time n. Construct the tpm of the Markov chain.

Find the long-run distribution of the cab driver being at the airport, and hotels H1, H2, and H3.

The long-run distribution of the cab driver being at H3.

4/15

The long-run distribution of the cab driver being at the airport,

1/5

The long-run distribution of the cab driver being at H2.

4/15

Question **3**Complete

Marked out of 1.00

Consider a Markov chain with state space $S = \{0,1,2,3\}$ and transition probability matrix P given by

	0	1	2	3
0	0	1	0	0
1	0	0	1	0
2	0	0	0	1
3	1/2	0	1/2	0

Obtain the period of state 0.

Answer: 2

Question 4

Complete

Marked out of 4.00

Three children (denoted by 1,2,3) arranged in a circle play a game of throwing a ball to one another. At each stage, the child having the ball is equally likely to throw it into any one of the other two children. Suppose that X_0 denotes the child who had the ball initially and X_n ($n \ge 1$) denotes the child who had the ball after n throws.

(Answer the following questions and then match the answer with the one given in the box))

- 1. Find the TPM of the Markov chain $\{X_n, n \ge 1\}$.
- 2. Calculate P($X_2=2 \mid X_0=3$).
- 3. Find the conditional probability of child 2 having the ball after 2 throws given that initially, he has the ball.

TPM entry p(3,2) =

Find the conditional probability of child 2 having the ball after 2 throws given that initially, he has the ball.

1/2

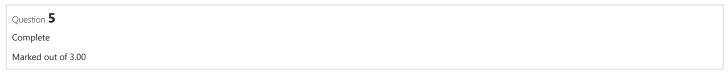
1/2

TPM entry p (1,3) =

1/2

 $P(X_2=2 | X_0=3) =$

1/4



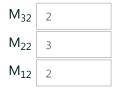
Consider a Markov chain with state space $S = \{1, 2, 3\}$ and transition probability matrix P given by

1/2 1/2 0

1/4 0 3/4

1/2 1/2 0

Find the mean recurrence time M_{22} and mean entrance times: M_{12} , M_{32}



Question **6**Complete

Marked out of 4.00

Consider a Markov chain with state space S={1, 2, 3, 4, 5, 6, 7} and tpm P=

	1	2	3	4	5	6	7
1	2/3	1/3	0	0	0	0	0
2	3/4	1/4	0	0	0	0	0
3	0	0	0	2/3	1/3	0	0
4	0	0	1	0	0	0	0
5	0	0	1	0	0	0	0
6	1/6	0	1/6	1/6	0	1/4	1/4
7	0	0	0	0	0	0	1

Classify the states of the Markov chain (Recurrent/transient/Absorbing)

Recurrent state $\{3,4,5\}$ Recurrent state $\{1,2\}$ Absorbing State $\{7\}$ Transient state $\{6\}$

Question 7	
Complete	
Marked out of 1.00	
Consider a Markov chain with state space S = { 1, 2, 3 } and tra matrix P given by 0 1 0 0 0 1 1 0 0 Find the mean recurrence time of state 1.	nsition probability
Question 8 Complete	
Marked out of 4.00	
Consider a Markov chain with state space S = { 1, 2, 3 } and tra matrix P given by 0.2 0.5 0.3 0.6 0.2 0.2 0.3 0.2 0.5	nsition probability
The 3-step first passage probabilities from state 1 to state 2 is $f(1,2)^{(2)}$	0.107
The 2-step first passage probabilities from state 1 to state 2 is	0.16
f(1,2)^(2)	
a 0	
Question 9 Complete	
Marked out of 2.00	
A state i of the Markov chain of state space S, is called recurrer Answer: 1	nt if the probability of ever returning to state i is equal to

Question 10	
Complete	
Marked out of 5.00	

Let {Xn , $n \ge 1$ } be a Markov chain with state space S={0,1,2,3,4} with tpm

	0	1	2	3	4
0	2/5	3/5	0	0	0
1	1/4	3/4	0	0	0
2	0	0	1	0	0
3	0	0	1/3	2/3	0
4	1	0	0	0	0

Then as $n \rightarrow \infty$, p(i,j) super subscript $n \ (x^n)$

is n step transition probability.

p(4,0) super subscript n	5/17
p(3,0) super subscript n	0
p(0,0) super subscript n	5/17
p(2,0) super subscript n	5/17
p(1,0) super subscript n	5/17

→ Announcements (hidden)

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