DEPARTMENT OF MATHEMATICS, IIT KHARAGPUR

Integral Equations and Variational Methods
Spring 2021 * Exam 1/3 * Date: 29.01.2021 * FM = 15
Time-duration: 45 minutes * Mode: Online

Instructions:

- Each of the 3 questions carries 5 marks.
- Show each step of calculations. Without showing proper step, no marks will be awarded.
- Give your file name as **rollno_exam1_29012021,** only PDF files will be accepted.
- Write your **name, roll number and serial number** on the top of every page.
- Make sure that there is no shadow in your photo that will be uploaded. If a student fails to upload clear picture of each page, his/ her exam will be considered as cancelled. NO equivalent exam will be taken.
- Your video shall be ON and audio shall be OFF. Focus your camera on the paper on which you will write. Failure of this will lead to cancellation of your exam.
- NO EMAIL/ MOODLE SUBMISSION – link will be provided to you where you have to submit.
- Time-duration is 45 minutes, 40 minutes for writing the paper and 5 minutes for taking photo and uploading.

1. Convert the following integral equation to an equivalent boundary value problem:

$$y(x) = 2x^2 + \int_0^1 K(x,t)y(t)\mathrm{d}t$$

where

$$K(x,t) = \begin{cases} \dfrac{\cos\left(t-x+\frac{1}{2}\right)}{2\sin\frac{1}{2}} & 0 \le t < x, \\ \dfrac{\cos\left(x-t+\frac{1}{2}\right)}{2\sin\frac{1}{2}} & x < t \le 1. \end{cases}$$

2. If the initial value problem given by

$$y'''(x) - xy''(x) + \frac{xy'(x)}{1+2x} + \frac{y(x)}{1+x} = \frac{x}{1+2x} ;$$

$$y(0) = -\frac{1}{\sqrt{2}} , y'(0) = \frac{1}{\sqrt{2}} , y''(0) = \sqrt{2}$$

is reduced to the integral equation $u(x) = \dfrac{1}{A(Bx+C)} + \int_0^x K(x,t)u(t)dt$,

find the constants A, B, C and the kernel $K(x,t)$.

3. Solve the following integral equation by the method of direct computation:
$$y(x) = \cos x + \frac{1}{2\pi} \int_0^\pi \sin(x - t)\, y(t)dt \ .$$

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