Indian Institute of Technology Kharagpur
Department of Mathematics
Integral Equations and Variational Methods
Assignment - 1

Dead line: 20-02-2022 at 11.59 pm

1. Classify the following equations as Volterra/Fredholm, first/second kind, singular/non-singular, homogenous/non-homogenous.

$$(a)\ u(x) = \sin x + 2\int_0^x \cos(x-\xi)\ u(\xi)\ d\xi$$

$$(b)\ u(x) = \int_0^x \frac{u(\xi)}{\sqrt{x-\xi}}\ d\xi$$

$$(c)\ f(x) = x - \int_0^1 \sinh(x-t)\sqrt{f(t)}\ dt$$

$$(d)\ y^2(x) = x + \frac{1}{2}\int_0^\infty e^{-(x-\xi)^2} y(\xi)\ d\xi$$

$$(e)\ u(x) = \int_0^1 (x-t)^2\ u(t)\ dt.$$

2. Check whether given function is solution to the given integral equation.

$$u(x) = \cos 2x; \qquad u(x) = \cos x + 3\int_0^x K(x,\xi)\ u(\xi)\ d\xi,$$

$$\text{where } K(x,\xi) = \begin{cases} \sin x \cos \xi, & 0 \le x \le \xi \\ \cos x \sin \xi, & \xi \le x \le \pi. \end{cases}$$

3. Reduce the following IVP to Volterra Integral equation (VIE)

$$y^{iv} + y'' + y = x; \quad y(0) = y'(0) = 1; \quad y''(0) = y'''(0) = 0.$$

4. Convert the following VIE to IVP. Hence solve for $u(x)$.

$$u(x) = 1 - \cos x + 2\int_0^x (x-t)^2\ u(t)\ dt$$

5. Convert the following BVP to Fredholm integral equation (FIE).

$$y^{(iv)} = y + 1; \quad y(0) = y'(0) = 0; \quad y''(1) = y'''(1) = 0.$$

6. Convert the following FIE to BVP. Hence solve for $u(x)$.

$$u(x) = \sinh x + \int_0^1 K(x,t)\ u(t)\ dt$$

$$\text{where } K(x,t) = \begin{cases} 4t\ (1-x), & 0 \le t \le x, \\ 4x\ (1-t), & x \le t \le 1. \end{cases}$$

7. Solve the following Integral Equation by the method of direct computation.

$$u(x) = e^{2x} - \frac{1}{4}(e^2+1)x + \int_0^1 xt\ u(t)\ dt.$$

8. Solve the following Integral Equation by the method of successive substitution.

$$u(x) = \frac{9}{10}x^3 + \frac{1}{2}\int_0^1 x^3 t\ u(t)\ dt$$

9. Solve the following Integral Equations by the method of successive approximation.

$$(a)\ u(x) = -\frac{1}{4} + \sec x\ \tan x + \frac{1}{4}\int_0^{\frac{\pi}{3}} u(t)\ dt;\ \text{take } u_0(x) = 1.$$

$$(b)\ u(x) = e^x + \int_0^x e^{(x-t)}\ u(t)\ dt;\ \text{take } u_0(x) = 1.$$

10. Find the eigenvalues and eigenfunctions for the Integral Equation. Hence verify the statements of Fredholm Alternatives.

$$u(x) = \lambda \int_0^1 (3 - \frac{3}{2}x)t\ u(t)\ dt.$$

11: Find the resolvent kernal corresponding to $K(x,t) = e^{-(x-t)}\sin(x-t)$. Hence find the solution of

$$u(x) = e^{-x} + \int_0^x e^{-(x-t)} \sin(x-t)\ u(t)\ dt.$$

12: Solve the following Abel IE's.

$$(a)\ \frac{8}{3}x^{\frac{3}{2}} + \frac{16}{5}x^{\frac{5}{2}} = \int_0^x \frac{1}{\sqrt{x-t}}\ u(t)\ dt,$$

$$(b)\ 2\pi\sqrt{x} = \int_0^x \frac{u(t)}{\sqrt{x-t}}\ dt.$$