DEPARTMENT OF MATHEMATICS, IIT KHARAGPUR
Integral Equations and Variational Methods,
Spring 2022 * Exam 1 * Date: 21.02.2022 * FM = 40
Department of Mathematics
Time-duration: 110 minutes * Mode: Online

Instructions:

- Show each step of calculations. Without showing proper step, no marks will be awarded.
- Give your file name as Rollno-IEVME1-21022022, only PDF files will be accepted.
- Write your name, roll number and page number on the top of every page.
- Make sure that there is no shadow in your photo that will be uploaded. If a student fails to upload clear picture of each page, his/ her exam will be considered as cancelled. NO equivalent exam will be taken.
- Your video shall be ON and audio shall be OFF. Focus your camera on the paper on which you will write. Failure of this will lead to cancellation of your exam.
- Only MOODLE SUBMISSION – link will be open between 4 and 5 pm.
- 5 marks will be deducted if not submitted by 5 pm.

(1) For what values of μ the integral equation

$$u(x) + \mu \int_0^1 e^{(x-t)}\ u(t)\ dt = 2\ xe^x$$

has a solution

$$u(x) = (2\ x - 5)\ e^x.$$

[3 Marks]

(2) Reduce the Fredholm integral equation

$$u(x) = \frac{1}{2} \int_0^1 K(x,t) u(t)\ dt$$
$$with\ K(x,t) = \begin{cases} t(1-x) + \frac{(x-1)}{2}, & 0 \le t < x \\ x(1-t) + \frac{x}{2}, & x < t \le 1 \end{cases}$$

to an equivalent boundary value problem. [5 Marks]

(3) Applying the method of separable kernel, solve the following integral equation

$$y(x) = x - 3\int_0^1 (xt^2 + x^2 t)\ y(t)\ dt$$

[7 Marks]

(4) Find the solution to the integral equation

$$u(x) = \frac{3.2}{4.2}\ x^{0.2}\ +\ \frac{2.2}{4.2}\int_0^1 x^{0.2}\ t\ u(t)\ dt$$

by the method of successive approximation taking $u_0(x) = 0$. Find u_1, u_2, u_3 and u_4 and write the general term u_n. [6 Marks]

(5) (a) Find eigenvalues, eigenfunctions of the integral equation

$$u(x) = \lambda \int_{-1}^1 xe^t\ u(t)\ dt$$

(b) Find nonzero solution to the associated homogeneous integral equation.

(c) Without solving the non-homogeneous equation

$$u(x) = x^{2m-1}e^{-x} + \lambda \int_{-1}^1 xe^t\ u(t)\ dt, \qquad m = 1, 2, 3...$$

discuss the existence of its solution. (Specify whether unique solution / no solution/ infinitely many solutions would exist.) [7 Marks]

(6) Solve the Volterra integral equation

$$y(x) = x + \int_0^x (t - x)\ y(t)\ dt$$

by the method of resolvent kernel. (You must find the iterated kernels K_1, K_2 and K_3 and write the general form K_n). [6 Marks]

(7) Find the solution to the integral equation

$$\int_0^x \frac{u(t)\ dt}{(x-t)^{\frac{1}{4}}} = x^{\frac{5}{6}} \qquad (0 < \alpha, n < 1)$$

in the form

$$u(x) = \frac{a\ \Gamma(a)\ x^{\mu}}{\Gamma(b)\ \Gamma(c)},$$

write down the values of a,b,c,μ. [6 Marks]