DEPARTMENT OF MATHEMATICS, IIT KHARAGPUR

Integral Equations and Variational Methods
Spring 2021 * Exam 2/3 * Date: 26.02.2021 * FM = 25
Time-duration: 75 minutes * Mode: Online

Instructions:

- Show each step of calculations. Without showing proper step, no marks will be awarded.
- Give your file name as **rollno_exam2_26022021,** only PDF files will be accepted.
- Write your **name, roll number and serial number** on the top of every page.
- Make sure that there is no shadow in your photo that will be uploaded. If a student fails to upload clear picture of each page, his/ her exam will be considered as cancelled. NO equivalent exam will be taken.
- Your video shall be ON and audio shall be OFF. Focus your camera on the paper on which you will write. Failure of this will lead to cancellation of your exam.
- NO EMAIL/ MOODLE SUBMISSION – link will be provided to you where you have to submit.
- Time-duration is 75 minutes, 70 minutes for writing the paper and 5 minutes for taking photo and uploading.

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1. Consider the Fredholm integral equation
$$u(x) = 1 + \frac{1}{2\pi}\int_0^{\pi} \sin(x+t)\, u(t)dt.$$
   a. Find the iterated kernels $K_n(x,t)$ $(n = 1, \ldots, 6)$ corresponding to the kernel $K(x,t)$. Hence write down the general form(s) of $K_n(x,t)$.
   b. Hence find the resolvent kernel $R(x,t;\lambda)$.
   c. Finally find the solution to the given integral equation using $R(x,t;\lambda)$. (6m)

2. Let $f(x) = \int_0^x (x-t)^{\alpha}\, u(t)dt$; $0 < \alpha < 1$; $f(x)$ and α are known and $f(x)$ is integrable. If the solution $u(x)$ is given as
$$u(x) = g(\alpha)\frac{d^2}{dx^2}\int_0^x K(x,t)\, f(t)dt,$$
find $g(\alpha)$ and $K(x,t)$.
Hint: Multiply both sides of given equation by $(y-x)^{-\alpha}$ and integrate w.r.to x between 0 and y. (5m)

3. Find the eigenvalues and eigenfunctions of the homogenous integral equation

$$u(x) = \lambda \int_1^2 \left[xt + \frac{1}{xt}\right] u(t)dt.$$

Express the entries in your answer correct to **4** decimal places.

(7m)

4. Solve the following Boundary Value Problem using Green's function technique:

$$y'' - y = -2\cos x\,;\ \ y(0) = y'(0),\ y(l) + y'(l) = 0.$$

(7m)

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