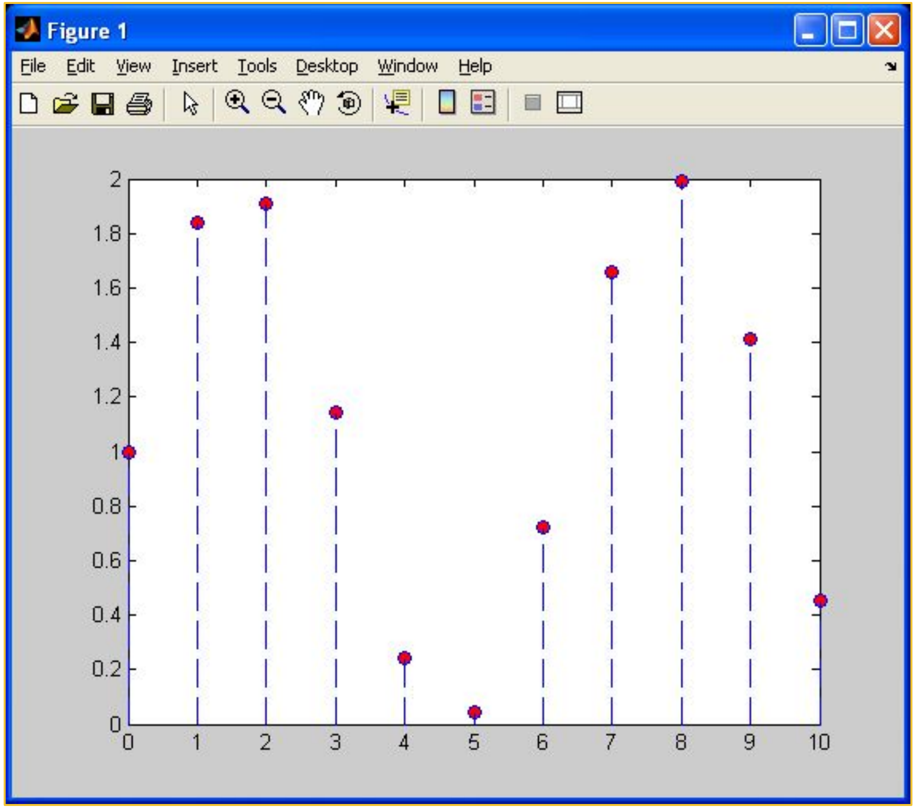
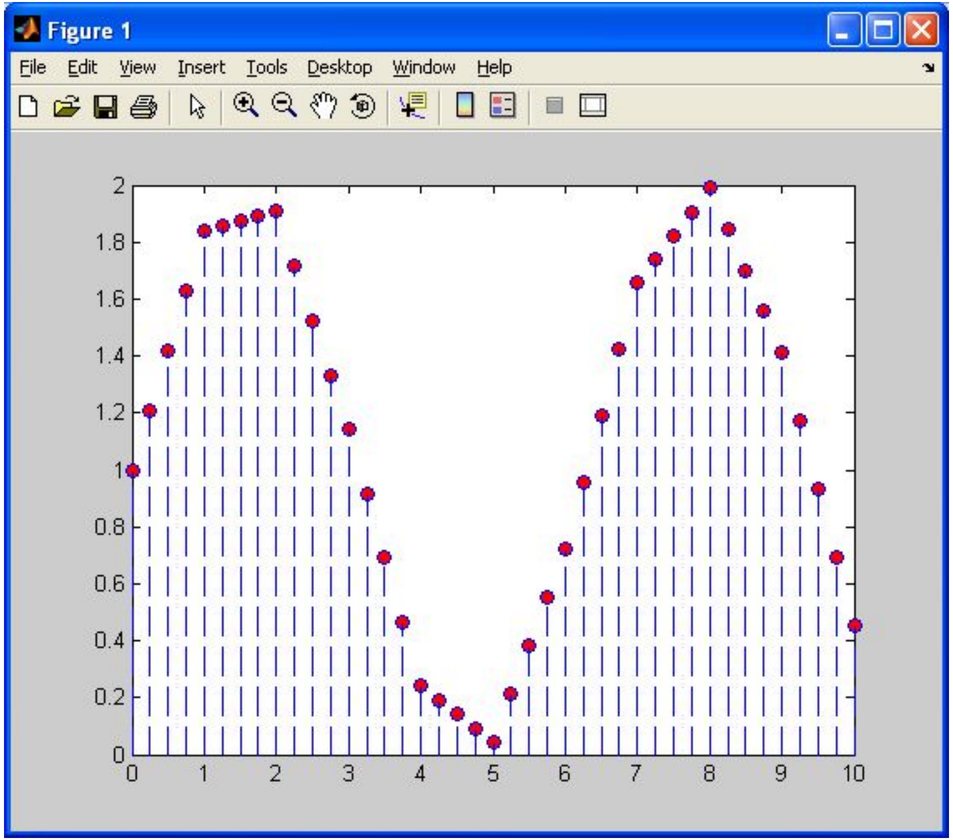
Interpolation

the estimation of the value of $f(x)$, or a function of x, from certain known values of the function. If $x_o < \dots < x_n$ and $y_o = f(x_o), \dots, y_n = f(x_n)$ are known, and if $x_o < x < x_n$, then the estimated value of $f(x)$ is said to be an interpolation. If $x < x_o$ or $x > x_n$, the estimated value of $f(x)$ is said to be an extrapolation.

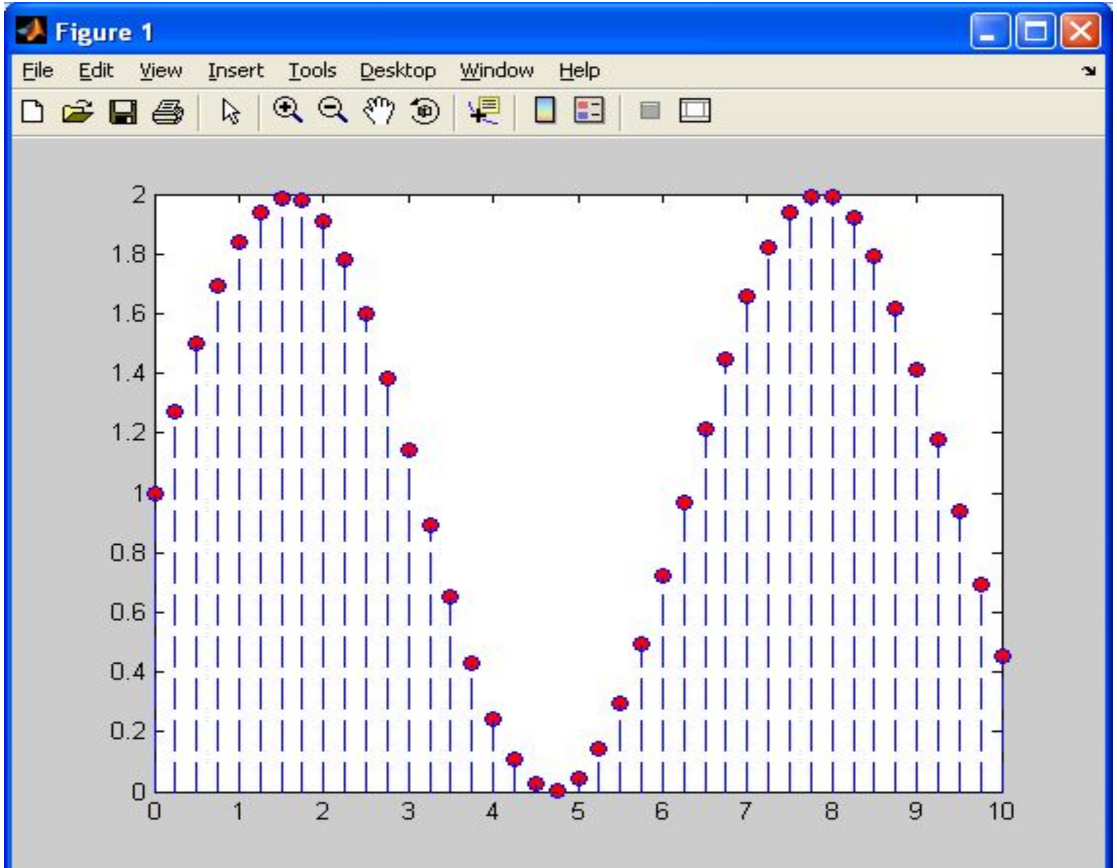
1 dimension data (pixel value of 11 points are given in graph)



Linear interpolation



Cubic interpolation



B spline interpolation

A spline is a piecewise polynomial representation of a smooth curve which connects a set of knots. Each piece of the spline between two consecutive knots is called a patch . On each patch, the spline is represented by a polynomial function of degree d.

The polynomials on patches are written as a function of an intrinsic parameter t which follows the curve of the spline. Given a set of n knots ti delimiting the ends of the patches, n − 1 patches can be defined between knots t1 and tn

All patches should be continuous and derivatives at knots.

In above images x axis can be view as intrinsic parameter t .

Knots are at t = [0 1 2 3 4 5 6 7 8 9 10]

We can see that at knots point pixel values are defined . patches should be defined in between the these values like between [0 , 1) , [1,2) …… [9, 10)

B spline interpolation

Patch is built as a linear combination of basis functions (B-spline functions) whose influence is driven by a set of neighboring control points $\boldsymbol{P_i}$

$$c_t = \Sigma_{i=0}^{n} P_i B_{i,d}(t)$$

t is the intrinsic parameter of a spatial system that follows the spline curve,

c(t) = spline curve coordinates evaluated at t in the Cartesian coordinate system

$\boldsymbol{P_i}$ are the control point coordinates

n is the number of fit knots,

d is the degree of the parametric curve,

$B_{i,d}(t)$ are the d-degree blending functions or the d-degree B-spline functions that can be defined by the Cox-de-Boor recursive algorithm.

B spline function

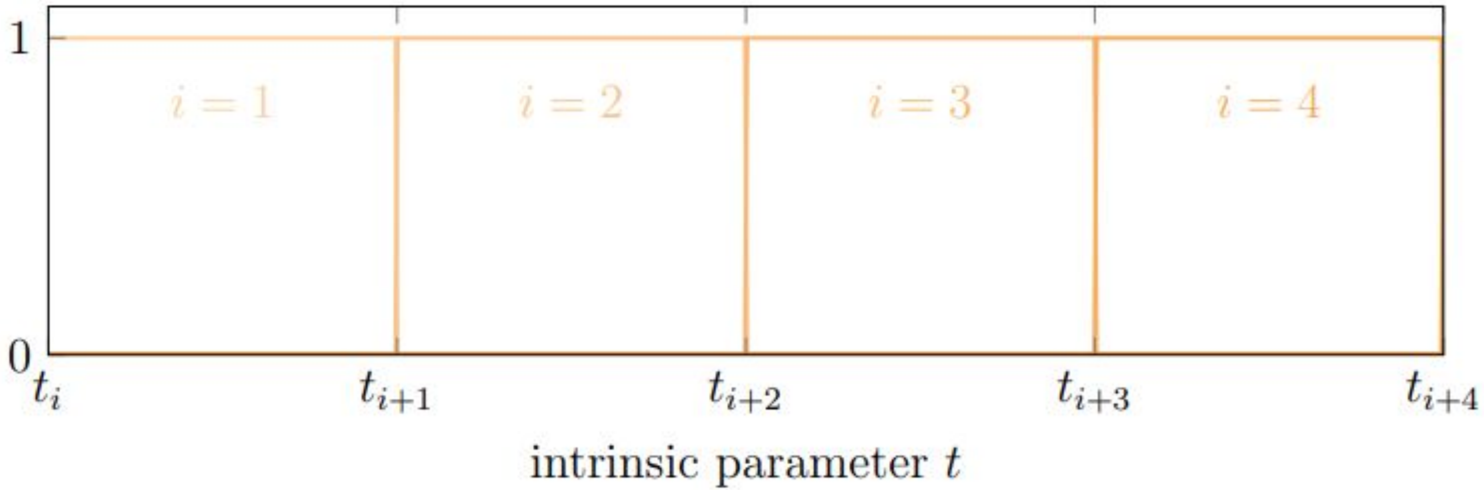
$$B_{i,0}(t) = \begin{cases} 1 \text{ if } t \in [t_i, t_{i+1}] \\ 0 \text{ if } t \notin [t_i, t_{i+1}] \end{cases}$$

$$B_{i,d}(t) = \frac{t - t_i}{t_{i+d} - t_i} B_{i,d-1}(t) + \frac{t_{i+d+1} - t}{t_{i+d+1} - t_{i+1}} B_{i+1,d-1}(t)$$

Constant B spline function for (0-degree)

$$B_{i,0}(t) = \begin{cases} 1 \text{ if } t \in [t_i, t_{i+1}] \\ 0 \text{ if } t \notin [t_i, t_{i+1}] \end{cases}$$

Constant B spline function for (0-degree)



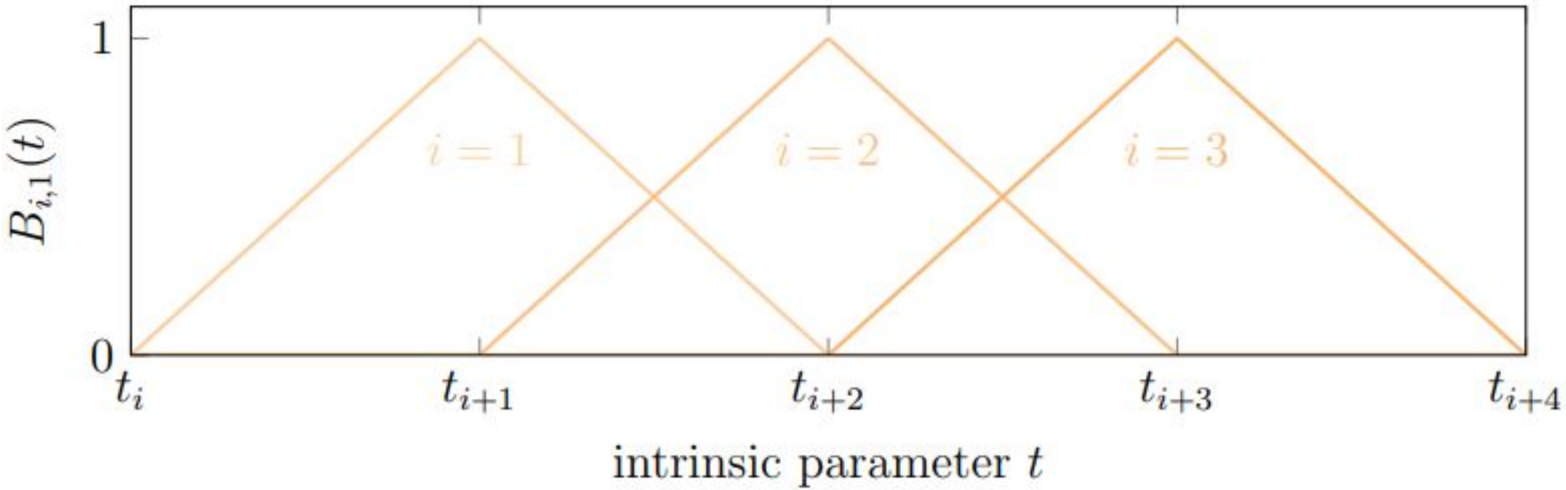
(a) order 0 B-spline functions

Linear B spline function for (1-degree)

$$B_{i,1}(t) = \frac{t - t_i}{t_{i+1} - t_i} B_{i,0}(t) + \frac{t_{i+2} - t}{t_{i+2} - t_{i+1}} B_{i+1,0}(t)$$

$$B_{i,1}(t) = \begin{cases} \frac{t-t_i}{t_{i+1}-t_i} & \text{if } t \in [t_i, t_{i+1}] \\ \frac{t_{i+2}-t}{t_{i+2}-t_{i+1}} & \text{if } t \in [t_{i+1}, t_{i+2}] \\ 0 & \text{otherwise} \end{cases}$$

Linear B spline function for (1-degree)



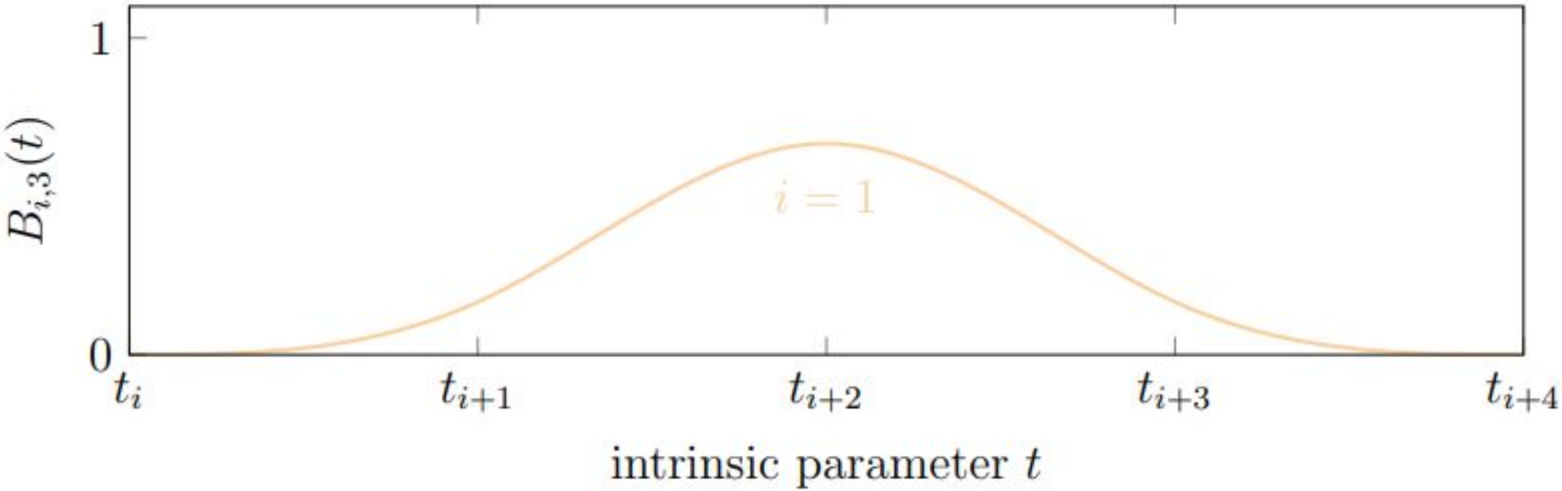
(b) linear B-spline functions

cubic B spline function for (3-degree)

$$B_{i,3}(t) = \frac{t - t_i}{t_{i+3} - t_i} B_{i,2}(t) + \frac{t_{i+4} - t}{t_{i+4} - t_{i+1}} B_{i+1,2}(t) \tag{10}$$

$$B_{i,3}(t) = \begin{cases} \frac{t-t_i}{t_{i+3}-t_i} \frac{t-t_i}{t_{i+2}-t_i} \frac{t-t_i}{t_{i+1}-t_i} & \text{if } t \in [t_i, t_{i+1}] \\ \frac{t-t_i}{t_{i+3}-t_i} \left[\frac{t-t_i}{t_{i+2}-t_i} \frac{t_{i+2}-t}{t_{i+2}-t_{i+1}} + \frac{t_{i+3}-t}{t_{i+3}-t_{i+1}} \frac{t-t_{i+1}}{t_{i+2}-t_{i+1}} \right] + \frac{t_{i+4}-t}{t_{i+4}-t_{i+1}} \frac{t-t_{i+1}}{t_{i+3}-t_{i+1}} \frac{t-t_{i+1}}{t_{i+2}-t_{i+1}} & \text{if } t \in [t_{i+1}, t_{i+2}] \\ \frac{t-t_i}{t_{i+3}-t_i} \frac{t_{i+3}-t}{t_{i+3}-t_{i+1}} \frac{t_{i+3}-t}{t_{i+3}-t_{i+2}} + \frac{t_{i+4}-t}{t_{i+4}-t_{i+1}} \left[\frac{t-t_{i+1}}{t_{i+3}-t_{i+1}} \frac{t_{i+3}-t}{t_{i+3}-t_{i+2}} + \frac{t_{i+4}-t}{t_{i+4}-t_{i+2}} \frac{t-t_{i+2}}{t_{i+3}-t_{i+2}} \right] & \text{if } t \in [t_{i+2}, t_{i+3}] \\ \frac{t_{i+4}-t}{t_{i+4}-t_{i+1}} \frac{t_{i+4}-t}{t_{i+4}-t_{i+2}} \frac{t_{i+4}-t}{t_{i+4}-t_{i+3}} & \text{if } t \in [t_{i+3}, t_{i+4}] \\ 0 & \text{otherwise} \end{cases}$$

cubic B spline function for (3-degree)



(d) cubic B-spline functions

Assignment 2 Q1

FInd value of f(2.5) using b spline interpolation 0, 1 and 3 degree.:

$f(0) = 2$

$f(1) = 3$

$f(2) = 4$

$f(3) = 2$

$f(4) = 5$

Assignment 2 Q2

2.Scale the following image (1 x 10 pixel) 2 times in x-direction with the help of

a. 0-degree b-spline function
b. 1-degree b-spline function
c. 3-degree b-spline function

Intensity of each pixel value is given corresponding to its index.

Pixel value	10	20	30	40	50	60	70	80	90	100
Index	0	1	2	3	4	5	6	7	8	9