Convex Optimisation
C1 Review Test
IIT2018031

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Answers

(2) Soln

Given →

$$f(x) = 3x^3 - 16$$

After differentiating w.r.t x.

$$f'(x) = 9x^2 \quad | \quad f'(x) = 0 \Rightarrow 9x^2 = 0 \Rightarrow x = 0$$

Now again differentiating w.r.t x.

$$f''(x) = 18x \quad | \quad f''(x) = 0 \Rightarrow 18x = 0 \Rightarrow x = 0$$

For inflection points →

$\therefore f'(x) = 0$ at $x = 0$

also $f''(x) = 0$ at $x = 0$

Now, we check the concavity of the curve

$f''(0^-) < 0$

$\therefore f(x)$ is concave down in this region

$f''(0^+) > 0$

$\therefore f(x)$ is concave up in this region

also $f''(x)$ is defined everywhere

Hence, $f''(x)$ changes concavity at $x = 0$ and it is defined as $f''(x)$.

Therefore, at $x = 0$, $f(x)$ has an inflection point.

Also, a cubic function always have one inflection point.

Ques 1 Sol:

Given :-

$$y = x^T A x + b^T$$

Now, differentiating the above eqn w.r.t x

$$\frac{dy}{dx} = \frac{d(x^T A x)}{dx} + \frac{d(b^T)}{dx} = 0$$

$$= \frac{d\left(\sum_{i=1}^{n} A x_i - x_i\right)}{dx} = \frac{d\left(\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_j x_i\right)}{dx}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i dx_j + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_j dx_i$$

$$= \sum_{i=1}^{n} (Ax) dx_i + \sum_{j=1}^{n} (A \cdot dx) x_i = (dx)^T Ax + x + A dx$$

$$= (dx)^T Ax + (dx)^T A^T = (dx)^T (A + A^T) x$$

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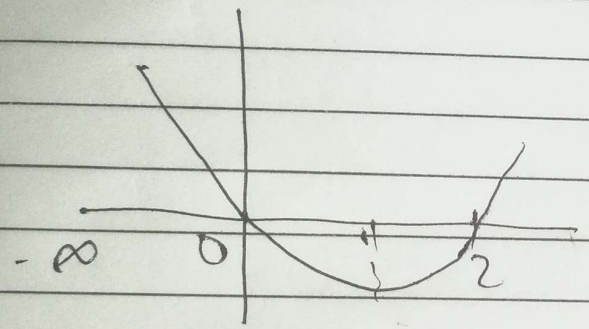
Ques 5 Soln

Given →

$$f(x) = x^2/2 - x \; ; \; x \in [0, 1000]$$

Iteration	a	b	a+b/2	f(a)	f(b)	$\frac{f(a)+f(b)}{2}$
1	0	1000	500	0	499000	124500
2	0	500	250	0	124500	31000
3	0	250	125	0	31000	7687.5
4	0	125	62.5	0	7687.5	1890.625
5	0	62.5	31.25	0	1890.625	457.03
6	0	31.25	15.625	0	457.03	106.49
7	0	15.625	7.8	0	106.44	22.62
8	0	7.8	3.9	0	22.6	3.7
9	0					



$[0, 1000]$

$$c^1 = \frac{0+1000}{2} = 500 \text{ & } f'(c) > 0$$

$\therefore [0, 500]$

$$c^2 = \frac{0+500}{2} = 250 \text{ & } f'(c^2) > 0$$

⋮

till

$[0, 1953 \simeq 2]$

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Ques 1

First order optimality is a measure of how close a point x is to its optima. Most optimization solvers use this measure though it has different definitions for a different algorithm. First order optimality is a necessary condition but it's not a sufficient condition.