

Convex Optimization

C3 Review

May 15, 2021

Time: 2 hrs

MM: 40

Note: *Attempt all questions*

1. Derive the optimal step factor for a line search on a quadratic function

$$\min_{\alpha} f(\mathbf{x} + \alpha \mathbf{d})$$

2. Consider

$$\text{Minimize } f(x, y) = x^4 + 2y^2 - 4xy$$

- a) Find the points that satisfy first order optimality conditions. Are they local minima?
- b) Is f a convex function?

3. Consider

$$\text{Minimize } x_1^2 + x_2^2 + x_3^2$$

- a) If the steepest decent method is used to find the minimum from the starting point $\mathbf{x}_0 = (1, -1, 0)^T$, how many iterations will it take to reach the optimum?
- b) Is the number of iterations same irrespective of the starting point? If so, why?

4. Consider

$$\text{Minimize } f(x) = x^4$$

What will be the convergence rate of Newton's method if it is started from $x_0 = 1$.

5. Consider

$$\text{Minimize } -x_1$$

$$\text{Subject to } x_1^2 + x_2^2 \leq 1; x_1, x_2 \geq 0$$

- a) Verify whether $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ satisfies first order KKT conditions or not.
- b) Is the point minimum. Show graphically.

6. Consider

Maximize $f(x_1, x_2) = x_1 + x_2$

subject to $g(x_1, x_2) = -(x_1^2 + x_2^2) \geq 0; x_1, x_2 \geq 0$

- a) Write the lagrangian and the conditions for an optimum.
- b) Is the Slater condition satisfied?
- c) Do we have a solution?

7. Discuss

- a) Barrier method
- b) Duality