Convex Optimization

Practice Problems

1. Consider the function

$$f(x, y) = x^2 + y^2 + \beta xy + x + 2y$$

For what values of β, does this function have a unique global minimum?

2. Suppose the one dimensional function $f(\mathbf{x}_k + \alpha \mathbf{d}_k)$ is unimodal and differentiable. Let α^* be the minimum of the function. If any $\alpha > \alpha^*$ is selected, show that $\nabla f(\mathbf{x}_{k+1})^T \mathbf{d}_k > 0$.

3. Consider the following problem.

$$\text{maximize} \quad f(x, y) = \exp(-\frac{1}{3}x^3 + x - y^2)$$

Suppose you want to do it using pure Newton's method. Is $\mathbf{x}_0 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ a good starting point?

4. Find the rectangle of given perimeter that has greatest area by using Lagrange multiplier theorem. Verify it using second order conditions.

5. Consider

$$\begin{aligned} \text{minimize} \quad & f(x, y) = -x \\ \text{subject to} \quad & y - (1 - x)^3 \leq 0 \\ & -y \leq 0 \end{aligned}$$

(a) Find the optimal solution by solving it graphically.

(b) Do Lagrange multipliers exist? How could you say without actually solving the problem?