

1)

$$A \quad B$$

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 5 \\ -4 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\text{scale} = [5, 5, 4]$$

$$\text{index} = [1, 2, 3]$$

Forward elimination part

Step 1.

$$\text{Ratios} = \left\{ \frac{|1|}{5}, \frac{|2|}{5}, \frac{|-4|}{4} \right\} \quad \text{max one is } l_3$$

swap (l_1, l_3)

$$\text{index} = [3, 2, 1]$$

$$\text{pivot} = 3$$

Let's update A and B.

$$\text{factor}_1 = 2 / -4 = -0.5$$

$$\text{factor}_2 = 1 / -4 = -0.25$$

$$\begin{bmatrix} 0 & 4.25 & 5.75 \\ 0 & 2.5 & 4.5 \\ -4 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.75 \\ 3.5 \\ 3 \end{bmatrix}$$

step 2:

$$\text{Ratios} = \left\{ \frac{14,25}{5}, \frac{12,5}{5} \right\}$$

max one is 1,

$$\text{swap}(l_2, l_1)$$

$$\text{index} = [3, 2, 2]$$

$$\text{pivot} = 1$$

Let's update A and B.

$$\text{factor 1} = 2,5 / 1,25 = 0,588$$

$$\begin{bmatrix} 0 & 4,25 & 5,75 \\ 0 & 0 & 3,117 \\ -4 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1,75 \\ 2,47 \\ 3 \end{bmatrix}$$

Backward substitution part

Step 1:

$$3,117x_3 = 2,47$$

$$x_3 = 0,792$$

~~$3,117x_3 = 2,47$~~
 ~~$x_3 = 0,792$~~

$$4,25x_2 = 1,75 - 5,75 \cdot x_3 = -2,765$$

$$x_2 = -0,66$$

$$-4x_1 = 3 - 3x_3 - x_2 = 1,277$$

$$x_1 = -0,32$$

$$\text{Result list: } \begin{bmatrix} -0,32 \\ -0,66 \\ 0,79 \end{bmatrix}$$