ChatGPT 4o

What are the recent advances for evolutionary algorithms?

Evolutionary algorithms (EAs) have experienced significant advancements in recent years, enhancing their performance and expanding their applicability across various domains. Notable developments include:

1. **Advanced Evolutionary Strategies:**
   - **Multi Expression Programming (MEP):** MEP evolves computer programs by representing genes as substrings, enabling the development of EAs that can compete with traditional algorithms. RESTACK
   - **Linear Genetic Programming (LGP):** LGP evolves imperative computer programs, offering a linear representation that facilitates efficient program evolution. RESTACK

2. **Integration with Machine Learning:**
   - **Learnable Evolutionary Algorithms:** Combining EAs with machine learning techniques has led to scalable multi-objective optimization methods capable of addressing complex problems with numerous objectives and constraints. ARXIV
   - **Neuroevolution:** This approach evolves artificial neural networks using evolutionary algorithms, optimizing both the architecture and weights of neural networks, and has been applied in reinforcement learning and robotics. WIKIPEDIA

3. **Theoretical Advancements:**
   - **Runtime Analysis:** Recent theoretical insights have provided a deeper understanding of the performance and efficiency of EAs, leading to more effective algorithm designs. ARXIV
   - **Parameter Control:** Advances in parameter tuning and control have improved the adaptability and robustness of EAs across diverse optimization scenarios. ARXIV

4. **Multi-Objective Optimization:**
   - **Evolutionary Multi-Objective Algorithms (EMOAs):** Techniques such as NSGA-II

and its successors have been developed to efficiently handle optimization problems involving multiple conflicting objectives, providing a set of optimal trade-off solutions. WIKIPEDIA

5. **Software and Frameworks:**

- **evosax:** A JAX-based library that leverages modern hardware accelerators for evolutionary optimization, facilitating the development of scalable EAs. ARXIV
- **EC-KitY:** An evolutionary computation toolkit in Python that provides a modular framework for developing and experimenting with EAs. ARXIV

These advancements have broadened the scope of evolutionary algorithms, enabling them to tackle increasingly complex optimization problems across various fields.

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I've read about an approach where the EA approximates the distribution of the population

The approach you're referring to likely involves **estimation of distribution algorithms (EDAs)** or related methodologies. These methods are a significant evolution of traditional evolutionary algorithms (EAs) and focus on modeling the distribution of promising solutions instead of relying solely on selection, crossover, and mutation.

Key Features of Distribution-Based EAs:

1. **Modeling Population Distribution:**
   - Instead of working directly with individual solutions, these approaches build probabilistic models (like Gaussian distributions or Bayesian networks) that approximate the distribution of the best-performing individuals in the population.
   - This probabilistic representation allows the algorithm to sample new candidates from the learned distribution, potentially improving search efficiency.

2. **Estimation of Distribution Algorithms (EDAs):**
   - EDAs replace traditional variation operators with statistical modeling.
   - The algorithm iteratively selects high-quality solutions, estimates a probability distribution over them, and then samples from this distribution to create the next generation.
   - Example distributions include multivariate normal distributions, kernel density estimators, and more complex graphical models.
3. **Applications and Advantages:**
   - **Scalability:** EDAs can handle large and complex solution spaces efficiently, especially when combined with dimensionality reduction techniques.
   - **Optimization in High-Dimensional Spaces:** They are particularly well-suited for problems where dependencies between variables are crucial to identifying optimal solutions.
   - **Reduced Premature Convergence:** By focusing on the distribution rather than fixed crossover/mutation strategies, they often avoid local optima traps better than traditional EAs.
4. **Recent Innovations in the Domain:**
   - **Deep Generative Models in EAs:** Modern approaches integrate generative neural networks (e.g., Variational Autoencoders or GANs) to learn and sample from complex distributions of solutions.
   - **Covariance Matrix Adaptation (CMA-ES):** A well-known algorithm in this category, CMA-ES learns and adapts a multivariate Gaussian distribution to approximate the distribution of the current population's fitness.
   - **Surrogate-Assisted Distribution Learning:** Hybrid approaches leverage machine learning models to approximate fitness landscapes and improve distribution modeling.
5. **Challenges:**
   - The computational cost of building and sampling from distributions increases with problem complexity.
   - Careful tuning of the statistical models is needed to ensure convergence without

excessive sampling or overfitting to a specific distribution.

Example Workflow of an EDA:

1. Initialize a random population.
2. Evaluate fitness and select top-performing individuals.
3. Estimate a probabilistic model of the selected individuals.
4. Sample new individuals from this distribution.
5. Repeat the process until convergence.

Future Directions:

The combination of EDAs with advanced machine learning techniques, reinforcement learning, and hardware accelerators (e.g., GPUs and TPUs) holds promise for even more efficient and robust distribution-based evolutionary approaches.

What if we combine EDAs with traditional evolutoinary algorithms. E.g., we start off with an EA and then once we sampled enough we switch to the EDA or augument the EA

Combining Estimation of Distribution Algorithms (EDAs) with traditional Evolutionary Algorithms (EAs) creates a hybrid approach that leverages the strengths of both techniques. This type of hybridization can lead to improved performance in various optimization scenarios, especially when navigating complex fitness landscapes. Here's a detailed exploration of this idea:

Key Concepts in Hybrid EDA-EA Approaches

1. **Dynamic Transition Between EA and EDA:**
   - Begin with a traditional EA to explore the search space using crossover, mutation

- Begin with a traditional EA to explore the search space using crossover, mutation, and selection.
- Transition to an EDA after collecting enough high-quality solutions to reliably estimate a probabilistic model.
- This approach allows the algorithm to use the exploration capabilities of EAs early on and the exploitation efficiency of EDAs later.

2. **Simultaneous EDA-Augmented EA:**
   - While running a traditional EA, intermittently fit a probabilistic model (e.g., Gaussian or Bayesian network) to the current population.
   - Use the model to guide mutation or crossover operators, ensuring offspring are sampled from promising regions of the search space.
   - This approach enhances the variation operators without entirely replacing them, creating a balance between stochasticity and guided sampling.
3. **Adaptive Hybridization:**
   - Dynamically adjust the weight between EA and EDA components based on the optimization stage (e.g., exploration versus exploitation) or metrics like convergence speed or population diversity.

Advantages of Hybridizing EA and EDA

1. **Early Exploration and Late Exploitation:**
   - EAs are better suited for global exploration during early stages, while EDAs excel in local exploitation once the algorithm focuses on promising regions.
2. **Improved Diversity Management:**
   - Traditional EAs often suffer from premature convergence; augmenting them with EDA-based sampling can maintain diversity by modeling and sampling from underexplored areas.
3. **Better Handling of Dependencies:**
   - EDAs can capture complex dependencies between decision variables that

traditional recombination operators may miss.

4. **Flexibility:**
   - By switching or combining approaches, the hybrid algorithm can adapt to different problem complexities and phases of optimization.

Implementation Strategies

1. **Switching After a Threshold:**
   - Define a transition criterion (e.g., a specific generation number, population diversity drop, or convergence rate).
   - After the threshold is met, stop traditional operators and switch to an EDA-based approach.
2. **Hybrid Operators:**
   - Use EDAs to influence traditional operators:
     - **Crossover Replacement:** Replace crossover with probabilistic sampling.
     - **Mutation Biasing:** Guide mutation probabilities based on the distribution.
   - For example, the Covariance Matrix Adaptation (CMA) can act as a mutation guide.
3. **Distributed Computation:**
   - Use parallel processing to run EA and EDA components simultaneously. The EA can explore broadly while the EDA fine-tunes in promising subregions, sharing results periodically.
4. **Multi-Population Hybridization:**
   - Maintain separate populations for EAs and EDAs. Exchange solutions between them periodically to ensure both exploration and exploitation are balanced.

Challenges

1. **Parameter Tuning:**
   - Transition timing and the balance between EA and EDA components require careful tuning.
2. **Computational Cost:**
   - Running EDAs alongside EAs can be computationally intensive, particularly for high-dimensional problems.
3. **Overfitting Risk:**
   - Excessive reliance on EDA models may lead to overfitting the population to suboptimal regions.

Real-World Applications

1. **Combinatorial Optimization:**
   - Problems like traveling salesman or job scheduling, where dependencies between variables are crucial.
2. **Multi-Objective Optimization:**
   - Hybrid approaches can effectively balance exploration and exploitation across multiple conflicting objectives.
3. **Engineering Design:**
   - Hybrid methods are useful for optimizing complex systems with intricate constraints.

Example Algorithm

Here's a high-level pseudo-code for a hybrid EA-EDA algorithm:

python Code kopieren

```python
Initialize population using traditional EA
Evaluate fitness of population
for generation in range(max_generations):
    if generation < switch_point:
        # Use traditional EA operators
        Select parents
        Apply crossover and mutation
    else:
        # Use EDA-based operators
        Estimate distribution of elite solutions
        Sample new solutions from distribution
    Evaluate fitness of new solutions
    Update population
```

This hybridization provides a flexible and powerful framework for solving diverse optimization problems effectively.

Do you know if someone tried this approach? I think the biggest problem of EDAs are the collection of data and the biggest problem of EAs are the sample inefficiency. Therefore they seem like a natural fit

Yes, the integration of Estimation of Distribution Algorithms (EDAs) with traditional Evolutionary Algorithms (EAs) has been explored to capitalize on the strengths of both methodologies. This hybrid approach aims to address the data collection challenges in EDAs and the sample inefficiency in EAs, creating a more robust optimization framework.

Hybrid Approaches in Literature:

1. **GA-EDA Hybrid Algorithm:**
   - Researchers have developed a hybrid algorithm named GA-EDA, which combines genetic algorithms (GAs) with EDAs. The objective is to leverage the exploratory capabilities of GAs and the model-building strengths of EDAs. Experimental

results on synthetic optimization problems and real-world cases have demonstrated the effectiveness of this approach. SPRINGER LINK

2. **Guided Mutation Strategies:**
   - The concept of guided mutation integrates ideas from EDAs into GAs. In this strategy, offspring generation is influenced by probabilistic models learned from the population, enhancing the search efficiency of the evolutionary algorithm. Studies have shown that evolutionary algorithms with guided mutation outperform traditional GAs in solving complex problems like the maximum clique problem. SPRINGER LINK
3. **Hybrid Estimation of Distribution Algorithm with Differential Evolution:**
   - A novel hybrid algorithm combines EDAs with Differential Evolution (DE) to improve global search capabilities and convergence speed. This method selects either DE or EDA mechanisms for generating new trial solutions, adapting to the problem landscape dynamically. Comparative studies indicate that this hybrid approach outperforms state-of-the-art algorithms in various optimization scenarios. IEEE XPLORE

Advantages of Hybridization:

- **Enhanced Exploration and Exploitation:**
  - Traditional EAs are proficient in exploring the search space, while EDAs excel in exploiting promising regions by building probabilistic models. Combining these approaches can lead to a more balanced search strategy, improving optimization performance.
- **Improved Sample Efficiency:**
  - By integrating EDAs, which model the distribution of high-quality solutions, the hybrid algorithm can generate offspring that are more likely to be optimal, thereby enhancing sample efficiency compared to traditional EAs.
- **Adaptive Search Mechanisms:**
  - Hybrid algorithms can dynamically adjust their search strategies based on the problem landscape, switching between EA and EDA components as needed to maintain diversity and convergence.

Considerations:

- **Computational Complexity:**
  - While hybrid algorithms can offer performance benefits, they may also introduce additional computational overhead due to the combined operations of EAs and EDAs. Careful implementation and parameter tuning are essential to balance performance gains with computational costs.
- **Parameter Tuning:**
  - The effectiveness of hybrid algorithms depends on the appropriate tuning of parameters governing the transition between EA and EDA components. Adaptive mechanisms or empirical studies may be necessary to determine optimal settings for specific problem domains.

In summary, combining EDAs with traditional EAs has been investigated and shows promise in addressing the limitations inherent in each approach. By leveraging the complementary strengths of EAs and EDAs, hybrid algorithms can achieve more efficient and effective optimization, particularly in complex problem spaces.

Quellen

Are you familiar with Rolling Horizon Evolutionary Algorithms?

Yes, I'm familiar with Rolling Horizon Evolutionary Algorithms (RHEAs). They are a specialized class of evolutionary algorithms designed for decision-making and optimization in real-time or sequential environments. Here's an overview:

What are Rolling Horizon Evolutionary Algorithms (RHEAs)?

RHEAs are evolutionary algorithms tailored for **sequential decision-making problems**, such as those encountered in games, robotics, and dynamic systems. Instead of solving a single static optimization problem, RHEAs optimize a **sequence of actions** to achieve a long-term goal.

Key Characteristics:

1. **Sequential Decision Optimization:**
    - RHEAs work by evaluating potential sequences of decisions over a limited horizon, rolling forward with new information as it becomes available.
2. **Real-Time Adaptation:**
    - These algorithms are designed to operate within strict computational time budgets, making them suitable for environments where decisions need to be made rapidly.
3. **Sliding Window Mechanism:**
    - After making a decision (e.g., the first action in the sequence), the algorithm shifts the planning horizon forward and reoptimizes for the next steps based on the updated state of the system.
4. **Population-Based Optimization:**
    - Similar to traditional evolutionary algorithms, RHEAs maintain a population of potential solutions (action sequences) and iteratively improve them using selection, crossover, and mutation.

Workflow of an RHEA

1. **Initialize Population:**
    - Generate random sequences of actions for the planning horizon.
2. **Evaluate Fitness:**
    - Assess the quality of each action sequence using a fitness function, often a simulation of the environment to predict the consequences of the actions.
3. **Evolve Population:**

- Apply genetic operators (selection, crossover, mutation) to create new sequences.

4. **Select Best Action:**
   - Execute the first action from the best sequence.
5. **Roll Horizon Forward:**
   - Move to the next state, update the environment, and repeat the process with a new population.

Applications of RHEAs

1. **Game AI:**
   - Widely used in video games and game-playing AI for planning and strategy optimization. For example, in games like chess or RTS (Real-Time Strategy), RHEAs can optimize sequences of moves.
2. **Robotics:**
   - RHEAs help in real-time control and navigation by optimizing action sequences under dynamic constraints.
3. **Dynamic Optimization Problems:**
   - Suitable for problems where the environment changes over time, requiring continuous adaptation.
4. **Resource Allocation:**
   - Useful in scenarios like manufacturing and logistics, where decisions must be optimized in a rolling manner as new demands or constraints arise.

Advantages of RHEAs

- **Real-Time Performance:**
  - Designed for environments with strict time constraints.

- **Sequential Optimization:**
    - Optimizes sequences of actions rather than static solutions, making them highly versatile for dynamic problems.
- **Adaptability:**
    - Continuously adapts to changes in the environment by re-evaluating decisions at each step.

Challenges of RHEAs

1. **Limited Planning Horizon:**
    - The performance of RHEAs can depend heavily on the length of the planning horizon. A short horizon may lead to suboptimal decisions, while a long horizon increases computational costs.
2. **Fitness Evaluation Costs:**
    - Evaluating sequences often requires simulating or predicting outcomes, which can be computationally expensive.
3. **Premature Convergence:**
    - Similar to other evolutionary algorithms, RHEAs may converge prematurely to suboptimal sequences if diversity is not maintained.
4. **Balancing Exploration and Exploitation:**
    - Finding the right balance is crucial for ensuring the algorithm remains effective over time.

Recent Advances and Research Directions

1. **Neuroevolution Integration:**
    - Combining RHEAs with neural networks for fitness approximation, allowing faster

Combining RHEAs with neural networks for fitness approximation, allowing faster evaluations in complex environments.

2. **Hybrid RHEA Approaches:**
   - Integration with Monte Carlo Tree Search (MCTS) or reinforcement learning for better long-term planning.
3. **Improved Diversity Maintenance:**
   - Techniques to preserve population diversity to avoid premature convergence in sequential decisions.
4. **Dynamic Fitness Functions:**
   - Adapting fitness functions to better capture the long-term consequences of decisions.

Rolling Horizon Evolutionary Algorithms are a powerful tool for dynamic and real-time optimization problems, especially in scenarios requiring continuous adaptation and sequential decision-making. If you have a specific application or aspect in mind, I can delve deeper into it!

As far as I understood, the RHEA (Rolling Horizon Evolutionary Algorithms) takes a subset of the decision and searches for the best solutions in a given time frame. Once the time frame is over the first decision of the best solutions are applied. Afterwards we substract the decision that has been made and add another decision and advance to the next state of the system. Is this correct?

Yes, your understanding of Rolling Horizon Evolutionary Algorithms (RHEAs) is spot on! Here's a step-by-step breakdown to confirm and elaborate on your interpretation:

RHEA Process Overview

1. **Decision Subset Selection (Planning Horizon):**
   - The algorithm focuses on a subset of decisions (action sequences) within a **limited planning horizon**, typically looking a few steps ahead into the future.
   - This planning horizon is chosen based on computational limits, domain constraints, or the dynamic nature of the system.
2. **Optimization Within a Time Frame:**
   - During the given computational budget (time frame), the RHEA evolves a population of potential sequences of actions within the planning horizon.
   - Standard evolutionary operations (e.g., selection, crossover, mutation) are applied to find the best sequence of decisions in this limited time.
3. **Application of the First Decision:**
   - Once the time frame is over, the **first action** from the best-performing sequence is selected and applied to the system.
   - This first action represents the immediate decision the algorithm believes is optimal based on the predicted consequences of the sequence.
4. **Advance to the Next State:**
   - After the first action is executed, the system transitions to a new state (e.g., the game board updates, the robot moves, etc.).
5. **Sliding Window (Rolling Horizon):**
   - The decision sequence is updated by:
     - **Removing the executed decision** (first action of the sequence).
     - **Adding a new decision slot** at the end of the sequence to maintain the planning horizon length.
   - This ensures the algorithm continuously optimizes over the most recent state and adapts to any changes in the environment.
6. **Repeat the Process:**

- The updated decision sequence becomes the new population for the next iteration, and the process repeats until a termination criterion (e.g., problem solved or maximum iterations) is met.

Example to Illustrate the Workflow

Scenario: Pathfinding in a Grid

- **Goal:** Navigate a robot from a start point to a goal point on a grid.
- **Planning Horizon:** 5 moves ahead.

1. **Initial State:**
   - The robot starts at (0, 0).
   - The RHEA generates and evaluates sequences of 5 moves (e.g., `up, right, right, down, left`).
   - The best sequence, according to a fitness function, is chosen.
2. **Execute First Move:**
   - The robot applies the first action (`up`) and moves to (0, 1).
3. **Roll Horizon Forward:**
   - The decision `up` is removed, and a new empty slot is added at the end.
   - The planning horizon now evaluates sequences starting from the new state (e.g., `right, right, down, left, ?`).
4. **Continue:**
   - The process repeats: generate, evaluate, execute, and roll forward until the robot reaches the goal.

Advantages of This Framework

- **Real-Time Decision Making:**
  - By focusing on a subset of actions and executing the first decision immediately, the algorithm is well-suited for real-time or dynamic systems.
- **Adaptability:**
  - As the system state evolves, RHEA adjusts its optimization focus, allowing it to handle changing environments effectively.
- **Scalable Horizons:**
  - The length of the planning horizon can be tuned based on the computational budget and problem complexity.

Challenges

- **Short-Sightedness:**
  - Since only the first decision is executed and the horizon is limited, RHEAs may sometimes make suboptimal long-term decisions.
- **State Transitions:**
  - The algorithm assumes that transitions between states are predictable or can be simulated accurately during optimization.
- **Balance Between Time and Horizon Length:**
  - Longer horizons provide better planning but require more computational resources, potentially reducing the number of iterations available within the time budget.

In summary, your description accurately captures the essence of RHEAs. They iteratively optimize over a sliding window of decisions, applying the first action of the best sequence at each step and adapting to the updated state of the system. This combination of rolling horizon and evolutionary optimization makes RHEAs both powerful and flexible for real-time, sequential decision-making tasks.

Are there any studies that compare the performance for different parameters, e.g., horizon length, computation time, etc.

Yes, several studies have examined how different parameters, such as horizon length and computation time, affect the performance of Rolling Horizon Evolutionary Algorithms (RHEAs). Notable findings include:

1. Impact of Population Size and Individual Length:

- **Study Overview:** An analysis of the vanilla RHEA on 20 games from the General Video Game AI (GVGAI) corpus focused on varying population sizes and individual lengths.
- **Findings:**
  - Increasing population size generally improved performance, with larger populations leading to higher win rates in most games.
  - The effect of individual length was more nuanced; while longer lengths could enhance performance, excessively long sequences sometimes resulted in decreased win rates, especially when combined with small population sizes.
  - The study also noted that RHEA could outperform Monte Carlo Tree Search (MCTS) in certain configurations, particularly with higher population sizes.

  Source: *Analysis of Vanilla Rolling Horizon Evolution Parameters in General Video Game Playing*

2. Influence of Computational Budget:

- **Study Overview:** The same study also explored the implications of increased computational budgets on RHEA performance.
- **Findings:**

    - With a higher computational budget, RHEA's performance improved across various games.
    - This suggests that given more time for evaluations, RHEA can explore the search space more effectively, leading to better decision-making.

  Source: *Analysis of Vanilla Rolling Horizon Evolution Parameters in General Video Game Playing*

3. Hybrid Approaches and Parameter Optimization:

- **Study Overview:** Research combining various modifications of RHEA employed a parameter optimizer to identify the best parameter combinations across multiple games.
- **Findings:**
    - The optimization process revealed that certain parameter settings led to state-of-the-art solutions in several games.
    - The study emphasized the importance of automatic exploration of the parameter space to enhance RHEA performance.

  Source: *Rolling Horizon Evolutionary Algorithms for General Video Game Playing*

4. Population Seeding Techniques:

- **Study Overview:** An investigation into different population initialization methods for RHEA examined their impact on performance across various games.
- **Findings:**
    - The effectiveness of seeding methods varied depending on the game type.
    - Some games benefited from greedy initialization, while others performed better with statistical approaches.

  Source: *Population Seeding Techniques for Rolling Horizon Evolution in General Video Game Playing*

Key Takeaways:

- **Population Size:** Larger populations can enhance performance, but the benefits may plateau or even decline if the individual length is not appropriately matched.

- **Individual Length (Horizon Length):** While longer action sequences can provide a more comprehensive search, excessively long sequences may lead to inefficiencies, especially with limited computational resources.
- **Computational Budget:** Increased computation time generally improves RHEA performance, allowing for more extensive exploration and refinement of action sequences.
- **Game Dependency:** Optimal parameter settings can vary significantly between different games, indicating the need for adaptive or game-specific tuning of RHEA parameters.

These studies highlight the importance of carefully tuning RHEA parameters to balance exploration and exploitation effectively, considering the specific characteristics of the problem domain and computational constraints.

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So what about if we proceed to the next state. Is the population then intialized randomly or is the population just a continuation of the population of the last state just with slightly changed decision variables, e.g., the first decision is removed and another decision is added.

When proceeding to the next state in a Rolling Horizon Evolutionary Algorithm (RHEA), the population can be initialized in different ways depending on the design and the problem requirements. Both approaches you mentioned—random initialization and continuation of the previous population—are commonly used, and each has its advantages and trade-offs.

1. Continuing the Population from the Previous State

In this approach, the algorithm reuses the population from the previous state with adjustments to account for the transition:

How It Works:

1. The **first decision** from each individual (sequence) in the population is removed because it has already been executed.
2. A **new decision** slot is appended to the end of each sequence, which can be initialized randomly or using heuristics.

Advantages:

- **Exploration Continuity:** The population retains knowledge from the previous optimization, preserving promising patterns in the sequences.
- **Faster Convergence:** Since the population is not reinitialized, the algorithm can build upon prior results, requiring fewer generations to refine solutions.
- **Adaptability:** The rolling horizon naturally adapts the decision sequence to the updated state while maintaining search momentum.

Disadvantages:

- **Risk of Premature Convergence:** If diversity is not maintained, the population may converge too quickly to suboptimal solutions.
- **Accumulated Errors:** If the initial population contained biases or poor decisions, these may propagate over multiple states.

Use Cases:

- Problems where continuity in decision-making is critical (e.g., games, robotics, real-time navigation).
- Scenarios with limited computation time, where reusing the population can save resources.

2. Randomly Initializing the Population for the New State

In this approach, a fresh population is generated at each state transition:

How It Works:

1. When transitioning to the next state, the entire population is discarded.
2. A new population of sequences is generated randomly, with decisions tailored to the current state.

Advantages:

- **High Diversity:** Ensures that the population remains diverse, avoiding stagnation or premature convergence.
- **Flexibility:** Allows the algorithm to completely adapt to the new state without being influenced by prior biases.

Disadvantages:

- **Loss of Knowledge:** Previously discovered promising patterns are discarded, which can slow down convergence.
- **Higher Computational Cost:** A new population requires more time to explore the search space effectively.

Use Cases:

- Problems where each state significantly changes the optimization landscape (e.g., environments with high variability or uncertainty).
- Scenarios where maintaining diversity is crucial to avoid local optima.

3. Hybrid Initialization Approach

This combines the benefits of both methods:

How It Works:

- Use the existing population from the previous state as a base, but introduce diversity by:

- Randomly replacing a fraction of the individuals.
- Adding mutations to the retained sequences.
- Sampling new decisions for the appended slot using heuristics or randomness.

Advantages:

- **Preserves Knowledge:** Retains promising sequences from the previous population.
- **Maintains Diversity:** Introduces variability to explore new regions of the search space.

Disadvantages:

- Balancing retention and diversity requires careful tuning.
- May still carry over some biases from previous states.

Use Cases:

- Scenarios requiring a balance between exploration and exploitation.
- Problems where state transitions are gradual but introduce some variability.

Best Practices for Population Initialization

1. **Dynamic Strategy Selection:** Adapt the initialization approach based on the problem's characteristics and the state transition dynamics.
2. **Diversity Preservation:** If continuing the population, ensure mechanisms like mutation or random replacements maintain diversity.
3. **Heuristic Guidance:** Use domain knowledge or heuristics to generate or adjust new decisions for the appended slots.

Example Workflow for Continued Population

1. Current population:

```
[[a1, a2, a3, a4], [b1, b2, b3, b4], [c1, c2, c3, c4]]
```

2. Execute first action (`a1` , `b1` , `c1`).
3. Transition to new state.
4. Update population:
    - Remove first decision:
      `[[a2, a3, a4], [b2, b3, b4], [c2, c3, c4]]`
    - Add new random/heuristic action:
      `[[a2, a3, a4, a5], [b2, b3, b4, b5], [c2, c3, c4, c5]]`
5. Continue optimization with the updated population.

Choosing the right approach depends on the specific problem, computational budget, and the importance of diversity versus exploitation. In many cases, continuing the population (with diversity mechanisms) strikes the best balance for dynamic and real-time optimization tasks.

when did this method of RHEA came up? Is it a relativly new approach?

Rolling Horizon Evolutionary Algorithms (RHEAs) are relatively recent as a formalized approach, but they build upon concepts that have been part of evolutionary computation and decision-making for some time. Here's a brief historical perspective:

Origins and Early Concepts

1. **Sequential Decision Making:**
    - The idea of optimizing sequences of actions in a rolling or sliding window manner

has been present in fields like dynamic programming and operations research for decades.

- Early methods like receding horizon control (used in control theory) and Model Predictive Control (MPC) share a conceptual similarity with RHEAs, where only the first action of a sequence is executed before recalculating the next step.

2. **Evolutionary Optimization:**
   - Traditional evolutionary algorithms (EAs) have been used to optimize static problems since the 1960s and 1970s.
   - The extension of EAs to sequential and real-time decision-making tasks began to appear in the 2000s, particularly in game AI research.

Emergence of Rolling Horizon Evolutionary Algorithms

1. **Formalization in Game AI (2010s):**
   - The concept of RHEA as a distinct evolutionary algorithm gained traction within the game AI research community.
   - Researchers working on the **General Video Game AI (GVGAI)** framework popularized RHEAs for solving real-time decision-making problems in video games.
   - Papers published in the early 2010s discussed RHEAs as a method to plan sequences of actions over a fixed horizon and adapt dynamically as new game states emerge.
2. **Key Publications and Studies:**
   - **2012-2015:** Early mentions of RHEA appeared in the context of video game planning and control.
   - **2017:** Detailed analysis of RHEAs for the GVGAI competition was conducted, exploring parameter sensitivity and comparing RHEAs to Monte Carlo Tree Search (MCTS).
     - Example: "Rolling Horizon Evolutionary Algorithms for General Video Game

Playing" (`Perez et al., 2017`).

- **2020 and Beyond:** The method continued to evolve with enhancements like hybridization with MCTS, diversity-preserving techniques, and applications beyond games, such as robotics and dynamic systems.

Is It a Relatively New Approach?

- **Formalized RHEAs:** Yes, the explicit identification and systematic study of RHEAs as a standalone method are relatively recent, dating back to the 2010s.
- **Underlying Concepts:** The foundational ideas (rolling horizons, evolutionary optimization) have existed for decades, but their integration into the RHEA framework for real-time sequential optimization represents a newer synthesis.

Recent Advancements

Since its formalization, RHEAs have been refined and applied to diverse fields:

1. **Hybrid Approaches:** Combining RHEAs with techniques like reinforcement learning and Monte Carlo Tree Search (MCTS).
2. **Dynamic Environments:** Extending RHEAs to handle highly dynamic systems and large state spaces.
3. **Parameter Sensitivity:** Studies to better understand the impact of horizon length, population size, and computational budget.

In conclusion, while the **concepts underlying RHEAs** have been around for some time, their explicit **formalization and widespread study** as a distinct algorithmic approach are relatively new, emerging strongly in the 2010s and continuing to evolve.

So I came up with a very similar startegy which is used for real-time scheduling. The idea is that the EA and the model is initialized with the plant state of a production process. The EA than evolves dynamically just as the underlying production process and we set synchronization times in which we reinitlalize the EA with the newest state of the process. This leads to some decision being excluded from the EA since these decisions already have been made. Another feature is that we can reinitialize the scheduling system with everything that happens in the production. For example if machines are not workin or if new orders arrive we can update the EA. In this scenario we found that the preservation of the population beyond states is crucial for performance and convergance of the algorithm. However one thing which needs to be investigated is how the synchronization interval influences the performanc eof the EA.

Your strategy aligns closely with the principles of **Rolling Horizon Evolutionary Algorithms (RHEAs)**, but it extends them to a specific and practical application in **real-time scheduling for production processes.** This is an exciting adaptation that demonstrates how evolutionary algorithms can be effectively integrated into dynamic, real-world systems. Let’s break this down and explore your ideas further:

Key Features of Your Approach

1. **Real-Time Scheduling with Evolutionary Algorithms:**
    - The evolutionary algorithm (EA) operates dynamically alongside the production process, continuously evolving solutions.
    - At fixed **synchronization intervals**, the EA is updated with the latest state of the production process.

2. **State Synchronization and Reinitialization:**
    - The EA population is synchronized with the current state of the production process, which involves:
        - Removing decisions that have already been executed.
        - Adding new variables to the decision-making process, such as incoming orders or machine status updates.
3. **Population Preservation:**
    - Retaining the population across synchronization points ensures that the algorithm leverages prior knowledge, avoiding the need to restart from scratch.
    - This preservation is crucial for maintaining performance and convergence.
4. **Dynamic Adaptation to Process Changes:**
    - The EA adapts to real-time changes in the production process, such as:
        - Machine breakdowns.
        - New or canceled orders.
        - Changes in resource availability.

Advantages of Your Strategy

- **Real-Time Responsiveness:** The system remains agile, continuously adapting to new information and ensuring that schedules are as up-to-date as possible.
- **Improved Convergence:** By preserving the population, the algorithm builds on previous solutions, reducing the time needed to converge to high-quality schedules.
- **Flexibility and Robustness:** The system can handle dynamic and unforeseen events effectively, which is a critical requirement in production environments.

Open Question: Synchronization Interval Impact

You correctly identified that the **synchronization interval** is a crucial factor influencing the performance of the algorithm. Here are some considerations and research directions:

Factors Influencing Synchronization Interval:

1. **Interval Length:**
   - **Short Intervals:**
     - Frequent updates allow the EA to remain tightly synchronized with the production process, but may reduce the number of generations the EA can perform between updates.
     - Computational overhead increases due to frequent state reinitializations.
   - **Long Intervals:**
     - The EA has more time to evolve solutions, potentially improving convergence.
     - However, the algorithm risks drifting away from the actual state of the production process, especially if significant changes occur during the interval.
2. **Production Process Dynamics:**
   - Highly dynamic processes with frequent changes (e.g., machine breakdowns, order updates) may benefit from shorter synchronization intervals.
   - Stable processes with fewer disruptions may allow for longer intervals without loss of performance.
3. **Computational Resources:**
   - The available computational power determines how many generations the EA can complete within an interval. A balance must be struck between the interval length and the algorithm’s capacity to evolve solutions.

Experimental Directions to Investigate Synchronization Interval Impact

1. **Simulation Studies:**
   - Simulate production environments with varying levels of complexity and

- Simulate production environments with varying levels of complexity and dynamism.
- Test different synchronization intervals to measure their impact on:
  - Scheduling quality (e.g., completion time, resource utilization).
  - Algorithm convergence speed.

2. **Adaptive Interval Strategies:**
   - Develop mechanisms to dynamically adjust the synchronization interval based on:
     - The rate of changes in the production process.
     - The convergence speed of the EA in the current interval.
3. **Impact of Population Size and Diversity:**
   - Investigate how the population size and diversity influence the algorithm's ability to recover from changes when synchronization intervals are longer.
4. **Hybrid Approaches:**
   - Combine your strategy with other techniques like Monte Carlo Tree Search (MCTS) or surrogate modeling to enhance decision-making during longer intervals.

Potential Challenges and Considerations

- **Synchronization Overhead:** Frequent reinitializations might introduce latency. Optimization techniques, like partial reinitialization or population clustering, could mitigate this.
- **Handling Disruptions:** The algorithm must be robust enough to handle drastic changes (e.g., multiple simultaneous machine failures).
- **Real-Time Constraints:** In highly dynamic environments, ensuring that the EA can keep up with the production process is critical.

Conclusion

Your approach is highly innovative and aligns well with the evolving needs of modern production systems. The **preservation of the population** is indeed critical for maintaining convergence and performance in this dynamic setting. Investigating the **synchronization interval's impact** is a valuable next step, and it can provide insights into achieving an optimal balance between responsiveness and solution quality.

Would you like assistance in formulating a specific experimental framework or exploring strategies for adaptive synchronization intervals?

I would like to know if somebody already tried something similar for production processes

Yes, the integration of evolutionary algorithms (EAs) with real-time scheduling in production processes has been explored in various studies. These approaches align closely with your strategy of synchronizing EAs with the dynamic state of production systems. Notable examples include:

1. Rolling-Horizon Simulation Optimization for Multi-Objective Scheduling

- **Overview:** This study addresses complex scheduling in multi-product biomanufacturing systems, focusing on objectives like makespan and lateness. A rolling-horizon approach is employed, where the EA is periodically reinitialized with the current state of the production process. This method adapts to real-time changes, such as machine downtimes or new order arrivals.
- **Findings:** The rolling-horizon strategy effectively manages the dynamic nature of the production environment, leading to improved scheduling performance. INFORMS SIM

2. Reactive Real-Time Scheduling Using Simulation-Optimization and Evolutionary Algorithms

- **Overview:** This research presents a reactive scheduling framework that combines simulation-optimization with EAs for real-time decision-making in production settings.

The EA evaluates and improves schedules based on the current state of the production process, accommodating disruptions like machine failures or urgent orders.

- **Application:** Demonstrated in a pharmaceutical batch plant, the approach showcases the EA's ability to adapt to real-time changes, maintaining efficient production schedules. IEEE XPLORE

3. Scheduling and Batching with Evolutionary Algorithms in Simulation Optimization of an Industrial Formulation Plant

- **Overview:** This study explores the use of EAs for optimizing scheduling and batching in an industrial formulation plant. The EA is integrated with simulation models to dynamically adjust to the current state of the production process, enhancing both ecological and economic efficiency.
- **Findings:** The integration of EAs with real-time data from the production process leads to significant improvements in scheduling efficiency and resource utilization. SSRN

4. A Rolling-Horizon Approach for Multi-Period Optimization

- **Overview:** This research introduces a rolling-horizon framework for multi-period optimization in production planning. The approach involves periodically updating the optimization model with the latest state of the production process, similar to your strategy of synchronizing the EA with the plant state.
- **Application:** Applied to lot-sizing problems in production planning, the rolling-horizon approach effectively adapts to changes over time, leading to solutions close to the global optimum. OPTIMIERUNG ONLINE

Key Insights:

- **Population Preservation:** Maintaining the EA population across synchronization points is crucial for leveraging prior knowledge and ensuring convergence, as highlighted in your approach.
- **Synchronization Intervals:** The frequency of synchronization (i.e., how often the EA is updated with the current state) significantly impacts performance. Balancing the interval length is essential to adapt to changes without incurring excessive computational overhead.
- **Real-Time Adaptation:** Integrating EAs with real-time data from the production

process enables the scheduling system to respond promptly to disruptions, enhancing overall efficiency.

These studies demonstrate that combining EAs with real-time scheduling in production processes is a viable and effective strategy. Your approach aligns well with current research trends, emphasizing the importance of synchronization intervals and population preservation in dynamic production environments.

Quellen

The second reference is actually a conference paper that I submitted

That's fantastic! It sounds like you're deeply involved in advancing the field of real-time scheduling with evolutionary algorithms. Submitting a conference paper in this area is a significant achievement. If the second reference aligns closely with your work, it highlights the relevance and importance of the approach you're developing.

If you'd like, I can assist with:

1. **Exploring Further Applications:** Identifying new domains or industries where your methodology could make an impact.
2. **Parameter Sensitivity Analysis:** Suggesting frameworks to study factors like synchronization intervals or population preservation in detail.
3. **Refinement:** Helping with additional research directions or preparing extended versions of your paper for journals.
4. **Discussion or Dissemination:** Summarizing your paper's contributions to communicate them effectively to broader audiences.

Let me know how I can help!

The third paper is from a formar colleauge with home I did some research

That's fantastic! It seems you're deeply connected to the community working on real-time scheduling and evolutionary algorithms, which is an excellent advantage for collaborative and impactful research.

The connection between your work and that of your former colleague shows a robust foundation in this domain. Collaborating or referencing each other's work could help build a cohesive research narrative and expand the reach of your contributions.

Here are a few ways you could leverage this connection:

1. Collaborative Extensions

- Consider revisiting past work with your former colleague to explore new angles or extend findings into more comprehensive studies.
- Jointly explore areas like hybridization with other techniques (e.g., reinforcement learning or surrogate models) or cross-domain applications.

2. Citation and Cross-Promotion

- Reference each other's work in your respective publications to build credibility and context for future readers.
- Highlight how your approaches complement or build upon each other, which could strengthen the narrative for both contributions.

3. Community Building

- If you're attending conferences or workshops, presenting related work in a session or panel could spotlight your shared contributions to the field.
- Consider organizing a special session or track on **Real-Time Scheduling with Evolutionary Techniques** to gather like-minded researchers.

4. Open Problems to Explore

- Investigate areas that both your and your colleague's work identified as future challenges, such as:
    - Managing uncertainty in production scheduling (e.g., sudden disruptions).
    - Synchronization strategies and their impact on performance.
    - Multi-objective extensions balancing efficiency, cost, and resource utilization.

5. Formal Collaboration Opportunities

- If not already in touch, reconnect to discuss potential projects, grant proposals, or co-authored review articles summarizing advancements in this niche.

It's inspiring to see how your work and connections intertwine within such an exciting field! Let me know if you'd like help brainstorming ideas for collaboration or specific ways to link your research narratives.

1/2

So there is actually a real challange in brining scheduling to the industry. Mathematical approach are good because they are consistent in their solution but the mathematical models are difficult to set up, hard to maintain, and often you cannot model everything which is really needed to be modelled. The alternaive approach is metaheuristcs. Here you can use detailed models, but the solution quality is not always the same and the computiation time can actually be high for both approaches. However, in general the industry is interested in solutions that they can understand and adopt right away. Therefore using metaheuristics with detailed models is in my opinion the better approach. It is also not always necessary to solve for optimiality because in the real process things will change eitherway. Therefore a little bit of suboptimality can be advantageous because it gives the system more room to cope with disturbances

You've captured the core tension between **mathematical optimization** and **metaheuristic approaches** in industrial scheduling perfectly. Your insights about practicality, adaptability

approaches in industrial scheduling perfectly. Your insights about practicality, adaptability, and the realities of real-world processes are spot on and resonate with the challenges industries face when implementing advanced scheduling solutions.

Challenges of Mathematical Optimization

1. **Complexity in Modeling:**
    - Mathematical models require a high level of abstraction, often leading to the exclusion of crucial details.
    - Setting up and maintaining these models demands expertise and significant time investment.
2. **Inflexibility:**
    - Changes in the production process (e.g., new machines, order types, or constraints) require reworking the model, which can be resource-intensive.
3. **Computation Time:**
    - For large, complex scheduling problems, mathematical optimization (e.g., Mixed-Integer Linear Programming, MILP) may take too long to converge, especially when near-optimal solutions are sufficient.

Advantages of Metaheuristics in Industrial Scheduling

1. **Flexibility and Detailed Modeling:**
    - Metaheuristics can incorporate detailed, domain-specific constraints and objectives that are difficult to express mathematically.
    - They can be updated or adapted relatively easily as production processes evolve.
2. **Practicality Over Optimality:**
    - Achieving a "good enough" solution quickly is often more valuable in dynamic environments than finding a theoretically optimal solution.

  - Suboptimality allows room for the system to adapt to real-time disturbances, aligning with your point about flexibility.

3. **Intuitive Application:**
   - Metaheuristics, when well-designed, can be implemented with clear rules and parameters that operators and managers can understand and trust.

4. **Scalability:**
   - They are more scalable to large and complex problems where exact optimization methods struggle.

Bridging the Gap Between Industry Needs and Metaheuristics

To make metaheuristic-based scheduling solutions more appealing and practical for industry adoption, several strategies can address the concerns you've identified:

1. Enhancing Understandability

- **Visual Representation:**
  - Provide intuitive visualizations of schedules, decisions, and trade-offs to help stakeholders grasp the algorithm's outputs.
- **Simplified Interfaces:**
  - Build user-friendly tools that abstract away the complexity of the metaheuristic.
- **Explainability:**
  - Incorporate explainability features, such as sensitivity analysis, to show how inputs and constraints influence outcomes.

2. Hybrid Approaches

- Combine metaheuristics with simpler, rule-based methods for certain parts of the problem. For example:
  - Use rule-based methods to pre-allocate resources or sequence simple jobs.
  - Apply metaheuristics to fine-tune schedules or solve more complex subproblems.

3. Real-Time Responsiveness

- Leverage real-time data integration to ensure that metaheuristic algorithms remain aligned with the current state of the system.
- Adopt rolling horizon frameworks (like your described approach) to continuously refine solutions without requiring extensive recomputation.

4. Adaptive Metaheuristics

- Implement adaptive mechanisms that tune algorithm parameters dynamically based on the current state of the system (e.g., computational time, process variability).

5. Solution Quality Assurance

- Develop mechanisms to evaluate and compare the quality of metaheuristic solutions against benchmarks or known best practices.
- Use robust testing to demonstrate the consistency of results across scenarios.

6. Emphasizing Robustness Over Optimality

- Design algorithms that prioritize robustness—ensuring schedules are resilient to disruptions—over finding the absolute optimal solution.
- Build buffer time or slack into schedules to absorb process variability.

Case for Metaheuristics in Industry

Your observation about **suboptimality being advantageous** is insightful and aligns with the philosophy of **pragmatic optimization.** Many industries prefer solutions that:

- Adapt quickly to changing conditions.
- Are easy to update or modify as the system evolves.
- Provide results within a predictable timeframe, even if they are slightly suboptimal.

Metaheuristics are well-suited for these requirements, as they:

- Excel in dynamic and highly constrained environments.

- Can produce good solutions within fixed computational budgets.
- Are versatile enough to model complex, domain-specific requirements.

Future Directions

1. **Collaboration with Industry:**
   - Engage with industry stakeholders to identify key pain points and constraints that metaheuristics can address.
   - Focus on co-developing solutions that fit existing workflows and technologies.
2. **Simulation-Driven Metaheuristics:**
   - Integrate metaheuristics with simulation tools to evaluate solutions in realistic settings before deployment.
   - Use simulations to model disruptions and validate the robustness of schedules.
3. **Machine Learning Integration:**
   - Combine metaheuristics with predictive models to anticipate disruptions (e.g., machine failures) and proactively adjust schedules.
4. **Open Research Problems:**
   - Investigate the trade-offs between solution quality and computational time in highly dynamic settings.
   - Explore hybrid methods that leverage both metaheuristics and mathematical optimization for improved consistency.

Your approach and perspective align well with the industry's need for **practical, flexible, and understandable scheduling systems.** Metaheuristics offer a promising avenue, especially when coupled with techniques to enhance their robustness, explainability, and responsiveness. Would you like assistance in exploring any of these ideas further, or in framing them for publication or industrial application?

framing them for publication or industrial application?

How is a state defined in MCTS and RHEA. Are they different?

The concept of a **state** in Monte Carlo Tree Search (MCTS) and Rolling Horizon Evolutionary Algorithms (RHEA) is central to how each algorithm navigates and optimizes in a problem space. While both approaches involve decision-making and planning, the way they define and use states differs significantly due to their structural and operational differences.

State in Monte Carlo Tree Search (MCTS)

In MCTS, a **state** represents a node in a **tree structure**, corresponding to a specific configuration or snapshot of the environment or problem at a given time.

Definition of a State in MCTS:

- **What It Represents:** A specific point in the problem space, usually after a sequence of actions has been applied from the root of the tree.
  - Example in games: A board configuration after a series of moves.
  - Example in scheduling: The current allocation of resources and tasks.
- **State Information:** Includes all data needed to evaluate the state, such as:
  - Current decision variables.
  - State of the environment or system.
  - Constraints and goals relevant to the problem.

Key Features of States in MCTS:

1. **Tree Nodes:** Each state is associated with a node in the tree, linked to its parent (previous state) and children (possible next states).

2. **State Transitions:** States are derived by applying actions to a parent state, creating a path through the tree.
3. **Rollouts:** From a state, MCTS performs rollouts (random simulations) to estimate the value of that state for guiding future exploration.
4. **Static Definition:** Once a state is added to the tree, it is immutable; updates to the state happen by exploring new branches.

Usage of States:

- MCTS uses states to:
  - Guide exploration (e.g., Upper Confidence Bound (UCB) determines which state to expand).
  - Estimate rewards or outcomes via simulations.
  - Build a tree structure that represents possible future scenarios.

State in Rolling Horizon Evolutionary Algorithms (RHEA)

In RHEA, a **state** is a snapshot of the **current environment** or system configuration at a specific time, from which the algorithm optimizes a sequence of decisions over a fixed horizon.

Definition of a State in RHEA:

- **What It Represents:** The current situation of the environment at the time of optimization.
  - Example in games: The current game board and active player.
  - Example in scheduling: The current production status, including active orders and machine states.
- **State Information:** Includes everything necessary to:
  - Simulate the outcomes of action sequences.
  - Evaluate fitness values for candidate solutions.

Key Features of States in RHEA:

1. **Current and Fixed:** The algorithm works with a single state (the current state of the environment) during each iteration or synchronization interval.
2. **State Transitions:** Instead of a tree, RHEA simulates transitions directly by applying sequences of actions to the state.
3. **Dynamic Updating:** The state is updated at each synchronization point to reflect changes in the environment, making it inherently dynamic.

Usage of States:

- RHEA uses the current state to:
  - Simulate action sequences and predict outcomes.
  - Evaluate the fitness of solutions based on how they perform starting from the current state.
  - Define the starting point for the rolling horizon as decisions progress.

Differences Between MCTS and RHEA States

Aspect	MCTS	RHEA
State Representation	A node in a tree, representing a specific scenario.	A snapshot of the current environment at runtime.
Structure	Static tree; states are fixed once added.	Dynamic updates; state changes at synchronization.
Transitions	Derived by expanding tree nodes through actions.	Direct simulation of action sequences.
Exploration	Explores multiple states via tree structure.	Optimizes action sequences starting from one state.
Focus	Focuses on balancing exploration and exploitation.	Focuses on optimizing a sequence of decisions.
Context	Part of a broader search tree; multiple states exist.	Single starting state at a time, updated iteratively.

Rollouts	Simulates random actions from a state to estimate value.	Simulates deterministic or heuristic action sequences.

Which Approach is Better?

- **MCTS States:**
  - Suitable for problems where branching and exploration of multiple possibilities are critical.
  - Examples: Board games, complex decision trees.
- **RHEA States:**
  - Better for problems requiring optimization of sequential actions starting from a single state.
  - Examples: Scheduling, real-time strategy games.

Both approaches have strengths and are applicable depending on the problem's nature and requirements. If you'd like to explore a specific example or scenario, let me know!

For RHEA, I would also say that a state transition occurs when the decision space changes. For example in scheduling we start with a sequence of 5 orders. Once we have made our decision on which order to process first the state of the decision making process, and therefore the RHEA, changes to the next state which is 4 orders. So the essence of a state in RHEA is defined by the decision space

You are absolutely correct, and your explanation refines the concept of a state in **Rolling Horizon Evolutionary Algorithms (RHEA)** by tying it directly to the **decision space**. This perspective provides a clear and practical understanding, particularly for applications like scheduling.

Let’s elaborate on this idea:

State Transitions in RHEA

In RHEA, a **state transition** occurs when the **decision space** evolves due to a decision being made. This aligns perfectly with your example of scheduling, where:

1. At the start, the decision space includes all available orders (e.g., {Order1, Order2, Order3, Order4, Order5}).
2. Once a decision is made (e.g., choosing Order1 as the first to process), the decision space shrinks (e.g., {Order2, Order3, Order4, Order5}).
3. This change in the decision space inherently defines a **new state**, which becomes the basis for further optimization.

Essence of a State in RHEA

The **state** in RHEA can thus be defined as:

> **The current configuration of the decision space, which reflects the set of available options and constraints at a given time in the decision-making process.**

Key Elements of a State in RHEA:

1. **Decision Space:** The set of actions or decisions still available (e.g., the remaining orders to process in a scheduling problem).
2. **Environmental State:** Any external or contextual information affecting decision-making (e.g., machine statuses, resource availability, or order deadlines).
3. **Historical Context:** The sequence of decisions already made, as this impacts the remaining options and future simulations.

State Transitions in Scheduling

Let's revisit your scheduling example to illustrate this more clearly:

1. **Initial State:**
   - Decision Space: {Order1, Order2, Order3, Order4, Order5}.
   - Environmental State: All machines are operational; no orders are delayed.
   - Historical Context: No decisions made yet.
2. **Transition after Decision:**
   - Decision: Process Order1 first.
   - New State:
     - Decision Space: {Order2, Order3, Order4, Order5}.
     - Environmental State: Machine 1 is now processing Order1; others are idle.
     - Historical Context: Order1 is being processed.
3. **Next Transition:**
   - Decision: Process Order3 next.
   - New State:
     - Decision Space: {Order2, Order4, Order5}.
     - Environmental State: Machine 1 finishes Order1; Machine 2 begins Order3.
     - Historical Context: Order1 and Order3 have been processed.

Implications for RHEA

1. The State Shapes the Decision Space

- The available actions (decision space) directly define the scope of optimization in each iteration of RHEA.
- Each state transition fundamentally changes the problem RHEA is solving, requiring

the algorithm to adapt dynamically.

2. Rolling Horizon and State Updates

- As decisions are made and the system evolves, RHEA:
  - Removes already executed decisions.
  - Updates environmental information (e.g., machine statuses, resource usage).
  - Adjusts the decision space to reflect new constraints or opportunities.

3. Synchronization with Reality

- If new information becomes available during execution (e.g., machine breakdowns, rush orders), the state can be updated dynamically to ensure RHEA remains aligned with the real-world process.

Advantages of Defining States by the Decision Space

1. **Practicality:** This definition ties the algorithm's operation directly to the core problem: selecting actions from a set of possibilities.
2. **Adaptability:** It naturally incorporates changes in the environment, such as disruptions or new constraints, making RHEA well-suited for dynamic problems.
3. **Incremental Optimization:** The algorithm operates iteratively on a progressively smaller and more constrained decision space, allowing for focused and efficient search.

Comparison with MCTS

This decision-space-driven definition of a state in RHEA contrasts with MCTS, where:

- MCTS defines a state as a **node** in a tree, representing a specific scenario (including a path of decisions).
- State transitions in MCTS occur when actions are applied to nodes, creating branches

in the search tree.

- In RHEA, the "branching" effect is not explicit since the focus is on optimizing sequences within the evolving decision space.

Your insight that **states in RHEA are defined by the decision space** is both intuitive and powerful. This perspective emphasizes how RHEA dynamically adapts to the evolving nature of real-world problems, making it particularly valuable for practical applications like scheduling. If you'd like, I can help formalize this concept further or explore how to model it in other domains!

I would add to this definition: "The current configuration of the decision space, which reflects the set of available options and constraints at a given time in the decision-making process." the following: "The current configuration of the decision space, which reflects the set of available options, constraints at a given time, and goals in the decision-making process." Since in theory the goal or objective could be dynamic as well

You're absolutely right! Including **goals** in the definition acknowledges that the objectives of the decision-making process can be dynamic and may evolve over time. This addition enhances the definition by making it more comprehensive and applicable to real-world scenarios where goals often shift due to changing priorities, constraints, or contexts.

Updated Definition:

> **"The current configuration of the decision space, which reflects the set of available options, constraints at a given time, and goals in the decision-making**

> **process."**

Why Include Goals?

1. **Dynamic Objectives:**
    - In many real-world problems, the goals of the decision-making process are not static. For example:
        - In production scheduling, minimizing makespan might initially be the goal, but it could shift to prioritizing high-value orders if a disruption occurs.
        - In games, the immediate objective could change based on the opponent's actions or evolving game states.
2. **Alignment with Real-World Applications:**
    - Dynamic objectives are especially relevant in:
        - **Supply Chain Management:** Priorities can change based on customer demands, market conditions, or resource availability.
        - **Manufacturing:** Goals may shift between cost minimization, on-time delivery, or maximizing machine utilization.
        - **Dynamic Environments:** In robotics or real-time systems, objectives like energy efficiency or task prioritization can evolve with the environment.
3. **Guiding Evolution:**
    - The goals directly influence the fitness function used in RHEA. By allowing the goal to be dynamic, the algorithm can adapt its optimization focus as the state changes.

Practical Implications of Dynamic Goals in RHEA:

1. **Fitness Function Updates:**

- The fitness function must reflect the current goals to ensure the algorithm evaluates solutions in alignment with the evolving priorities.

2. **State Transitions Incorporate Goal Changes:**
   - Each state transition may bring a shift not only in the decision space and constraints but also in the goals.
   - For example:
     - After processing a critical order, the goal may shift from minimizing lateness to balancing workloads.
3. **Improved Flexibility:**
   - Including goals in the state definition makes RHEA more robust for applications where objectives are context-dependent.

Example Application: Production Scheduling

1. **Initial State:**
   - Decision Space: {Order1, Order2, Order3, Order4}.
   - Constraints: Machine1 and Machine2 are operational; maximum workload is 8 hours.
   - Goals: Minimize makespan.
2. **State Transition:**
   - After Order1 is processed:
     - Decision Space: {Order2, Order3, Order4}.
     - Constraints: Machine1 is under maintenance; Machine2 is operational.
     - Goals: Shift to prioritizing high-priority orders due to a rush job (dynamic goal change).

By explicitly including goals in the state definition, the framework becomes more flexible and better suited for dynamic, real-world problems. Would you like assistance in exploring how to model this formally or in designing experiments to test RHEA with dynamic goals?

We already talked about the importance of population preservance in RHEA during state transitions. However it is also important that the population stays diverse as well. How can we ensure that the population does not converge to the common best solution at the moment and stays diverse?

Maintaining **diversity** in the population during state transitions in **Rolling Horizon Evolutionary Algorithms (RHEA)** is indeed crucial for avoiding premature convergence and ensuring robust exploration of the search space. Diversity helps the algorithm to:

- Avoid getting stuck in local optima.
- Explore alternative solutions, which can be critical in dynamic environments.

Here are several strategies to ensure population diversity in RHEA:

1. Fitness Sharing

- **Concept:** Adjust the fitness of individuals based on their similarity to others in the population. The more similar an individual is to others, the lower its effective fitness.
- **Implementation:**
  - Define a similarity metric (e.g., based on Hamming distance for discrete sequences or Euclidean distance for continuous parameters).
  - Penalize fitness scores for clusters of similar individuals to encourage exploration of less-visited regions.
- **Effect:** Prevents the population from converging to a single dominant solution.

2. Diversity Preservation Mechanisms

- **Maintain Diverse Archives:**
  - Keep an archive of diverse solutions seen during previous generations.
  - Periodically introduce solutions from this archive back into the population.
- **Enforced Diversity:**
  - Restrict the number of offspring similar to their parents or other individuals in the population.

3. Adaptive Mutation and Crossover

- **Dynamic Mutation Rates:**
  - Increase the mutation rate when diversity drops, introducing more randomness into the population.
  - Reduce mutation rates once sufficient diversity is detected to allow for refinement of solutions.
- **Novelty-Based Mutation:**
  - Apply mutations that aim to create offspring as different as possible from existing individuals.
- **Diverse Crossover:**
  - Use crossover operators that combine dissimilar individuals to create offspring with higher variability.

4. Novelty Search

4. Novelty Search

- **Concept:** Instead of focusing purely on fitness, evaluate individuals based on how different (novel) they are from the rest of the population or previously visited solutions.
- **Implementation:**
  - Define a novelty metric (e.g., distance in the decision space).
  - Select individuals based on both novelty and fitness.
- **Effect:** Encourages the exploration of new regions in the search space, balancing exploitation and exploration.

5. Multi-Objective Approaches

- **Introduce Diversity as an Objective:**
  - Treat diversity as an explicit objective in addition to the primary fitness goal.
  - Use a multi-objective optimization framework to balance between fitness and diversity.
- **Pareto Front Preservation:**
  - Maintain a diverse set of non-dominated solutions in the population.

6. Restart Mechanisms

- **Partial Restarts:**
  - Periodically replace a portion of the population with random or archived individuals to inject diversity.
- **State-Triggered Restarts:**
  - If diversity metrics fall below a threshold, reset the population partially or fully while retaining the best solution.

7. Diversity Metrics for Monitoring

- **Measure Population Diversity:**
  - Use metrics such as Shannon entropy, Hamming distance, or variance in decision variables to quantify diversity.
  - Monitor these metrics to detect and respond to convergence early.

8. Hybrid Approaches

- Combine RHEA with other techniques to maintain diversity:
  - **Monte Carlo Sampling:** Occasionally introduce random sequences into the population.
  - **Neuroevolution:** Use neural networks to learn diverse patterns and integrate them into the population.

Example: Scheduling Application

Let's apply these strategies to a scheduling problem where RHEA is optimizing task sequences:

1. **Fitness Sharing:**
   - Penalize sequences that prioritize tasks in the same order as many others.
2. **Adaptive Mutation:**
   - Increase mutation rate for sequences with overlapping tasks to generate alternative schedules.
3. **Novelty Search:**

- Reward solutions that explore task assignments not yet heavily investigated.

4. **Diversity Archive:**

   - Maintain a library of previously generated diverse schedules and reintroduce them when diversity drops.

Best Practices for Implementation

1. **Track Diversity:**

   - Regularly monitor the population's diversity using metrics like entropy or pairwise dissimilarity.

2. **Balance Exploration and Exploitation:**

   - Adjust mutation rates and selection pressure dynamically based on the diversity level.

3. **Integrate Domain Knowledge:**

   - Use problem-specific heuristics to guide diversity-preserving mechanisms, ensuring they are meaningful to the application.

By combining these strategies, you can maintain a diverse population in RHEA, ensuring the algorithm remains robust and adaptable, particularly in dynamic or complex environments. Let me know if you'd like to dive deeper into any of these strategies or develop an implementation plan!

So what I did in my approach is to track the diversity of the population by calculating the levensthein distance with respect to the best individual. This would guarantee that all the other individuals are different from the best, however it would not say anything about how they would be different to each other. With this metric I set some parameters, e.g., the average levensthein distance and if it would reach a level of e.g., 60% (where 0% would be the same solution and 100% a total different solution) I would change the mutation rate and the mutation operator.

Your approach to measuring and maintaining diversity using the **Levenshtein distance** relative to the best individual is a thoughtful and practical way to ensure that the population remains sufficiently distinct from the dominant solution. However, as you correctly point out, while this ensures diversity relative to the best individual, it doesn't necessarily capture how diverse the rest of the population is relative to each other.

Here's a breakdown of your approach and suggestions for refining it further:

1. Strengths of Your Current Approach

- **Focus on the Best Solution:**
  - By comparing all individuals to the best, you ensure the algorithm explores alternatives to the dominant solution, which is essential for avoiding local optima.
- **Dynamic Mutation Adjustment:**
  - Adjusting mutation rates and operators based on diversity is an excellent way to balance exploration and exploitation dynamically.
- **Simplicity:**
  - The Levenshtein distance is computationally efficient for sequence-based problems like scheduling, making it well-suited for real-time applications.

2. Limitations of the Current Metric

- **Inter-Individual Diversity:**
  - The current metric ensures diversity relative to the best individual but doesn't account for diversity within the rest of the population.
  - This could lead to clusters of similar solutions, even if they are distinct from the best solution.
- **Blind Spots:**
  - If multiple individuals are similarly diverse from the best solution but converge around a suboptimal region, the diversity metric might fail to capture this clustering.

3. Enhancements to Consider

To address these limitations, you could incorporate additional metrics or combine them with your current approach.

a. Pairwise Levenshtein Distance (Inter-Individual Diversity)

- Calculate the Levenshtein distance between all pairs of individuals, not just against the best individual.
- Use the average pairwise distance or a related statistic (e.g., standard deviation) to capture overall population diversity.

b. Combined Metric

- Combine your current approach with pairwise diversity:
  - Example: Weight 70% on the diversity relative to the best individual and 30% on pairwise diversity.

c. Dynamic Thresholds

- Instead of a fixed threshold (e.g., 60%), make the diversity threshold adaptive based on:
  - The stage of the optimization process (early stages need higher diversity; later stages can tolerate less)

stages can tolerate less).

- The dynamics of the problem (more complex or dynamic problems need higher diversity).

4. Advanced Mutation Strategies

Your idea to modify the mutation rate and operator is excellent. You could expand it by:

- **Adaptive Mutation Operators:**
  - Dynamically switch between operators based on diversity metrics.
  - Example:
    - Low diversity → High-disruption operators (e.g., larger mutations, reordering tasks).
    - High diversity → Fine-tuning operators (e.g., small perturbations).
- **Non-Uniform Mutation:**
  - Apply higher mutation rates to individuals closer to the best solution while preserving others to maintain diversity.

5. Monitoring Diversity with Enhanced Metrics

Here are a few metrics you might incorporate alongside Levenshtein distance:

a. Shannon Entropy

- Treat each decision or sequence element as a symbol.
- Calculate the entropy of the population to measure its diversity.
- Low entropy → Similar population.
- High entropy → Diverse population.

b. Cluster Analysis

- Use clustering techniques to group similar individuals.
- The number of clusters and their spread can provide insights into diversity.

c. Variance in Fitness

- While not directly related to sequence differences, variance in fitness values often correlates with solution diversity.
- High variance suggests a more exploratory population.

6. Example Workflow for Enhanced Diversity Management

1. **Diversity Tracking:**
   - Calculate:
     - Levenshtein distance relative to the best individual.
     - Pairwise Levenshtein distances within the population.
     - Optional: Entropy or clustering metrics.
2. **Diversity Thresholds:**
   - Define thresholds for both relative and inter-individual diversity.
3. **Action Triggers:**
   - If relative diversity < 60%:
     - Increase mutation rate.
     - Introduce a disruptive mutation operator.
   - If inter-individual diversity < 50%:
     - Add new random individuals.
     - Replace some individuals with archived or heuristic solutions.
4. **Reevaluation:**
   - Recompute diversity after the next generation to ensure diversity-preserving actions were effective.

7. Benefits of These Enhancements

- **Broader Exploration:** Capturing inter-individual diversity ensures exploration beyond the best solution.
- **Adaptability:** Dynamic mutation strategies enable the algorithm to respond effectively to diversity changes.
- **Robustness:** Combining metrics provides a more comprehensive view of the population, making the algorithm more resilient to convergence issues.

Would you like to dive deeper into implementing pairwise diversity metrics, dynamic mutation strategies, or another specific aspect? Let me know!

I think your suggestions are good but I dont agree with this point:

"c. Variance in Fitness

While not directly related to sequence differences, variance in fitness values often correlates with solution diversity.
High variance suggests a more exploratory population."

Because the variance in fitness really depends on the fitness function itself. There are fitness functions that react sensitivy to small changes. So the relative distance of the individual is really low but the difference in fitness value is really high. There might also be opposit instances where the fitness function is flat and differences in the fitness function are really hard to gain.

You're absolutely correct, and I appreciate your critique—this highlights an important nuance about the relationship between **fitness values** and **population diversity**. The sensitivity of the fitness function to changes in the solution space can indeed distort any direct correlation between variance in fitness and diversity. Let me clarify and refine that point:

Fitness Function Sensitivity and Its Implications

1. **Highly Sensitive Fitness Functions:**
   - Small changes in the decision variables (solutions) can lead to large differences in fitness values.
   - In such cases, even a population with very **low diversity** (similar solutions) might display **high fitness variance**.
2. **Flat Fitness Functions:**
   - Large changes in decision variables might result in only minor differences in fitness values.
   - In this scenario, a population with high **diversity** (distant solutions) might still show **low fitness variance**.
3. **Intermediate Cases:**
   - For fitness functions that are neither overly sensitive nor flat, fitness variance may correlate better with population diversity, but this is not universally true.

Revised Perspective on Fitness Variance

Given this variability, **fitness variance alone should not be relied upon as a measure of diversity.** It can provide supporting information but requires additional context to interpret effectively. Here's how it might be used more appropriately:

1. Supplementary Indicator:

- Combine fitness variance with diversity metrics (e.g., Levenshtein distance) to gain a holistic view.
- For example:
  - If fitness variance is high and Levenshtein diversity is low, it indicates high sensitivity in the fitness function.
  - If both fitness variance and Levenshtein diversity are low, the population may be converging prematurely.

2. Normalize by Fitness Function Sensitivity:

- Measure how sensitive the fitness function is to changes in decision variables.
- Use this sensitivity information to adjust the interpretation of fitness variance.

3. Use Local Sensitivity Analysis:

- In highly dynamic environments, evaluate local gradients of the fitness function to estimate the responsiveness of fitness values to small changes in the decision space.

Alternative Metrics for Measuring Diversity

To move away from relying on fitness variance, we can focus on metrics that measure structural or positional differences directly:

1. Structural Diversity:

- Levenshtein distance (your current approach).
- Pairwise diversity among all individuals.

2. Behavioral Diversity:

- Evaluate how different solutions behave when applied to the problem.
- Example in scheduling: How different schedules impact resource utilization or order completion times.

3. Genetic Diversity (for Metaheuristics):

- Measure diversity in genetic representations (e.g., task orderings, sequences).
- Example: Use entropy-based measures to analyze task distribution across solutions.

Actionable Takeaway

Your critique is valid, and it strengthens the argument that **fitness variance cannot reliably indicate diversity on its own.** Instead, diversity should be measured directly through structural or behavioral metrics, while fitness variance may serve as a supplementary indicator with appropriate contextualization.

If you'd like, I can help refine a specific framework to evaluate diversity for your RHEA implementation!