

LISTS IN HASKELL

SOME MORE LIST NOTATIONS

List notation, such as `[1,2,3]`, is in fact an abbreviation for a more basic form `1:2:3:[]` where `[]` is the null list and `(:)` is the cons operator

Example:

```
ghci> 3:[]
```

```
[3]
```

```
ghci> 2:3:[]
```

```
[2,3]
```

```
ghci> 1:2:3:[]
```

```
[1,2,3]
```

FORMS AND KINDS OF LISTS

- Every list of type `[a]` takes one of three forms:
 - The undefined list `undefined :: [a]`;
 - The empty list `[] :: [a]`;
 - A list of the form `x:xs` where `x :: a` and `xs :: [a]`.
- As a result there are three kinds of lists:
- A *finite* list, which is built from `(:)` and `[]`; for example, `1:2:3:[]`
- An *infinite* list, which is built from `(:)` alone; for example, `[1..]` is the infinite list of the non-negative integers.
- A *partial* list, which is built from `(:)` and `undefined`. For example, the list `filter (<4) [1..]` is the partial list `1:2:3:undefined`.

```
ghci> (filter odd [1,2,3,4,5])  
[1,3,5]
```

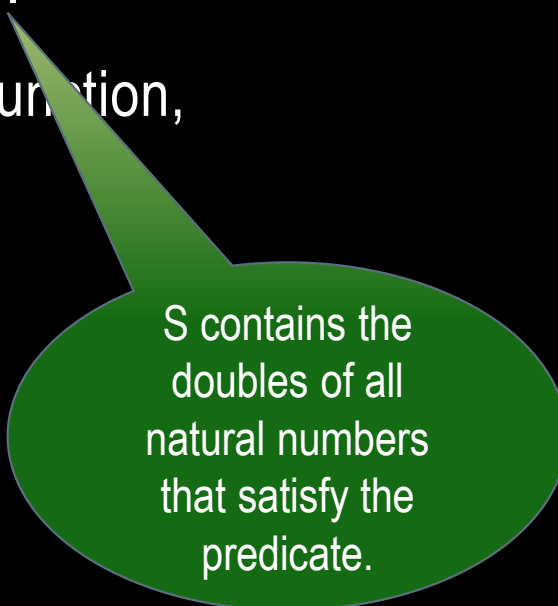
```
ghci> (filter (<4) [1..])  
[1,2,3]
```

SET COMPREHENSIONS TO LIST COMPREHENSIONS

- Set comprehensions: Normally used for building more specific sets out of general sets.
- Example:

$$S = \{2 \cdot x \mid x \in \mathbb{N}, x \leq 10\}.$$

- The part before the pipe is called the output function,
- x is the variable,
- \mathbb{N} is the input set
- and $x \leq 10$ is the *predicate*



S contains the doubles of all natural numbers that satisfy the predicate.

- Example in Haskell:
 - `ghci> take 10 [2,4..]`
 - `[2,4,6,8,10,12,14,16,18,20]`

LIST COMPREHENSION

- Creates a list from one or more other lists

Example 1:

```
ghci> [x*x | x <- [1..5]]  
[1,4,9,16,25]
```

Example 2:

```
ghci> [(i,j) | i <- [1..5], even i, j <- [i..5]]  
[(2,2),(2,3),(2,4),(2,5),(4,4),(4,5)]
```

Example 3:

```
ghci> [x*2 | x <- [1..10]]  
[2,4,6,8,10,12,14,16,18,20]
```

LIST COMPREHENSION

- Predicates go after the binding parts and are separated from them by a comma.
- Let's say we want only the elements which, doubled, are greater than or equal to 12.

```
ghci> [x*2 | x <- [1..10], x*2 >= 12]  
[12,14,16,18,20]
```



Weeding out lists
by predicates is
also called
filtering

Try it yourself:

- How to obtain all numbers from 50 to 100 whose remainder when divided with the number 7 is 3?

```
ghci> [ x | x <- [50..100], x `mod` 7 == 3]  
[52,59,66,73,80,87,94]
```

A COMPREHENSION SCENARIO USING FUNCTION !

- Let's say we want a comprehension that replaces each odd number greater than 10 with "**BANG!**" and each odd number that's less than 10 with "**BOOM!**". If a number isn't odd, we throw it out of our list.

```
--boomBangs.hs
```

```
boomBangs xs = [ if x < 10 then "BOOM!" else "BANG!" | x <- xs, odd x]
```

```
ghci> :l boomBangs.hs
```

```
[1 of 1] Compiling Main           ( boomBangs.hs, interpreted )
```

```
Ok, one module loaded.
```

```
ghci> boomBangs [7..13]
```

```
["BOOM!","BOOM!","BANG!","BANG!"]
```

INCLUDING SEVERAL PREDICATES

Example:

Obtain all numbers from 10 to 20 that are not 13, 15 or 19

```
ghci> [ x | x <- [10..20], x /= 13, x /= 15, x /= 19]
```

```
[10,11,12,14,16,17,18,20]
```


DRAWING THE OUTPUT FROM SEVERAL LISTS

- When drawing from several lists, comprehensions produce all combinations of the given lists and then join them by the output function we supply.
- A list produced by a comprehension that draws from two lists of length 4 will have a length of 16, provided we don't filter them.
- Example: To obtain the product of all possible combinations of two lists:

```
ghci> [ x*y | x <- [2,5,10], y <- [8,10,11]]
```

```
[16,20,22,40,50,55,80,100,110]
```
- Using Filter: To get all possible products that are more than 50

```
ghci> [ x*y | x <- [2,5,10], y <- [8,10,11], x*y > 50]
```

```
[55,80,100,110]
```

MORE EXAMPLES

- A list comprehension that combines a list of adjectives and a list of nouns

```
ghci> let nouns = ["hobo","frog","pope"]
```

```
ghci> let adjectives = ["lazy","grouchy","scheming"]
```

```
ghci> [adjective ++ " " ++ noun | adjective <- adjectives, noun <- nouns]
```

```
["lazy hobo","lazy frog","lazy pope","grouchy hobo","grouchy  
frog","grouchy pope","scheming hobo","scheming frog","scheming pope"]
```

- Nested list comprehensions:

```
ghci> let xxs = [[1,3,5,2,3,1,2,4,5], [1,2,3,4,5,6,7,8,9],  
[1,2,4,2,1,6,3,1,3,2,3,6]]
```

```
ghci> [ [ x | x <- xs, even x ] | xs <- xxs]
```

```
[[2,2,4],[2,4,6,8],[2,4,2,6,2,6]]
```

SOME COMMON LIST FUNCTIONS USING COMPREHENSIONS : MAP

```
--mymap.hs
```

```
mymap f xs = [ f x | x <- xs ]
```

Mymap applies the function f to each element x of the list xs

```
ghci> :l mymap.hs
```

```
[1 of 1] Compiling Main             ( mymap.hs, interpreted )
```

```
Ok, one module loaded.
```

```
ghci> mymap abs [1,-2,3,-4,5]
```

```
[1,2,3,4,5]
```

```
ghci> mymap odd [1,2,3,4,5]
```

```
[True,False,True,False,True]
```

FILTER FUNCTION USING COMPREHENSION

```
--myfilter.hs
```

```
myfilter p xs = [x | x <- xs, p x]
```

myfilter applies the
predicate p to each
element x of the list
xs

```
ghci> :l myfilter.hs
```

```
[1 of 1] Compiling Main             ( myfilter.hs, interpreted )
```

```
Ok, one module loaded.
```

```
ghci> myfilter odd [1,2,3,4,5]
```

```
[1,3,5]
```

```
ghci> myfilter (<4) [1,2,3,4,5]
```

```
[1,2,3]
```

CONCATENATION USING COMPREHENSION

```
--myconcat.hs
```

```
myconcat xss = [x | xs <- xss, x <- xs]
```

```
ghci> :l myconcat.hs
```

```
[1 of 1] Compiling Main (myconcat.hs, interpreted)
```

```
Ok, one module loaded.
```

```
ghci> myconcat [[1,2,3],[4,5,6]]
```

```
[1,2,3,4,5,6]
```

```
ghci> myconcat [['a','b','c'],['d','e','f']]
```

```
"abcdef "
```

```
ghci> myconcat [[1,2,3],[4,5,6],[7,8,9]]
```

```
[1,2,3,4,5,6,7,8,9]
```



xss is a list of lists

PYTHAGOREAN TRIAD FUNCTION - EXAMPLE

--Pythagorean triads function using list comprehension pythtriad.hs

```
triads :: Int -> [(Int,Int,Int)]
```

```
triads n = [(x,y,z) | x <- [1..n], y <- [1..n],  
  z <- [1..n], x*x+y*y==z*z]
```

```
ghci> :l pythtriad.hs
```

```
[1 of 1] Compiling Main          ( pythtriad.hs, interpreted )
```

```
Ok, one module loaded.
```

```
ghci> triads 15
```

```
[(3,4,5),(4,3,5),(5,12,13),(6,8,10),(8,6,10),(9,12,15),(12,5,13),(12,9,15)]
```

SOME BASIC OPERATIONS

- null :

`null :: [a] -> Bool`

`ghci> null []`

`True`

`ghci> null [1,2,3,4,5]`

`False`

- head:

`head :: [a] -> a`

- `head (x:xs) = x`

`ghci> head [1,2,3,4,5]`

`1`

- tail:

`tail :: [a] -> [a]`

- `tail (x:xs) = xs`

`ghci> tail [1,2,3,4,5]`

`[2,3,4,5]`

Haskell reports an error if we try to evaluate `head []` or `tail []`

`ghci> head []`

`*** Exception: Prelude.head: empty list`

`ghci> tail []`

`*** Exception: Prelude.tail: empty list`

SOME BASIC OPERATIONS

- last:

$\text{last} :: [a] \rightarrow a$

- $\text{last } [x] = x$

```
ghci> last [1]
```

1

- $\text{last } (x:y:ys) = \text{last } (y:ys)$

```
ghci> last [1,2,3,4,5]
```

5

- Concatenation:

$(++) :: [a] \rightarrow [a] \rightarrow [a]$

- $[] ++ ys = ys$

```
ghci> [] ++ [1,2,3,4,5]
```

[1,2,3,4,5]

SOME BASIC OPERATIONS

- Concatenation:

$(++) :: [a] \rightarrow [a] \rightarrow [a]$

- $[] ++ ys = ys$
- $(x:xs) ++ ys = x:(xs ++ ys)$

```
ghci> [1,2,3,4,5] ++ [6,7,8,9,10]  
[1,2,3,4,5,6,7,8,9,10]
```

- length:

$\text{length} :: [a] \rightarrow \text{Int}$

- $\text{length} [] = 0$
- $\text{length} (x:xs) = 1 + \text{length} xs$

```
ghci> length []  
0
```

```
ghci> length [1,2,3,4,5]  
5
```

DEFINING CONCAT USING PATTERN MATCHING

```
--concat1 using pattern matching  
concat1 :: [[a]] -> [a]  
concat1 [] = []  
concat1 (xs:xss) = xs ++ concat1 xss
```

The concat function takes a list of lists, all of the same type, and concatenates them into a single list

```
ghci> :l concat1.hs  
[1 of 1] Compiling Main          ( concat1.hs, interpreted )  
Ok, one module loaded.  
ghci> concat1 []  
[]  
ghci> concat1 [[1,2,3,4], [5,6,7,8]]  
[1,2,3,4,5,6,7,8]  
ghci> concat1 [[1,2,3,4], [5,6,7,8],[9,10,11,12]]  
[1,2,3,4,5,6,7,8,9,10,11,12]
```

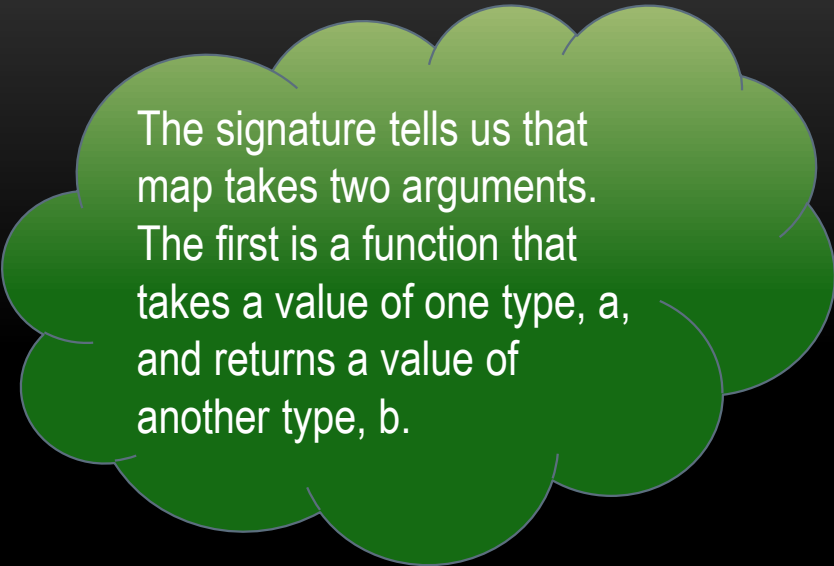
DEFINING MAP USING PATTERN MATCHING

--map1 using pattern matching

map1 :: (a -> b) -> [a] -> [b]

map1 f [] = []

map1 f (x:xs) = f x:map1 f xs



The signature tells us that map takes two arguments. The first is a function that takes a value of one type, a, and returns a value of another type, b.

ghci> :l map1.hs

[1 of 1] Compiling Main (map1.hs, interpreted)

Ok, one module loaded.

ghci> map1 abs [1,-2,3,-4,5]

[1,2,3,4,5]

ghci> map1 odd [1,2,3,4,5]

[True,False,True,False,True]

DEFINING FILTER USING PATTERN MATCHING

--filter1 using pattern matching

filter1 :: (a -> Bool) -> [a] -> [a]

filter1 p [] = []

filter1 p (x:xs) = if p x then x:filter1 p xs

else filter1 p xs

The filter function takes a predicate and applies it to every element in its input list, returning a list of only those for which the predicate evaluates to True.

ghci> :l filter1.hs

[1 of 1] Compiling Main (filter1.hs, interpreted)

Ok, one module loaded.

ghci> filter1 odd [1,2,3,4,5]

[1,3,5]

ghci> filter1 even [1,2,3,4,5]

[2,4]

ghci> filter1 (<=3) [1,2,3,4,5]

[1,2,3]

ZIP AND ZIPWITH

- The zip function takes two lists and “zips” them into a single list of pairs
- The resulting list is the same length as the shorter of the two inputs

```
ghci> :type zip
```

```
zip :: [a] -> [b] -> [(a, b)]
```

```
ghci> zip [12,72,93] "zippity"
```

```
[(12,'z'),(72,'i'),(93,'p')]
```

- zipWith takes two lists and applies a function to each pair of
- elements, generating a list that is the same length as the shorter of the two

```
ghci> :type zipWith
```

```
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]
```

```
ghci> zipWith (+) [1,2,3] [4,5,6]
```

```
[5,7,9]
```

STANDARD PRELUDE DEFINITIONS OF ZIP AND ZIPWITH

$\text{zip} :: [a] \rightarrow [b] \rightarrow [(a,b)]$

$\text{zip } (x:xs) (y:ys) = (x,y) : \text{zip } xs \ ys$

$\text{zip } _ _ = []$

$\text{zipWith} :: (a \rightarrow b \rightarrow c) \rightarrow [a] \rightarrow [b] \rightarrow [c]$

$\text{zipWith } f (x:xs) (y:ys) = f \ x \ y : \text{zipWith } f \ xs \ ys$

$\text{zipWith } f _ _ = []$

MORE LIST OPERATORS AND FUNCTIONS

- If you want to get an element out of a list by index, use `!!`. The indices start at 0.

```
ghci> "Steve Buscemi" !! 6
```

```
'B'
```

```
ghci> [9.4,33.2,96.2,11.2,23.25] !! 1
```

```
33.2
```

- Lists can also contain lists. They can also contain lists that contain lists that contain lists ...

```
ghci> let b = [[1,2,3,4],[5,3,3,3],[1,2,2,3,4],[1,2,3]]
```

```
ghci> b
```

```
[[1,2,3,4],[5,3,3,3],[1,2,2,3,4],[1,2,3]]
```

```
ghci> b ++ [[1,1,1,1]]
```

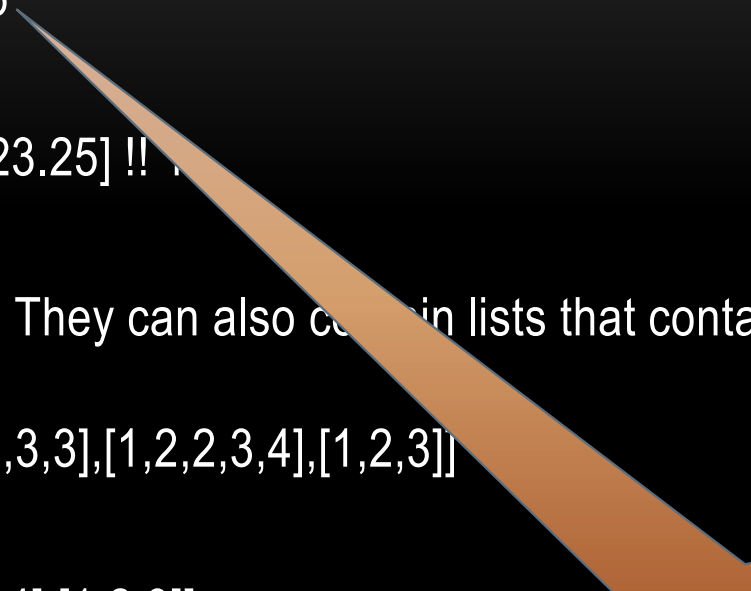
```
[[1,2,3,4],[5,3,3,3],[1,2,2,3,4],[1,2,3],[1,1,1,1]]
```

```
ghci> [6,6,6]:b
```

```
[[6,6,6],[1,2,3,4],[5,3,3,3],[1,2,2,3,4],[1,2,3]]
```

```
ghci> b !! 2
```

```
[1,2,2,3,4]
```



But if you try to get the sixth element from a list that only has four elements, you'll get an error so be careful!

LIST COMPARISONS

- Lists can be compared if the stuff they contain can be compared
- When using $<$, $<=$, $>$ and $>=$ to compare lists, they are compared in lexicographical order
- First the heads are compared
- If they are equal then the second elements are compared, etc.

```
ghci> [3,2,1] > [2,1,0]
```

```
True
```

```
ghci> [3,2,1] > [2,10,100]
```

```
True
```

```
ghci> [3,4,2] > [3,4]
```

```
True
```

```
ghci> [3,4,2] > [2,4]
```

```
True
```

```
ghci> [3,4,2] == [3,4,2]
```

```
True
```


SOME MORE BASIC FUNCTIONS TO OPERATE ON LISTS

- `init`: takes a list and returns everything except its last element

```
ghci> init [5,4,3,2,1]
```

```
[5,4,3,2]
```

- `reverse` reverses a list:

```
ghci> reverse [5,4,3,2,1]
```

```
[1,2,3,4,5]
```

- **`maximum`** takes a list of stuff that can be put in some kind of order and returns the biggest element and **`minimum`** returns the smallest.

```
ghci> minimum [8,4,2,1,5,6]
```

```
1
```

```
ghci> maximum [1,9,2,3,4]
```

```
9
```

SOME MORE BASIC FUNCTIONS TO OPERATE ON LISTS

- **Sum** takes a list of numbers and returns their sum.
- **Product** takes a list of numbers and returns their product.

```
ghci> sum [5,2,1,6,3,2,5,7]
```

```
31
```

```
ghci> product [6,2,1,2]
```

```
24
```

```
ghci> product [1,2,5,6,7,9,2,0]
```

```
0
```

SOME MORE BASIC FUNCTIONS TO OPERATE ON LISTS

- **elem** takes an element and a list of items and says if the element is an item in the list
- **elem** is usually called as an infix function for easy readability

```
ghci> 4 `elem` [3,4,5,6]
```

```
True
```

```
ghci> 10 `elem` [3,4,5,6]
```

```
False
```

- **cycle** takes a list and cycles it into an infinite list. If you just try to display the result, it will go on forever so you have to slice it off somewhere.

```
ghci> take 10 (cycle [1,2,3])
```

```
[1,2,3,1,2,3,1,2,3,1]
```

```
ghci> take 12 (cycle "LOL ")
```

```
"LOL LOL LOL "
```

SOME MORE BASIC FUNCTIONS TO OPERATE ON LISTS

- **repeat** takes an element and produces an infinite list of just that element. It's like cycling a list with only one element.

```
ghci> take 10 (repeat 5)
```

```
[5,5,5,5,5,5,5,5,5,5]
```

- **replicate** function also replicates some number of the same element in a list:

```
ghci> replicate 3 10
```

```
[10,10,10]
```