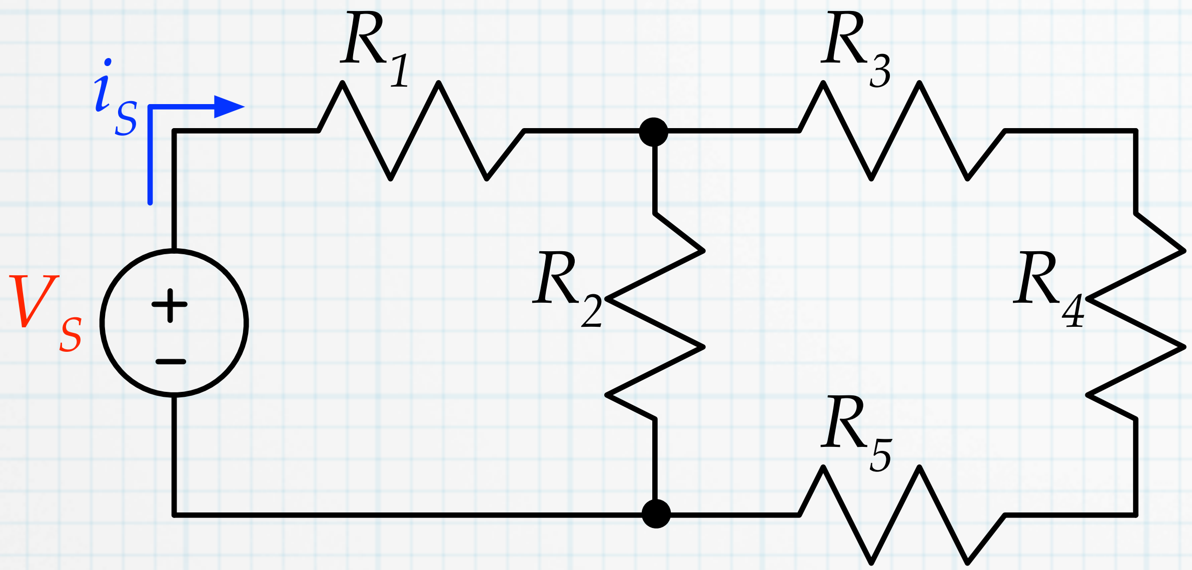
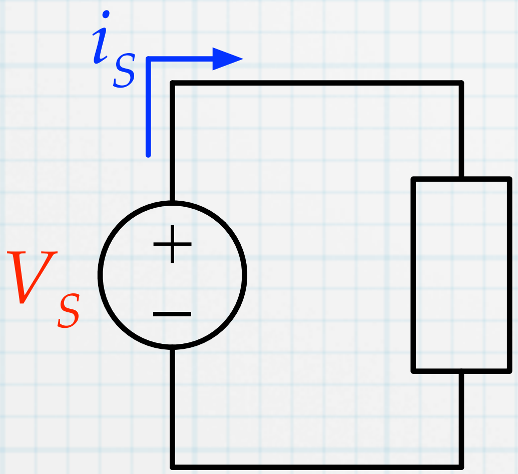
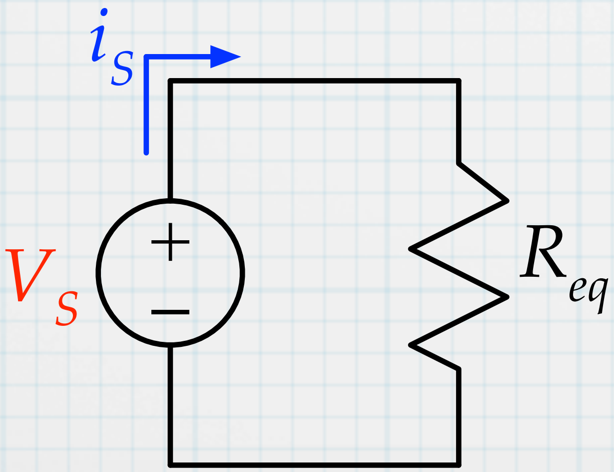
Equivalent resistance



Interested only in i_S. Not interested in details of individual resistor currents and voltages.

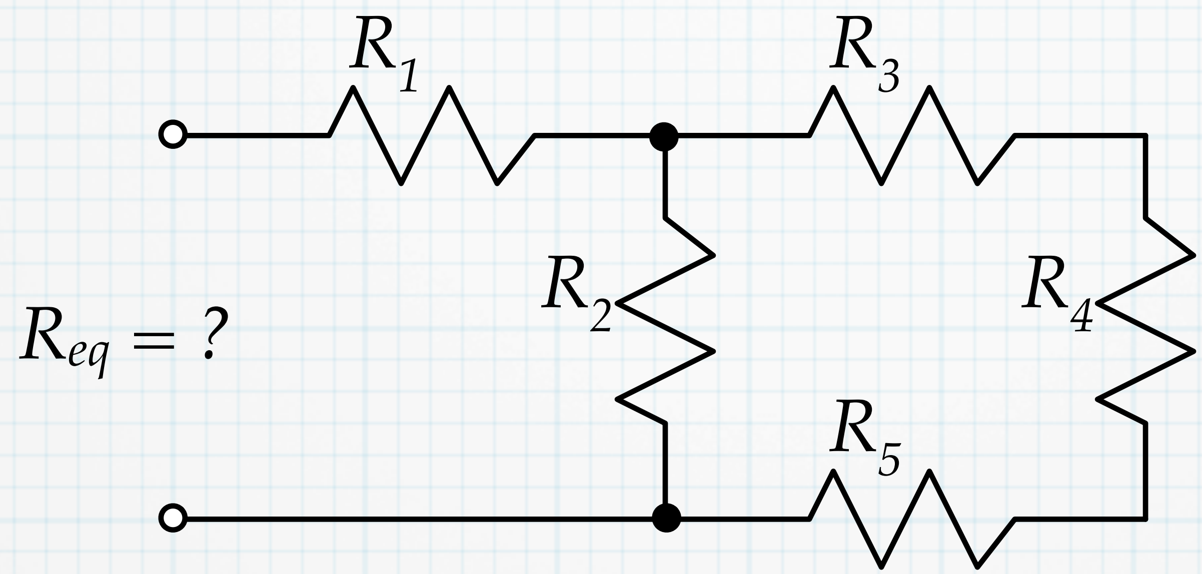


Same applied V_S must give same resulting i_S. (Same power supplied.)



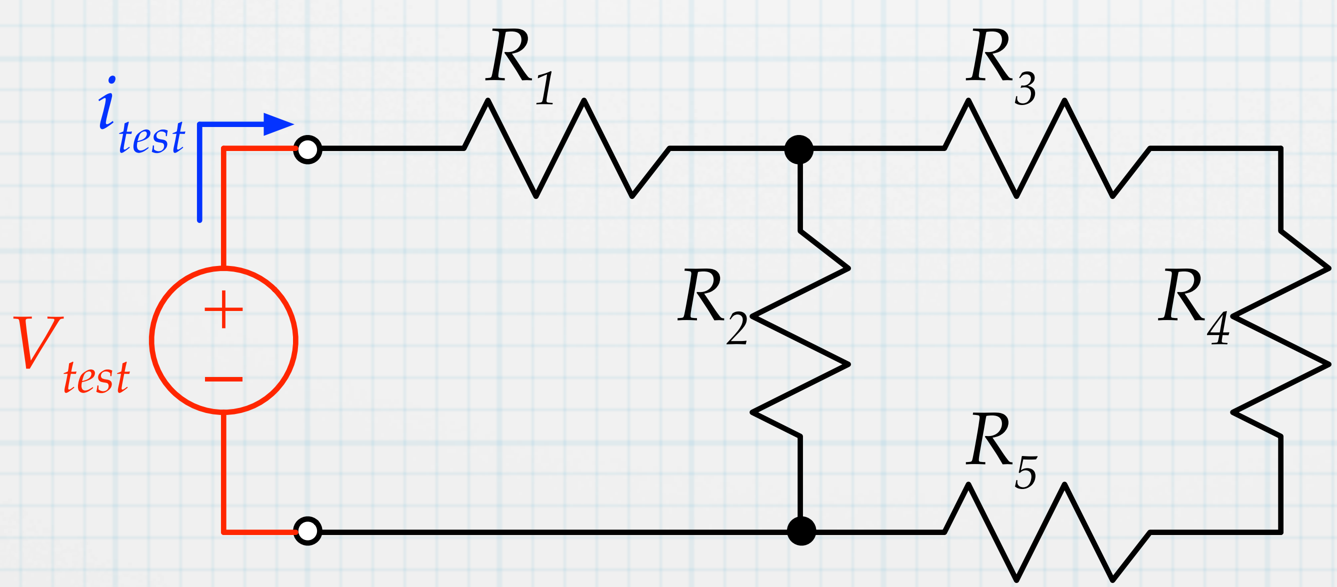
$$R_{eq} = \frac{V_S}{i_S}$$

Test generator (or test source) method



Equivalent resistance must be defined between 2 nodes of the network. A different pair of nodes gives different R_{eq}.

Apply a test generator between the two nodes of interest.

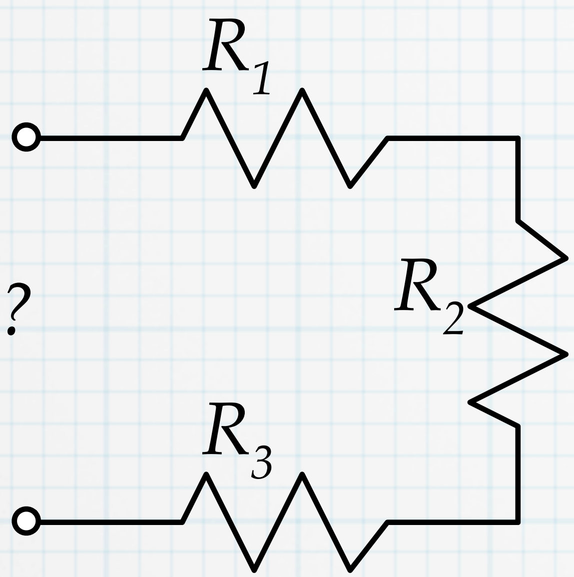


Apply V_{test}.

Determine i_{test}.

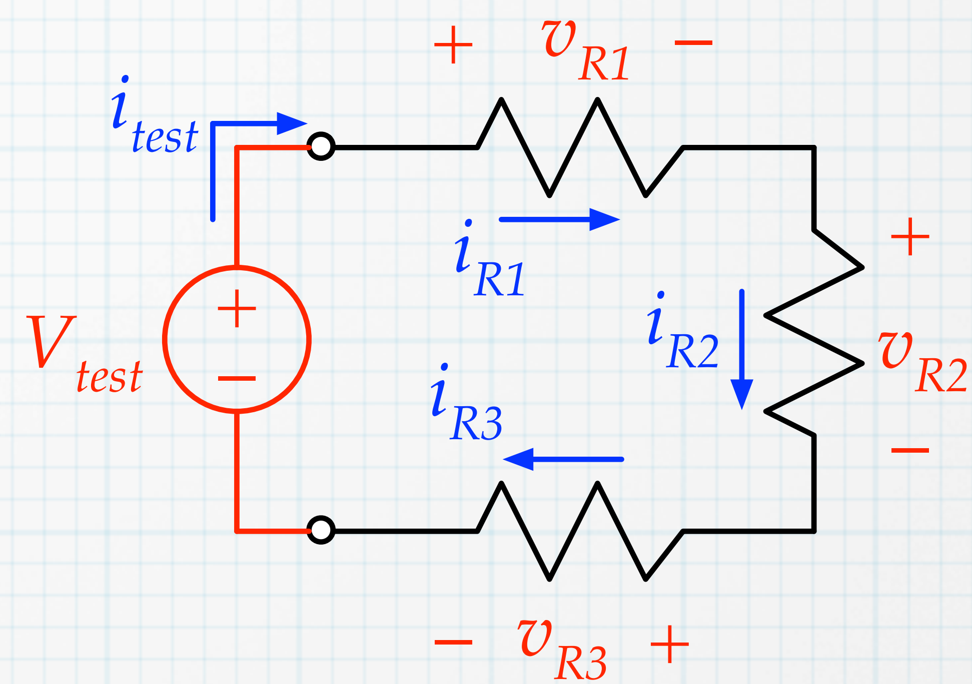
$$R_{eq} = \frac{V_{test}}{i_{test}}$$

Series combination



$R_{eq} = ?$

Apply test source.
Define voltages
and currents.



By KCL: $i_{test} = i_{R1} = i_{R2} = i_{R3}$ Series connection.

By KVL: $V_{test} - v_{R1} - v_{R2} - v_{R3} = 0$

use Ohm's law: $V_{test} - i_{R1}R_1 - i_{R2}R_2 - i_{R3}R_3 = 0$

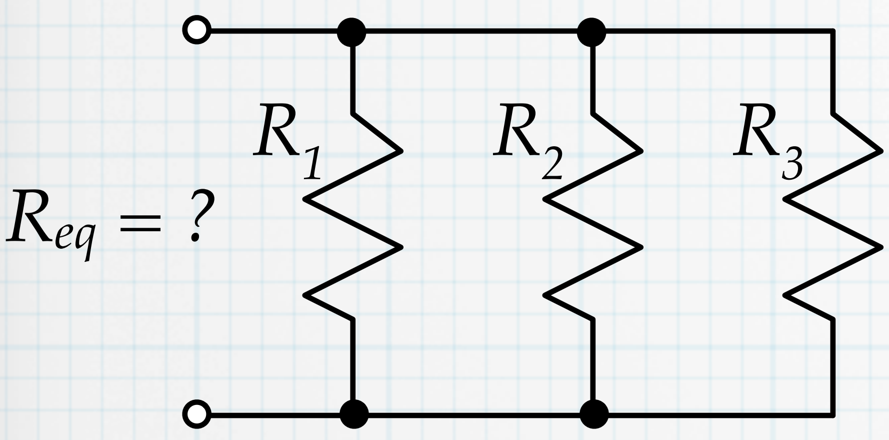
$$V_{test} - i_{test}\left(R_1 + R_2 + R_3\right) = 0$$

$$R_{eq} = \frac{V_{test}}{i_{test}} = R_1 + R_2 + R_3$$

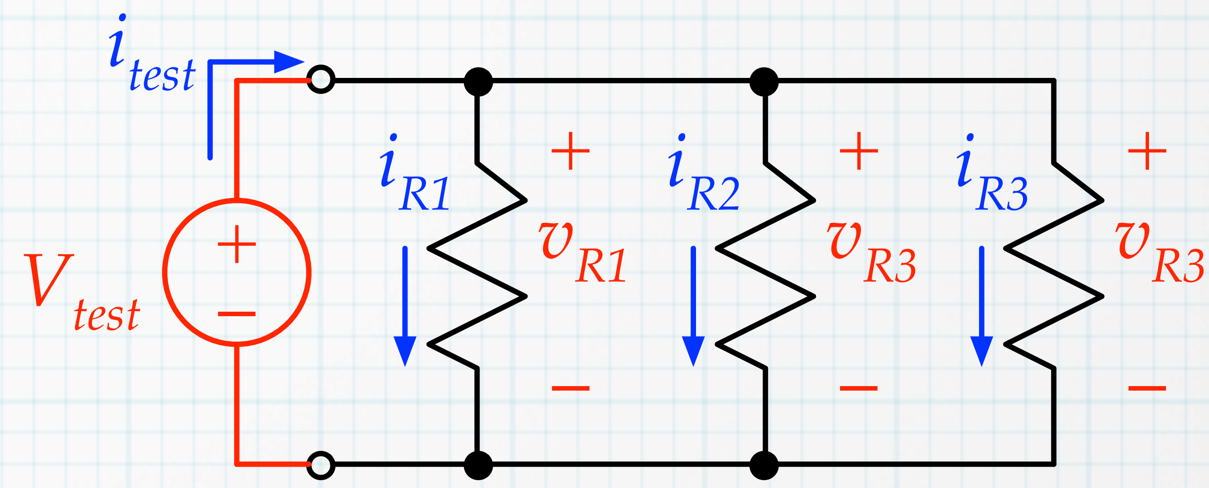
Series combination:

$$R_{eq} = \sum_{m=1}^{N} R_m$$

Parallel combination



Apply V_{test}. Define voltages and currents.



By KVL: $V_{test} = v_{R1} = v_{R2} = v_{R3}$ (Parallel connection)

By KCL: $i_{test} = i_{R1} + i_{R2} + i_{R3}$

use Ohm's law: $$i_{test} = \frac{v_{R1}}{R_1} + \frac{v_{R2}}{R_2} + \frac{v_{R3}}{R_3}$$

$$i_{test} = \frac{v_{test}}{R_1} + \frac{v_{test}}{R_2} + \frac{v_{test}}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{i_{test}}{V_{test}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Parallel combination:

$$\boxed{\frac{1}{R_{eq}} = \sum_{m=1}^{N} \frac{1}{R_m}}$$

Series combination: Easy to calculate.

Series: equivalent is always bigger than any resistor in the string.

$R_{eq} > R_m.$

Parallel: equivalent is always smaller than any single resistor the parallel branches.

$R_{eq} < R_m.$

Special cases for parallel combinations:

Two resistors only: $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$ (product over sum)

More special cases for parallel combinations:

Two resistors, $R_1 = R_2 = R$: $$R_{eq} = \frac{R^2}{2R} = \frac{R}{2}$$

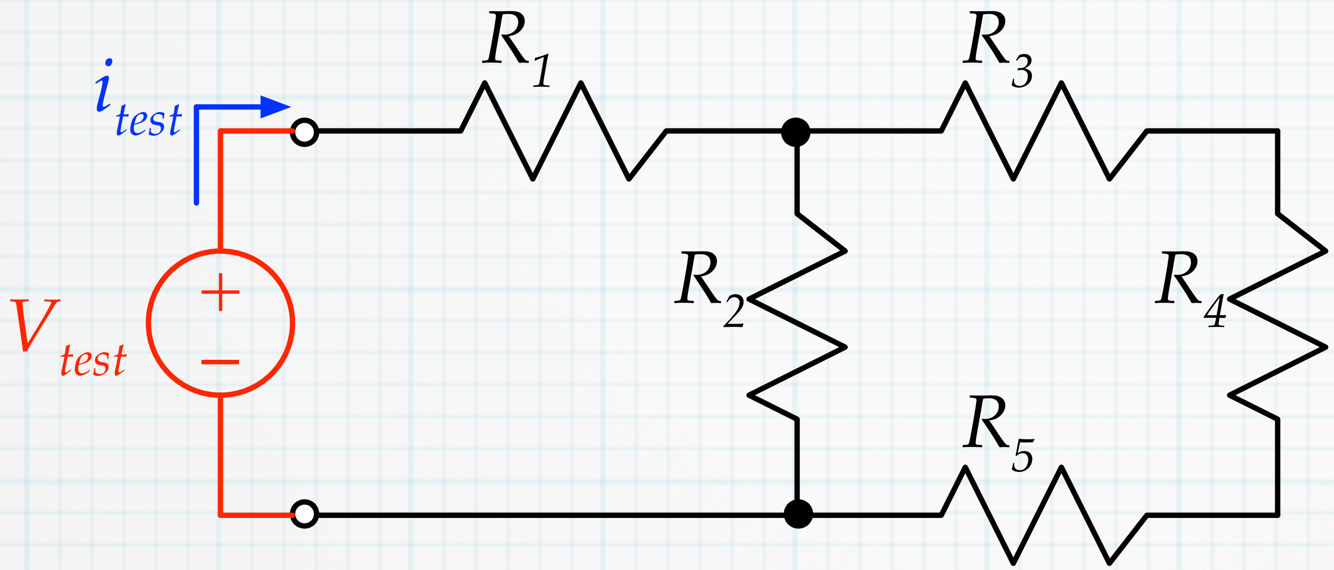
Two resistors, $R_2 = 2R_1$: $$R_{eq} = \frac{2R_1^2}{3R} = \frac{2}{3}R_1$$

One small resistor: $R_1 << R_2, R_3, R_4,...$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + ...$$
$$\approx \frac{1}{R_1}$$

$$R_{eq} \approx R_1$$ (Equivalent is approximately equal to smallest.

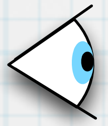
Combination circuits



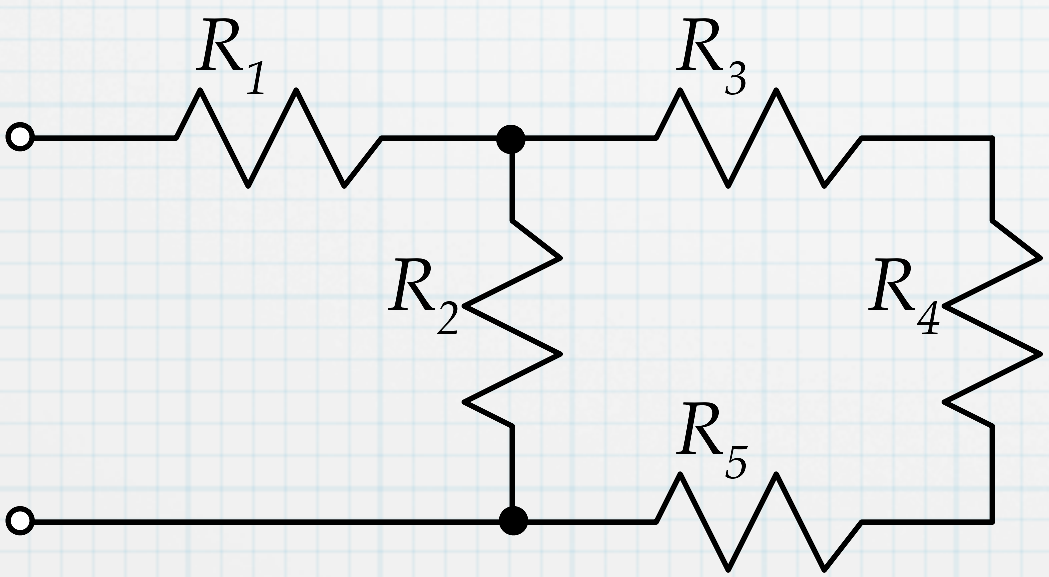
Test generator method always works.

Sometime necessary (with dependent sources in circuit).

For purely resistive circuits, there is a faster method – inspection.



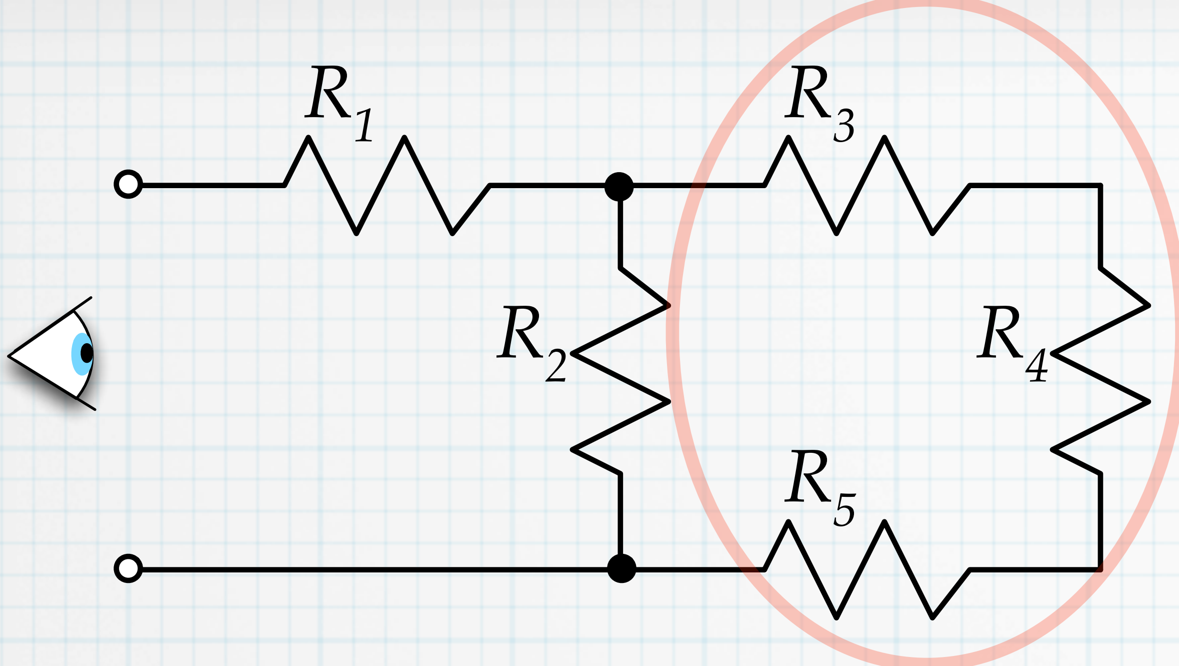
Ohm's eye



Inspect structure of network.

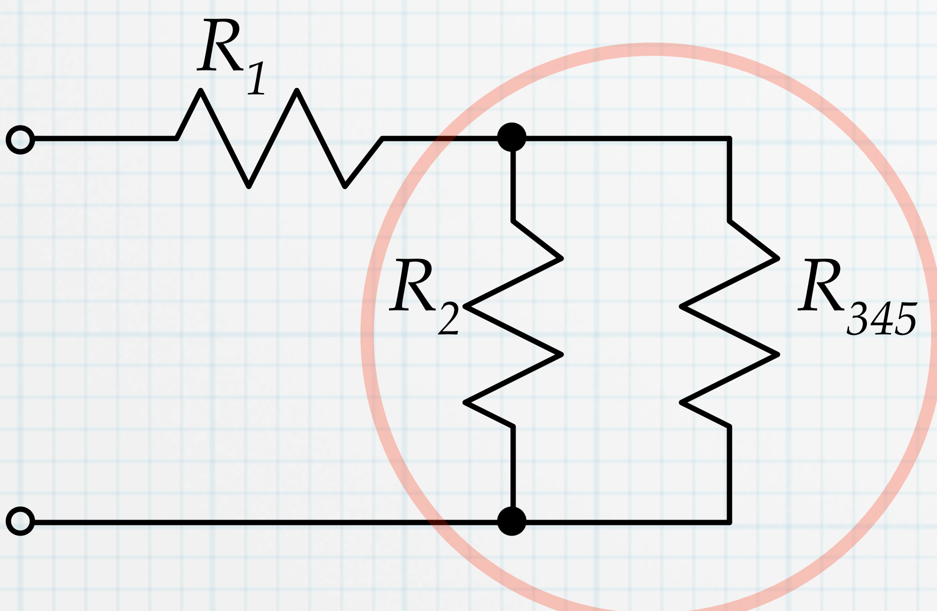
Use parallel & series combinations to sequentially reduce pieces of the network to single resistances.

With practice, you will be able to find R_{eq} in one step.



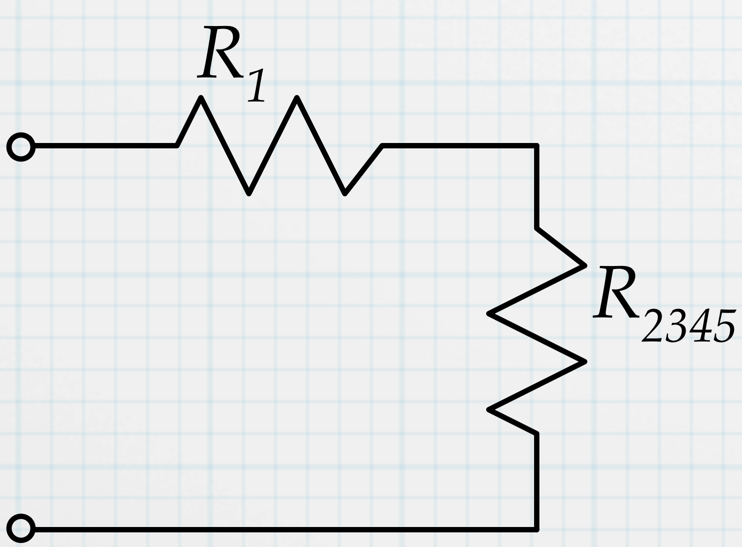
1. Recognize and replace the series branch with the three resistors.

$$R_{345} = R_3 + R_4 + R_5$$



2. Recognize and replace the parallel combination.

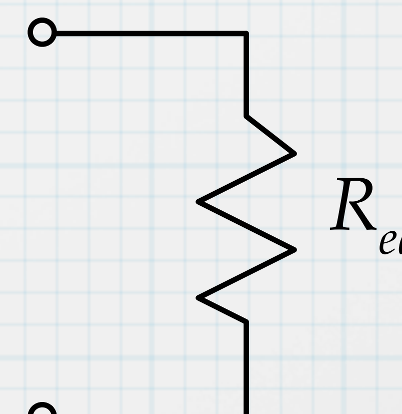
$$R_{2345} = R_2 || R_{345} = \frac{R_2 R_{345}}{R_2 + R_{345}}$$



3. We are left with a simple series pair.

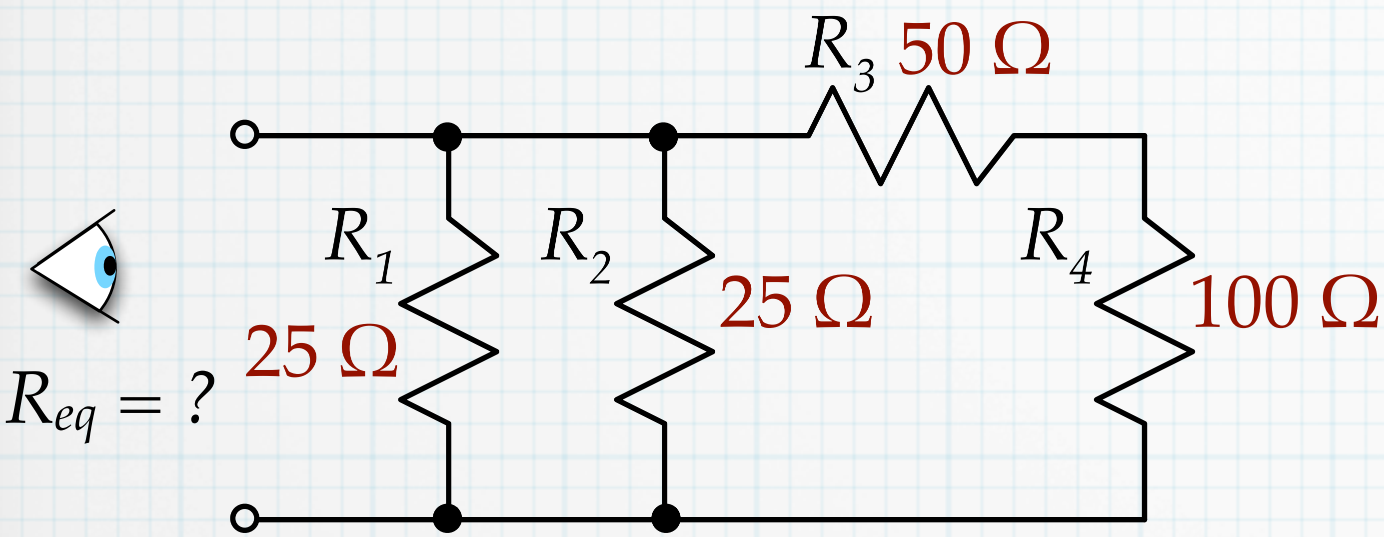
$$R_{eq} = R_1 + R_{2345}$$

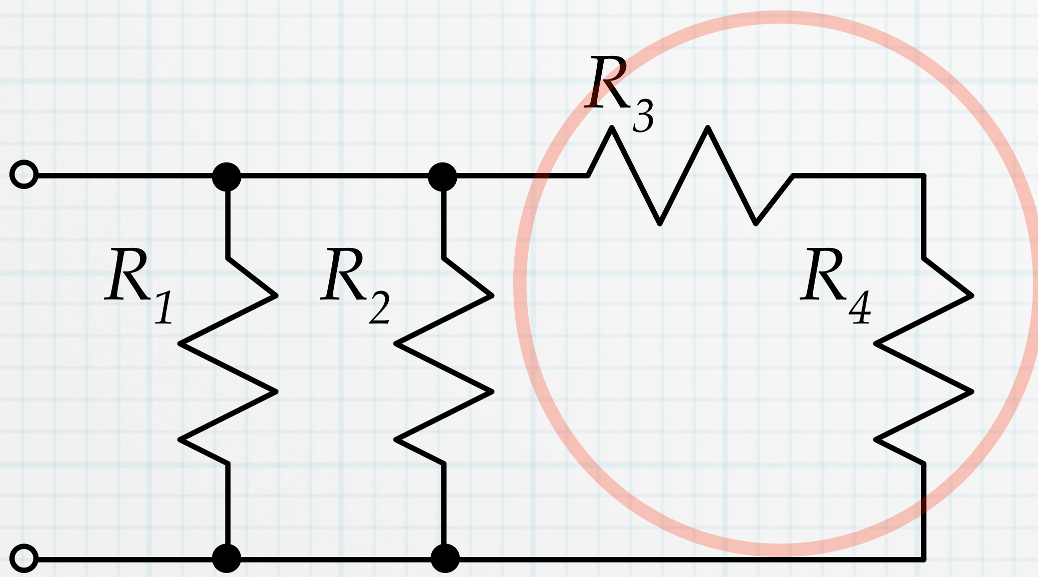
$$= R_1 + \frac{R_2 \left(R_3 + R_4 + R_5\right)}{R_2 + \left(R_3 + R_4 + R_5\right)}$$



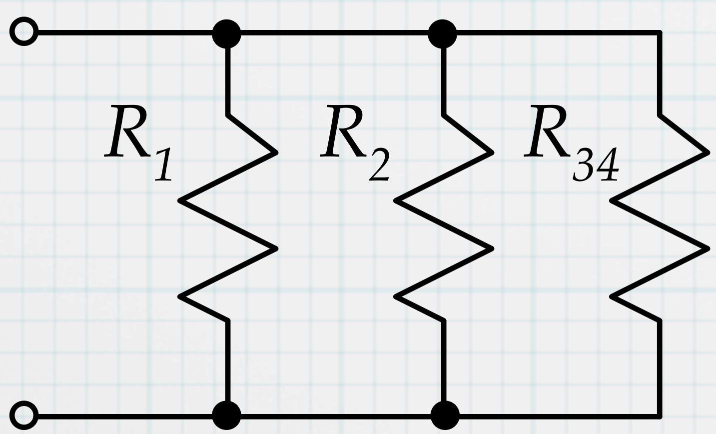
Example

$R_{eq} = ?$

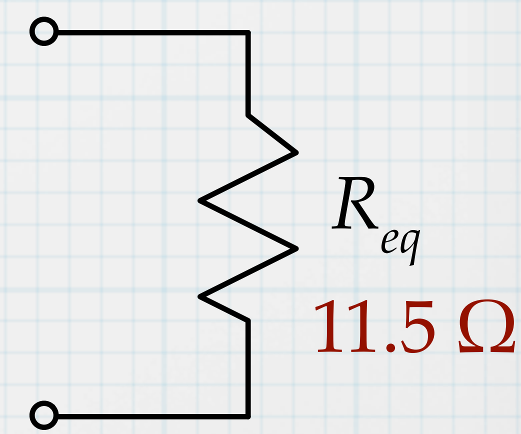




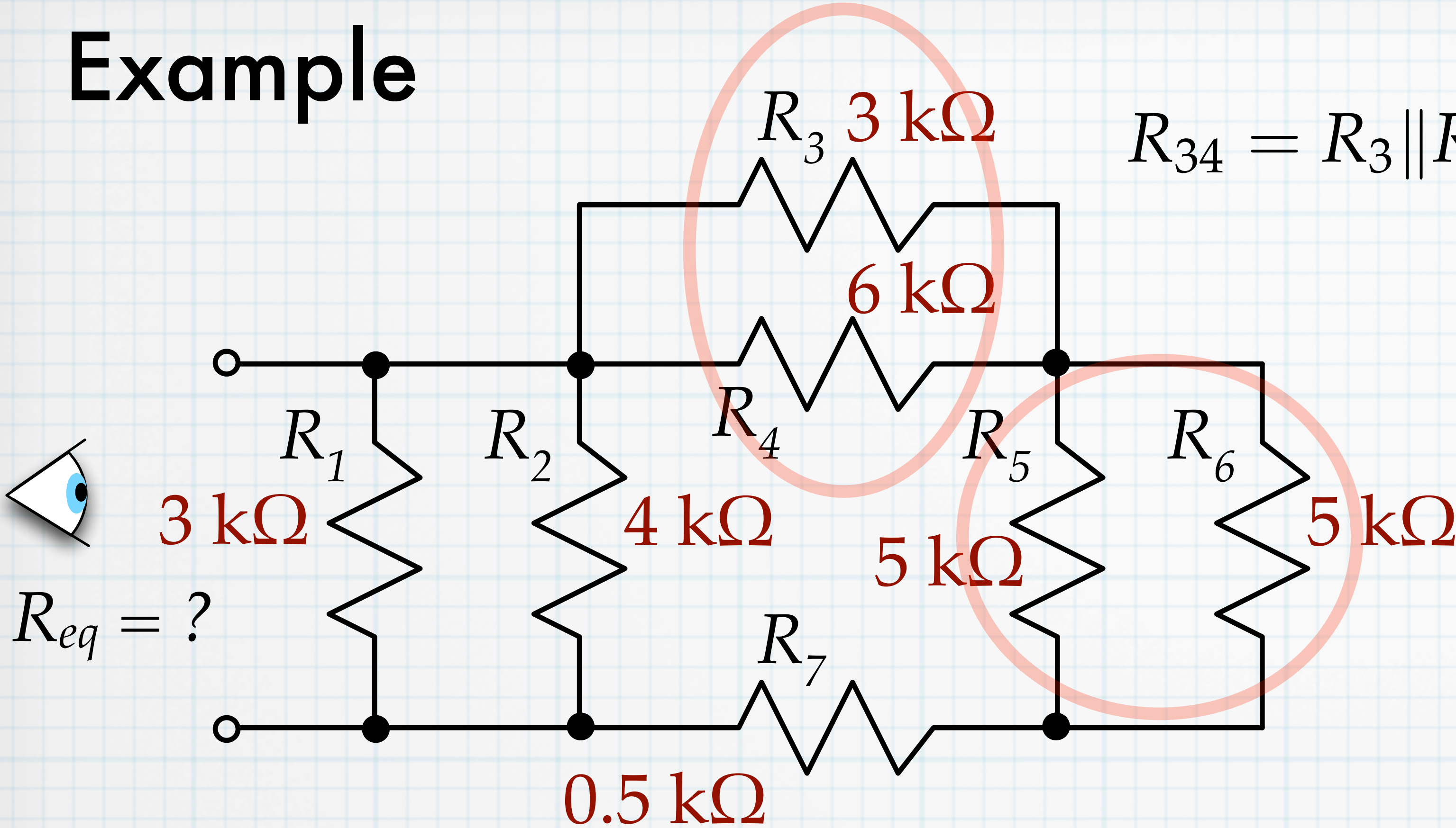
$$R_{34} = R_3 + R_4$$
$$= 50\Omega + 100\Omega = 150\Omega$$



$$R_{eq} = R_1 \| R_2 \| R_{34}$$
$$\frac{1}{R_{eq}} = \frac{1}{25\Omega} + \frac{1}{25\Omega} + \frac{1}{150\Omega}$$
$$R_{eq} = 11.5\Omega$$



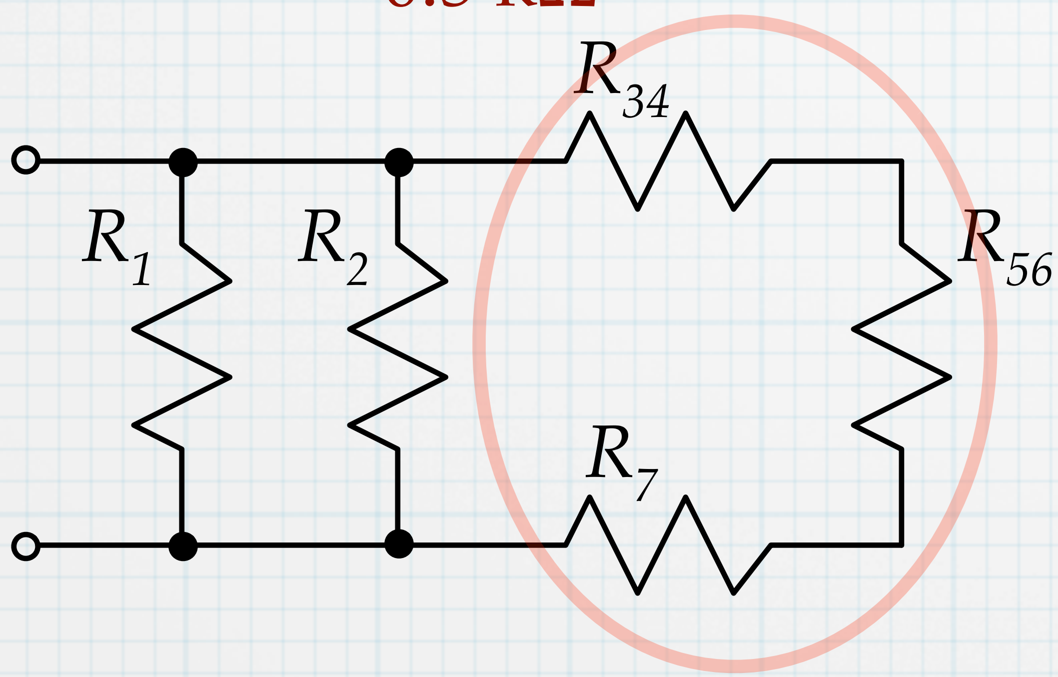
Example



$$R_{34} = R_3 \| R_4 = \frac{(3\text{k}\Omega)\,(6\text{k}\Omega)}{3\text{k}\Omega + 6\text{k}\Omega} = 2\text{k}\Omega$$

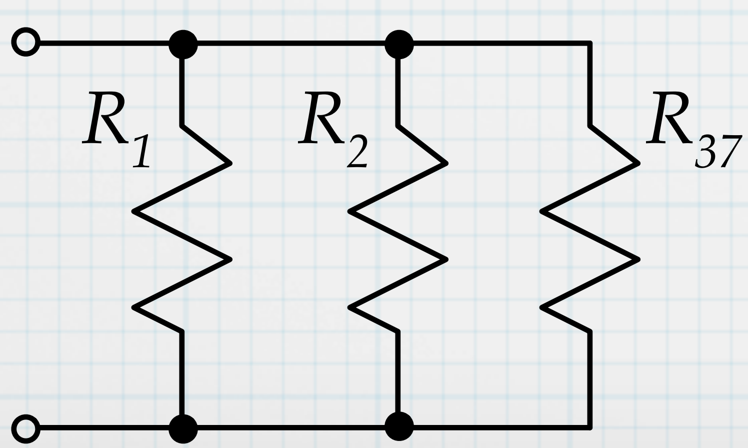
$$R_{56} = R_5 \| R_6$$

$$= \frac{(5\text{k}\Omega)\,(5\text{k}\Omega)}{5\text{k}\Omega + 5\text{k}\Omega} = 2.5\text{k}\Omega$$



$$R_{37} = R_{34} + R_{56} + R_7$$

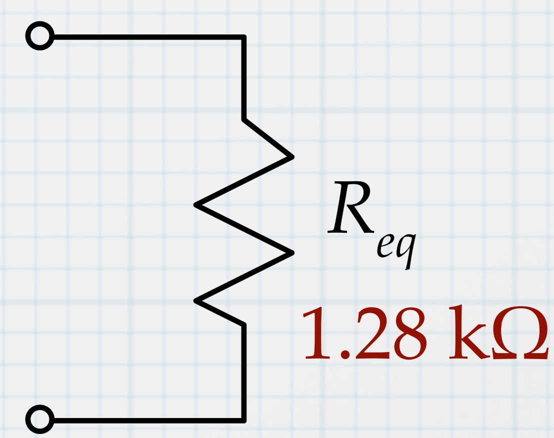
$$= 2\text{k}\Omega + 2.5\text{k}\Omega + 0.5\text{k}\Omega = 5\text{k}\Omega$$



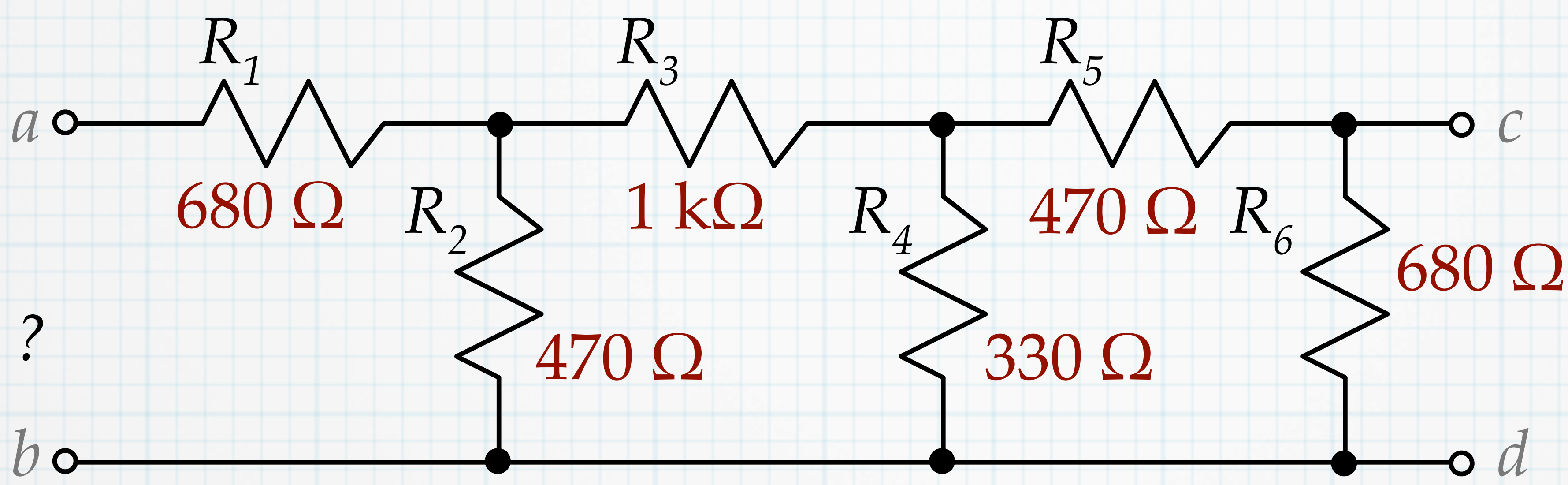
$$R_{eq} = R_1 \| R_2 \| R_{37}$$

$$\frac{1}{R_{eq}} = \frac{1}{3\text{k}\Omega} + \frac{1}{4\text{k}\Omega} + \frac{1}{5\text{k}\Omega}$$

$$R_{eq} = 1.28\text{k}\Omega$$

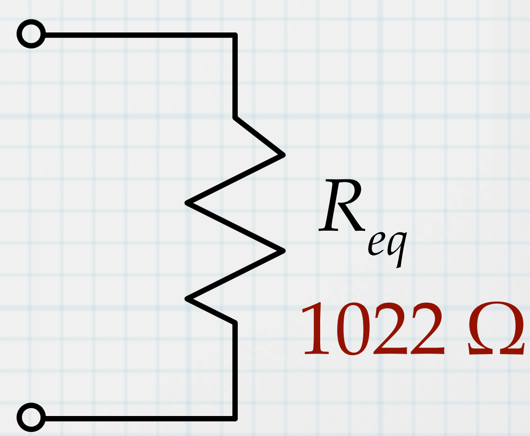


Example

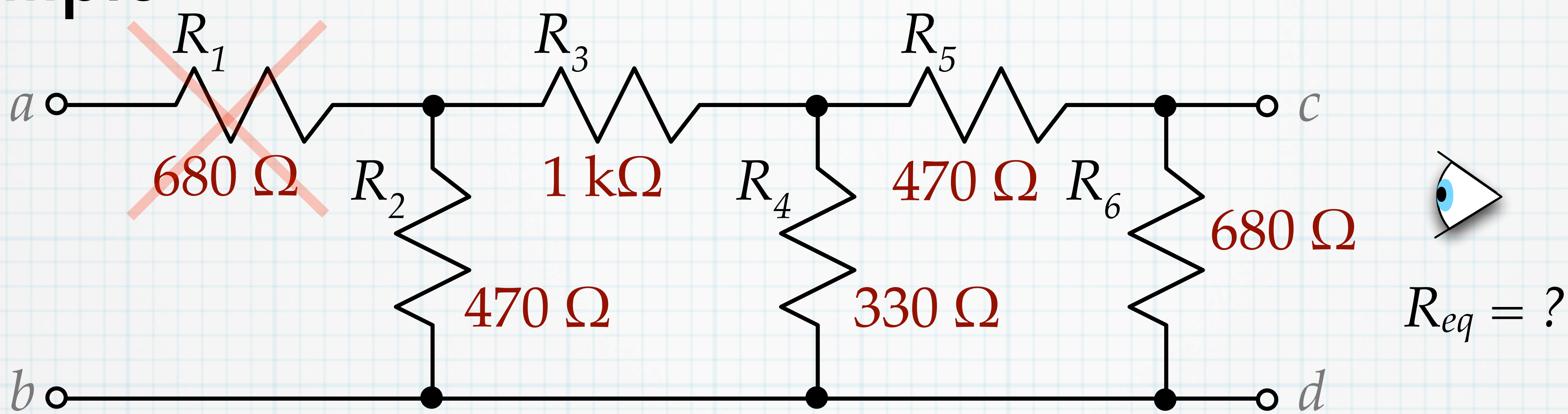


$R_{eq} = ?$

1. R_5 and R_6 are in series.
   $R_{56} = R_5 + R_6 = 1150\ \Omega$.
2. R_4 is in parallel with R_{56}.
   $R_{46} = R_4||R_6 = 256\ \Omega$.
3. R_3 is in series with R_{46}.
   $R_{36} = R_2 + R_{46} = 1000\ \Omega + 256\ \Omega = 1256\ \Omega$.
4. R_2 is in parallel with R_{36}.
   $R_{26} = R_2||R_{36} = 470\ \Omega\ ||\ 1256\ \Omega = 342\ \Omega$.
5. R_1 is in parallel with R_{26}.
   $R_{eq} = R_1 + R_{26} = 680\ \Omega + 342\ \Omega = 1022\ \Omega$.



Example



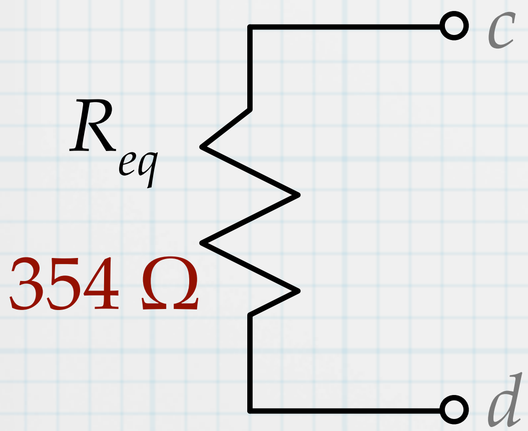
Find the R_{eq} referenced between the nodes c and d. Note that in this case R_1 is *dangling* (unconnected). No current will flow there – it has no effect on the rest of the circuit, and we can ignore it.

1. R_2 and R_3 are in series.
   $R_{23} = R_2 + R_3 = 470\ \Omega + 1000\ \Omega = 1470\ \Omega.$

2. R_{23} and R_4 are in parallel.
   $R_{24} = R_{23}||R_4 = 1470\ \Omega\ ||\ 330\ \Omega = 269.5\ \Omega.$

3. R_{24} and R_5 are in series.
   $R_{25} = R_{24} + R_5 = 269.5\ \Omega + 470\ \Omega = 739.5\ \Omega.$

4. R_{25} and R_6 are in parallel.
   $R_{eq} = R_{25}||R_6 = 739.5\ \Omega\ ||\ 680\ \Omega = 354\ \Omega.$



To study:

1. Work at least a dozen of the equivalent resistance practice problems on the web site, making sure you can get the correct answer each time.
2. Sketch out your own crazy resistor network and see if you can calculate the equivalent resistance.
3. "The equivalent resistance of a parallel combination is always less than the value of any of the individual resistors." Make sure that you understand this statement and why it is true.
4. Use a test generator alone with KCL and KVL to work any of the examples shown in this lecture. Show that you obtain the same result.
5. As noted, the test generator could be a current source. Then the goal would be to find the corresponding voltage. Re-work the series and parallel cases using a test current generator. Show that you obtain the same result.
6. Work through the first circuit (bottom of slide 2) using the test generator method. Show that you obtain the same equivalent resistance as the "inspection" method.