ELECTROSTATICS : Study of Electricity in which electric charges are static i.e. not moving, is called electrostatics

• STATIC CLING
• An electrical phenomenon that accompanies dry weather, causes these pieces of papers to stick to one another and to the plastic comb.
• Due to this reason our clothes stick to our body.

• ELECTRIC CHARGE : Electric charge is characteristic developed in particle of material due to which it exert force on other such particles. It automatically accompanies the particle wherever it goes.

• Charge cannot exist without material carrying it

• It is possible to develop the charge by **rubbing two solids having friction.**

• Carrying the charges is called **electrification**.

• Electrification due to friction is called **frictional electricity**.

 Since these charges are <u>not flowing</u> it is also called static electricity.

 There are two types of charges.   +ve and ‑ve.
• Similar charges repel each other,
• Opposite charges attract each other.

• Benjamin Franklin made this nomenclature of charges being +ve and ‑ve for mathematical calculations because adding them together cancel each other.

• Any particle has vast amount of charges.

• The number of positive and negative charges are **equal**, hence **matter is basically neutral.**

• Inequality of charges give the material a <u>net</u> charge which is equal to the difference of the two type of charges.

Electrostatic series :If two substances are rubbed together the former in series acquires the positive charge and later, the –ve.

(i) Glass (ii) Flannel (iii) Wool (iv) Silk (v) Hard Metal (vi) Hard rubber (vii) Sealing wax (viii) Resin (ix) Sulphur

Electron theory of Electrification

• Nucleus of atom is positively charged.
• The electron revolving around it is negatively charged.
• They are equal in numbers, hence atom is electrically neutral.
• With friction there is transfer of electrons, hence net charge is developed in the particles.

• It also explains that the charges are compulsorily developed in pairs
  equally . +ve in one body and –ve in second.
• It establish **conservation of charges in the universe.**
• The **loss** of **electrons** develops +ve charge. While **excess** of **electrons** develop ‑ve charge
• A **proton** is 1837 times heavier than electron hence it cannot be transferred. Transferring lighter electron is easier.
• Therefore for electrification of matter, only **electrons** are active and responsible.

Charge and Mass relation
• Charge cannot exist without matter.
• One carrier of charge is electron which has **mass** as well.
• Hence if there is charge transfer, mass is also transferred.
• Logically, negatively charged body is heavier then positively charged body.

Conductors, Insulators and Semiconductors
• Conductors : Material in which electrons can move easily and freely.
 Ex. Metals, Tap water, human body.
 Brass rod in our hand, if charged by rubbing the charge will move easily to earth. Hence Brass is a conductor.
The flow of this excess charge is called   **discharging**

• **Insulator** : Material in which charge cannot move freely. Ex . Glass, pure water, plastic etc.

- Electrons can be forced to move across an insulator by applying strong force (called electric field.) Then this acts like a conductor.

- ## dielectric strength.

The maximum electric field an insulator can withstand without becoming a conductor is called its dielectric strength.

- **Semiconductor** : is a material which under little stimulation (heat or Elect. Field) converts from insulator to a conductor.
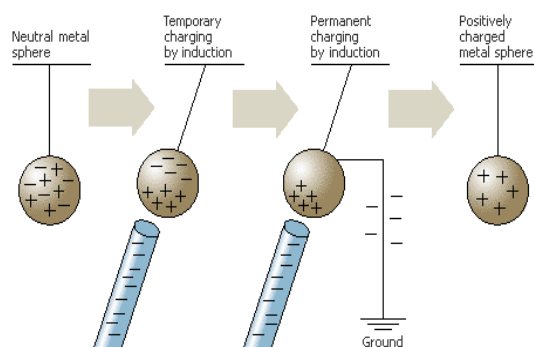
  Ex. Silicon, germanium.

- **Superconductor** : is that material which presents no resistance to the movement of the charge through it.

  The resistance is precisely zero.

Electrostatic Induction

- Phenomenon of polarization of charges in a body, when a charged body is present near it, is called electrostatic induction.

- In this process bodies are charged without touching them.

- **Charging   by   Induction**

Neutral metal sphere | Temporary charging by induction | Permanent charging by induction | Positively charged metal sphere

Ground

A charged object will induce a charge on a nearby conductor. In this example, a negatively charged rod pushes some of the negatively charged electrons to the far side of a nearby copper sphere because like charges repel each other. The positive charges that remain on the near side of the sphere are attracted to the rod.

- If the sphere is grounded so that the electrons can escape altogether, the charge on the sphere will remain if the rod is removed.

Basic properties of Electric charge

- Additivity of Electric charges
- Quantization of Electric charge
- Conservation of Electric Charge

Additivity of Charges...

- Charges can be added by simple rules of algebra. Addition of positive and negative charge makes Zero charge

Quantization of Electric charge

- Principle: **Electric charge is not a continuous quantity, but is an integral multiple of minimum charge (e).**
- Reason of quantization:
- Minimum charge e exist on an electron.
- The material which is transferred during electrification is an electron, in integral numbers.
- Hence **charge transferred has to be integral multiple of *e*.**
- Charge on an electron (-e) and charge on a proton (+e) are equal and opposite, and are the **minimum.**

 This minimum charge is 1.6×10^{-19} coulomb.

 one electron has charge $- 1.6 \times 10^{-19}$ C

One proton has charge $+ 1.6 \times 10^{-19}$ C

- Charge on a body Q is given by

    Q = $\pm$ ne

  Where n is a whole number 1,2,3.....

   and e = 1.6×10^{-19}

- since e is smallest value of charge, it is called Elementary Charge or Fundamental charge

- **(Quarks** : In new theories of proton and neutrons, a required constituent particles called Quarks which carry charges $\pm(1/3)e$ or $\pm(2/3)e$.

- But because free quarks do not exist and their sum is always an integral number, it does not violet the quantization rules.)
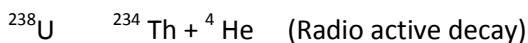
- **Conservation of Charges**
- Like conservation of energy, and Momentum, the electric charges also follow the rules of conservation.
1. Isolated (Individual) Electric charge can neither be created nor destroyed, it can only be transferred.
2. Charges in pair can be created or destroyed.

Example for 1.

At Nuclear level : Decay of U-238

^{238}U ^{234} Th + ^{4} He   (Radio active decay)

Atomic number Z of radioactive material U-238 is 92. Hence it has 92 protons hence charge is 92*e.* Thorium has Z= 90, hence charge is 90e, alpha particles have charge 2e. Therefore charges before decay are 92 and after decay are 90+2=92

Example for 2. (a) Annihilation (destruction in pair)
In a nuclear process an electron -e and its antiparticle positron +e undergo annihilation process in which they transform into two gamma rays (high energy light)
        e$^-$ + e$^+$ *y* + *y*
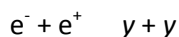
Example for 2 (b):Pair production:
 is converse of annihila tion, charge is also conserved when a gamma ray transforms into an electron and a positron
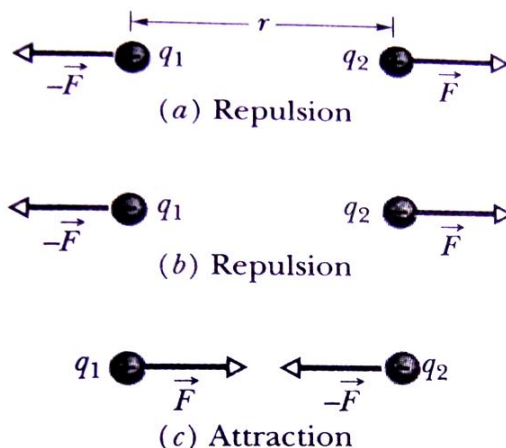   *y*   e$^-$ + e$^+$ (pair production)

Electric Force - Coulumb's Law
- Coulumb's law in Electrostatics :
Force of Interaction between two stationery point charges is

directly proportional to the product of the charges,
inversely proportional to the square of the distance between them and
acts along the straight line joining the two charges.

(a) Repulsion

(b) Repulsion

(c) Attraction

Fig. 22-6 Two charged particles, separated by distance r, repel each other if their charges are (a) both positive and (b) both negative. (c) They attract each other if their charges are of opposite signs. In each of the three situations, the force acting on one particle is equal in magnitude to the force acting on the other particle but has the opposite direction.

If two charges q_1 and q_2 are placed at distance r then,

$$F = c \frac{q_1 q_2}{r^2}$$

where c is a constant .

c is called Coulomb's constant and its value is

$$c = \frac{1}{4\pi\varepsilon_0} \quad \therefore \quad F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

The value of c depends upon system of units and on the medium between two charges
It is seen experimentally that if two charges of 1 Coulomb each are placed at a distance of 1 meter in air or vacuum, then they attract each other with a force (F) of 9 x 10^9 Newton.
Accordingly value of c is 9 x 10^9 Newton x m^2/coul^2

e_0 is permittivity of free space or vacuum and its value is $e_0 = 8.85 \times 10^{-12}$ coul^2 / N x m^2

If point charges are immersed in a dielectric medium, then e_0 is replaced by e a quantity-characteristic of the matter involved In such case. For vacuum $e = e_0$

$$F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}$$

Permittivity, Relative Permittivity and Dielectric Constant

Permittivity is a measure of the property of the medium surrounding electric charge which determine the forces between the charges.

Its value is known as Absolute permittivity of that Medium e

More is Permittivity of medium, Less is coulombs Force.

For water, permittivity is 80 times then that of vacuum, hence force between two charges in water will be 1/80 time force in vacuum (or air.)

Relative Permittivity(e_r) : It is a dimension-less characteristic constant, which express absolute permittivity of a medium w.r.t. permittivity of vacuum or air. It is also called

Dielectric constant (K) $K = e_r = e/e_0$

• This result leads to the calculation that

$$\varepsilon_r = K = \frac{F \text{ (Force in Vacuum)}}{F' \text{ (Force in Medium)}}$$

$$F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi \varepsilon_0 \varepsilon_r} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi \varepsilon_0 K} \frac{q_1 q_2}{r^2}$$

• Unit of charge:- In S.I. System of units, the unit of charge is **Coulomb.**

• **One coulomb** is defined as that charge, which, when placed at a distance of 1 m in air or vacuum from an equal and similar charge, repel it with a force of 9×10^9 Newton

• Charge on one electron is 1.6019×10^{-19} coul. Hence

• One coulomb is equivalent to a charge of 6.243×10^{18} electrons

Is electric charge a fundamental quantity?

• No, In S.I. System, the fundamental quantity is **Electric current** and its unit is Ampere. Therefore coulomb is defined in it's terms as under:

• Coulomb is that quantity of charge which passes across any section of a conductor per second when current of one ampere flows through it, i.e.

• 1 coulomb=1 Ampere x 1 sec

In **cgs electrostatic** system, the unit of charge is called as STATECOULUMB or esu of charge.

• In this system electrostatic constant c=1 for vacuum or air,

$$F = \frac{q_1 q_2}{r^2}$$

One stat coulomb is defined that amount of charge which when placed at a distance of 1 cm in air from an equal and similar charge repel it with a force of one dyne.

In **cgs electromagnetic** system, the unit of charge is called ABCOULOMB or emu of charge

  1 Coulomb = 3 x 10^9 statcoulomb

          = 1/10 abcoulomb

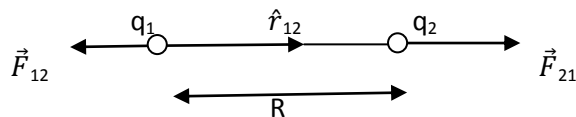\ 1 emu = 3x10^{10} esu of charge

Vector form of Coulumbs' Law

Equation of Coulumbs force showing magnitude as well as direction is called Vector form of coulumbs' law.

If $\hat{r}_{12}$ is unit vector pointing from q_1 to q_2, then as per diagram $\hat{r}_{12}$ and $\vec{F}_{21}$ will be in the same direction, then $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$ (vector equation)........ 1.

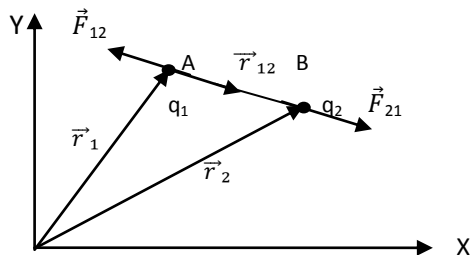Similarily $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$ 2

Since $\hat{r}_{21} = - \hat{r}_{12}$ $\therefore \vec{F}_{21} = - \vec{F}_{12}$

Electrostatic Force between two point charges in terms of their position vectors.

(i).Let there be two point charges q1 and q2 at points A & B in vacume. With reference to an origin O let their

position vectors be $\vec{r}_1$ (OA) and $\vec{r}_2$ (OB). Then AB=$\vec{r}_{12}$. According to triangle law of vectors :

$\vec{r}_1 + \vec{r}_{12} = \vec{r}_2$ $\therefore$ $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ and

$\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$

(ii) According to Coulumb's law, the Force $\vec{F}_{12}$ exerted on q_1 by q_2 is given by : $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_{21}|^2} \hat{r}_{21}$ where $\hat{r}_{21}$ is a unit vector pointing from q_2 to q_1 . We know that $\hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|} = \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$

Hence, general Vector forms of Coulomb's equation is

$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} (\vec{r}_1 - \vec{r}_2)$ and

$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^2} (\vec{r}_2 - \vec{r}_1)$

Comparison of Electrostatic and Gravitational Force

1. <u>Identical Properties</u> :
   - Both the forces are central forces, i.e., they act along the line joining the centers of two charged bodies.
   - Both the forces obey inverse square law, $F \propto \frac{1}{r^2}$
   - Both are conservative forces, i.e. the work done by them is independent of the path followed.
   - Both the forces are effective even in free space.
2. <u>Non identical properties</u> :
   a. Gravitational forces are always attractive in nature while electrostatic forces may be attractive or repulsive.
   b. Gravitational constant of proportionality does not depend upon medium, the electrical constant of proportionality depends upon medium.
   c. Electrostatic forces are extremely large as compared to gravitational forces

Qn. Compare electrostatic and gravitational force between one electron and one proton system.

Ans : $F_e = \frac{1}{4\pi\varepsilon_0} \frac{e.e}{r^2} = 9 \times 10^9 \frac{(1.6 \times 10^{-19})^2}{r^2}$ Newton

$F_g = G \frac{m_e \times m_p}{x r^2} = 6.67 \times 10^{-11} \frac{(9.1 \times 10^{-31}) \times (1.67 \times 10^{-27})}{r^2}$ Newton

$$\underline{F_e / F_g = 2.26 \times 10^{39}}$$

Principle of Superposition of Charges :

If a number of Forces F_{11}, F_{12}, F_{13},......F_{1n} are acting on a single charge q_1 then charge will experience force F_1 equal to vector sum of all these forces .

$F_1 = F_{11} + F_{12} + F_{13} + ... + F_{1n}$

The vector sum is obtained as usual by parallelogram law of vectors.

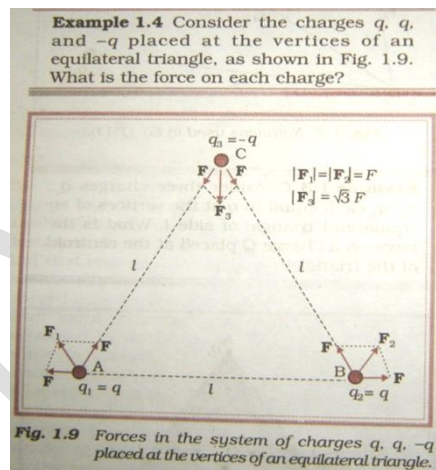All electrostatics is basically about Coulomb's Law and Principle of superposition.

Example 1.4 Consider the charges q, q, and $-q$ placed at the vertices of an equilateral triangle, as shown in Fig. 1.9. What is the force on each charge?

Fig. 1.9 Forces in the system of charges q, q, $-q$ placed at the vertices of an equilateral triangle.

NUMERICALS FOR PRACTICE

1. How many electrons must be removed from the sphere to give it a charge of   +2 μC . Is there any change in the mass when it is given this positive charge. How much is this change?

2. Two identical charged copper spheres A and B have their centers separated by a distance of 50 cm. A third sphere of same size but uncharged is brought in contact with the first, then brought in contact with the second and finally removed from both. What is the new force of repulsion between A and B?

3. A central particle of charge $-q$ is surrounded by two circular rings of charged particles, of radii r and R, such that R > r. What are the magnitude and direction of the net electrostatic force on the central particle due to other particles.
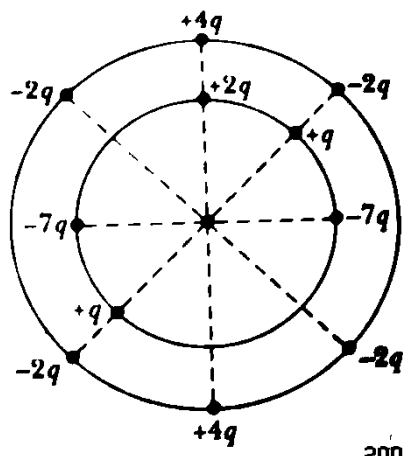
4.-Three equal charges each of 2.0 x 10-6 are fixed at three corners of an equilateral triangle of side 5 cm. Find the coulomb force experienced by one of the charges due to other two.

5.

★ -------o-----------------------o---------
   -3q   -q

Above two charged particles are free to move. At one point, however a third charged particle can be placed such that all three particles are in equilibrium.

(a) is that point to the left of the first two particles, to their right, or between them?

(b) Should the third particle be positively or negatively charged?

© Is the equilibrium stable or unstable?

6. A charge q is placed at the center of the line joining two equal charges Q. Show that the system of three charges will be in equilibrium if q = Q/4.

7. Two particles having charges 8q and –2q are fixed at a distance L. where, in the line joining the two charges, a proton be placed so that it is in equilibrium (the net force is zero). Is that equilibrium stable or unstable?

8. What are the horizontal and vertical components of the net electrostatic force on the charged particle in the lower left corner of the square if q = 1.0 x 10^7C and a = 5.0 cm?

+ q   a   -q

a   a

+2q   a   -2q
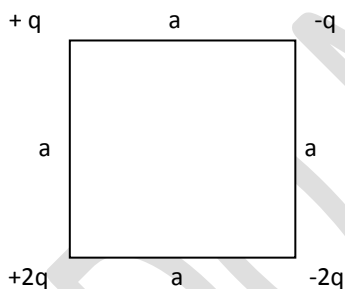
9. Two tiny conducting balls of identical mass m and identical charge q hang from non conducting threads of length L. Assume that θ is so small that tan θ can be replaced by sin θ; show that, for equilibrium,

$$X = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg} \right)^{1/3}$$
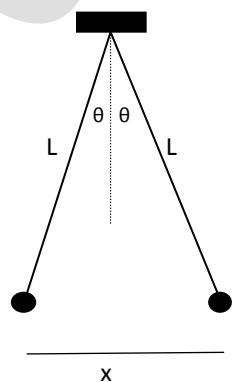
.8.A long non-conducting massless rod of length L, pivoted at its centre and balanced with a block of weight W at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive

charges q and 2q, respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q. a. Find the distance x when the rod is horizontal and balanced. (b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?
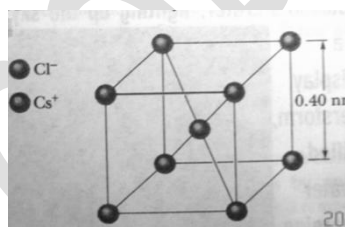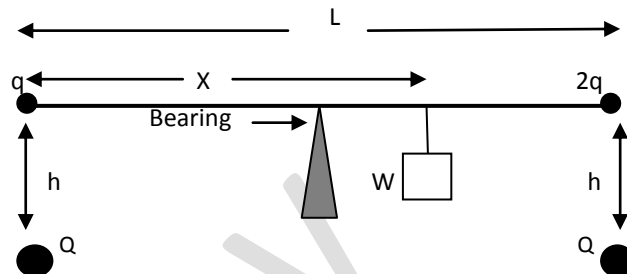
9. In the basic CsCl (Cesium chloride) crystal, Cs+ ions form the corners of a cube and a Cl- ion is at the centre of cube. Edge length is 0.40 nm.

(a) What is the magnitude of the net electrostatic force exerted on Cl- ion by the eight Cs+ ions.?

(b) If one of the Cs+ ion is missing the crystal is said to have defect. How much will be the force on chlorine ion in that case?

10. Two similar helium-filled spherical balloons tied to a 5 g weight with strings and each carrying a charge q float in equilibrium as shown. Find (a) the magnitude of q, assuming that the charge on each balloon is at its centre and (b) the volume of each balloon. Assume that the density of air =1.29 kg m^{-3} and the density of helium in the balloon is= 0.2 kg $^{m-3}$. Neglect the weight of the unfilled balloons. Ans: q = 5.5 x 10^{-7} V = 2.3 x 10^{-3} m^3
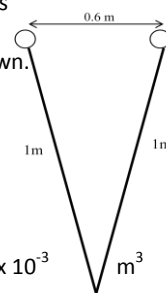
11. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30^0 with each other. When suspended in a liquid of density of 800 kg m^{-3}, the angle remain the same. What is the dielectric constant of the liquid? The density of the material of the sphere is 1600 kg m^{-3} Ans : K = 2

12. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane. Two beads of equal masses m each and carrying charges q_1 q_2 are connected by a cord of length l and can slide without friction on the wires. Considering the case when the beads are stationary, determine (a) angle a (b) the tension in the cord and (c) the normal reaction on the beads. If the cord is now cut what are the value of the charges for which the beads continue to remain stationary
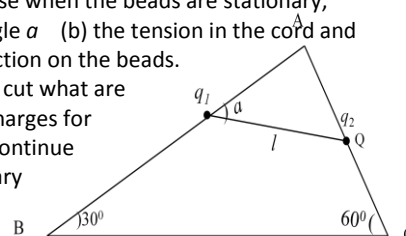
ELECTRIC FIELD

ELECTRIC FIELD-is the environment created by an electric charge (source charge) in the space around it, such that if any other electric charges(test charges)is present in this space, it will come to know of its presence and exert a force on it.

$\vec{E}$

$\vec{F}$ (Force on q)

Q ____●__ q

$\vec{E}$
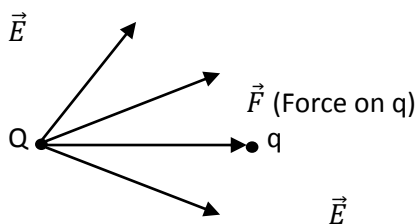
INTENSITY (OR STRENGTH) OF ELECTRIC FIELD AT A LOCATION Is the force exerted on a unit charge placed at that location

: if intensity of electric field at a location is E and a charge 'q' is placed ,then force experienced by this charges F is

$$\vec{F} = q.\vec{E} \quad\text{------- 1}$$

or

$$\boxed{\vec{E} = \frac{\vec{F}}{q}} \quad\text{------- 2}$$

Direction of force F is in direction of electric field E

$$F = \frac{1}{4\pi E} \frac{Q\,q}{r^2} \hat{r} \quad\text{-------}_3$$

By equ.1and 3 : Intensity of electric field due to Source charge Q is

$$\vec{E} = \frac{1}{4\pi E} \frac{Q}{r^2} \hat{r} \quad\text{-------}_4$$

$$Q \xrightarrow{\vec{r}} \overset{\vec{E}}{\underset{q}{}} \vec{F}$$

By coloumb's law we know that in similar situation if q=1 then

$$\vec{E} = \frac{1}{4\pi E} \frac{Q}{r^2}$$

Relation in F, E and Test charge q is

$$E = \frac{F}{q}$$

DISTRIBUTION OF CHARGE

Electric charge on a body may be concentrated at a point, then it is called a 'point charge'. If it is distributed all over, then it is called distribution of

charge. Depending on shape of it is given different names

1.Linear distribution: when charge is evenly distributed over a length. In such case we use a quantity Linear charge density λ. Which has relation

$$\lambda = \frac{Q}{L}$$, Where 'Q' is charge distributed over a long conductor of length 'L'
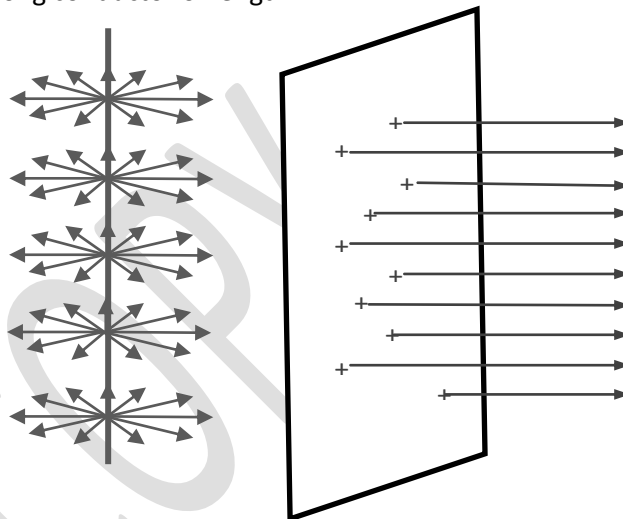
2- Areal distribution: charge is evenly distributed over a surface area,S.

The surface charge density is 'σ' given by $\sigma = \frac{Q}{S}$

Where Q is charge given to a surface of area 'S'.

3-volumetric distribution: charge is evenly distributed throughout the body having volume 'V'Volumetric charge density is 'ρ' $\rho = \frac{Q}{V}$

GENERAL DISTRIBUTION OF ELECTRIC FIELD DUE TO DIFFERENT DISTRIBUTION OF CHARGES

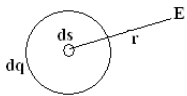1-Due to point change Q $\vec{E} = \frac{1}{4\pi E} \frac{Q}{r^2} \hat{r}$

2-E due to linear distribution of electric charge

$$dq = \lambda.dl$$

$$dE = \frac{1}{4\pi E_0} \frac{\lambda.dl}{r^2}$$

$$\boxed{E = \frac{1}{4\pi E_0} \int_L \frac{\lambda.dl}{r^2}}$$

3 - E due to areal distribution of charge:

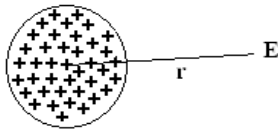$$dq = \sigma . ds$$

$$dE = \frac{1}{4\pi E_0} \frac{\sigma . ds}{r^2}$$

$$\boxed{E = \frac{1}{4\pi E_0} \int_S \frac{\sigma . ds}{r^2}}$$

4- E due to volumetric distribution of charge

SOLID SPHERE VOLUME V

$$\boxed{E = \frac{1}{4\pi E_0} \int_V \frac{\rho . ds}{r^2}}$$

DIPOLE

**1-Dipole** is a system of two equal and opposite charges at finite & fixed distance.

example: molecule of electrolytic compounds.
Example - HCl, H_2O.

2-CO_2 & CH_4 are non-polar because centers of –ve & +ve charges co-incide and there is no distance between them.

3-if non polar atom is placed in an elect.field a distance is created between +ve & -ve charge: it become polar.

Dipole moment:-the effectiveness or strength of a dipole is measured by the physical quantity .Dipole moment $\vec{P}$. it is calculated as $\vec{P}$= q x $2\vec{L}$

P=q x 2L(magnitude) or $\vec{P}$= q x $2\vec{L}$ (vector)
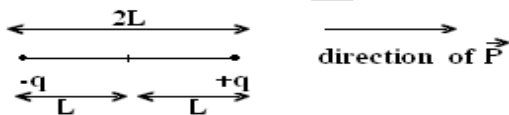
Where 'q' is each charge and '2L' is distance between them.(each charge is at a distance L from 'center' of dipole)

Dipole moment $\vec{P}$= q x $2\vec{L}$ is a vector quantity it has magnitude p=2qL

And its direction is along line from –q to +q.

ELECTRIC FIELD DUE TO DIPOLE

ON THE AXIAL LINE
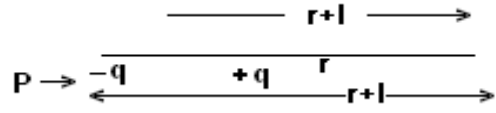
E DUE TO +q
ALONG $\vec{P}$

$$\vec{E_1} = \frac{q}{4\pi \in_0 (r+l)^2} \hat{r}$$

E DUE TO –q

$$\vec{E_1} = \frac{-q}{4\pi \in_0 (r+l)^2} \hat{r}$$

OPPOSITE TO $\vec{P}$

NET ELECTRIC FIELD

$$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{q}{4\pi \in_0} \left(\frac{1}{(r-l)} \frac{1}{(r+l)^2} \right)$$

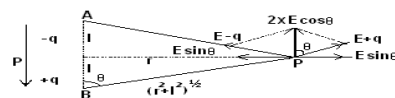$$= \frac{q}{4\pi \in_0} \frac{4rl}{(r^2 l^2)} \qquad \left\{ 2ql = P \right\}$$

$$\boxed{E = \frac{2P.r}{4\pi \in_0 (r^2 l^2)^2}}$$

SINCE $\vec{E_1} > \vec{E_2}$

$\vec{E}$ IS IN THE DIRECTION OF $\vec{P}$

IF R>>L THE, $E = \dfrac{2P}{4\pi \varepsilon_0 r^3}$

2 $\vec{E}$ ON EQUATORIAL LINE (TRANSVERAL LINE)

E due to +q , $E_{+q} = \dfrac{q}{4\pi \in_0 (r^2 l^2)} \hat{BP}$

E due to -q $E_{-q} = \dfrac{q}{4\pi \in_0 (r^2 l^2)} \hat{PA}$

|E+q| = |E-q| = Eq

each Eq is resolved in two direction. One along equatorial line and other in axial directions which are the Esinθ and normal direction E cosθ .

Esinθ in opposite direction cancel each other while E cosθ add up to two.

: net electric field E = 2E cosθ

E(net) = 2Ecosθ

$$= 2 \cdot \frac{q}{4\pi \in_0 (r^2 + l^2)} \cdot \frac{l}{(r^2 + l^2)^{1/2}}$$

$$E = \frac{P}{4\pi\varepsilon_0 \, (r^2 + l^2)^{3/2}}$$

IF R>>L Then, $E = \dfrac{P}{4\pi\varepsilon_0 r^3}$

The direction is opposite to that of P

<u>Electric Field at equatorial line is half of the field on axial line in strength and opposite in direction</u>.

z

Electric Field Intensity due to a Short Electric Dipole at some General Point

(i) Let AB be a short electric dipole of dipole moment $\vec{p}$ (directed from B to A). We are interested to find the electric field at some general point P whose polar coordinates are (r, θ). The distance of observation point P w.r.t. mid point O of the dipole is r and the angle made by the line OP w.r.t. axis of dipole is θ.

(ii) We know that dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components $\vec{p_1}$ and $\vec{p_2}$ as shown in Fig. 27, so that $\vec{p} = \vec{p_1} + \vec{p_2}$.
The magnitudes of $\vec{p_1}$ and $\vec{p_2}$ are $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$.

(iii) It is clear from figure that point P lies on the axial line of dipole with moment $\vec{p_1}$. Hence magnitude of the electric field intensity $\vec{E_1}$ at P due to $\vec{p_1}$ is
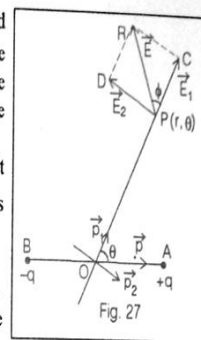
$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p\cos\theta}{r^3} \quad \text{(along } \vec{p_1}). \qquad ....(1)$$

Similarly, P lies on the equatorial line of dipole with moment $\vec{p_2}$. Hence, magnitude of electric field intensity $\vec{E_2}$ at P due to $\vec{p_2}$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\sin\theta}{r^3} \quad \text{(opposite to } \vec{p_2}) \qquad ....(2)$$

Hence resultant intensity at P is : $\vec{E} = \vec{E_1} + \vec{E_2}$

Magnitude of $\vec{E}$ is : $E = \sqrt{(E_1^2 + E_2^2)}$ (as $\vec{E_1}$ and $\vec{E_2}$ are mutually perpendicular).

or $E = \sqrt{\left(\dfrac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2} = \dfrac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$

or $\boxed{E = \dfrac{p}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\cos^2\theta}}$ ....(3)

(iv) If the resultant field intensity vector $\vec{E}$ makes an angle ϕ with the direction of $\vec{E_1}$, then

$$\tan\phi = \frac{E_2}{E_1} = \frac{(p\sin\theta/4\pi\varepsilon_0 r^3)}{(2p\cos\theta/4\pi\varepsilon_0 r^3)} = \frac{1}{2}\tan\theta$$

Electric Line of Force :
The idea of Lines of Force was given by Michel Faraday. These are imaginary lines which give visual idea of Electric field, its magnitude, and direction.

A line of force is continuous curve the tangent to which at a point gives the direction of Electric field, and its concentration gives the strength of Field.
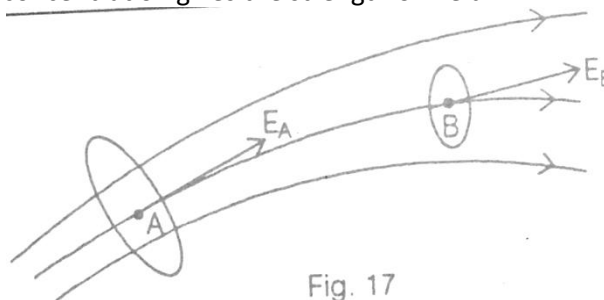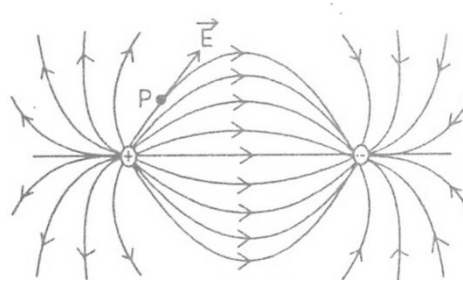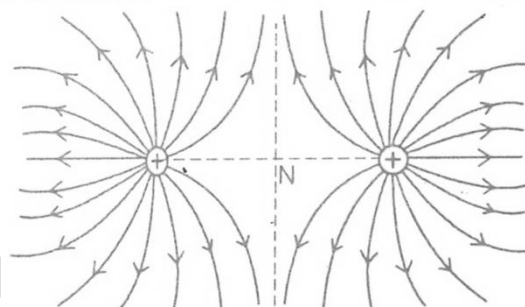
Electric Field at A is stronger than field at B.

Properties of Electric Lines of Force :

Electric Lines of Force :
1.start from positive charge and end at negative.
2.Electric Lines of forces are **imaginary** but Electric field they represent is **real.**
3.The tangent drawn at any point on the line of force gives the direction of force acting on a positive charge at that point.
4.**In SI system, the number of electric lines originating or terminating on charge q is q/ε₀ . That means lines associated with unit charge are 1/ ε₀**
5.Two lines of force never cross each other, because if they do so then at the point of intersection, intensity will have two directions which is absurd.
6. Electric Lines of force can never be a closed loop since they do not start and end at the same point. The lines are discontinuous, start from + and terminate at –
7. The electric line of force do not pass through a conductor as electric field inside a conductor is zero.
8. Lines of force have tendency to contract longitudinally like a stretched string, producing attraction between opposite charges and edge effect.
9.Electric Lines of force start and end **Normal to the surface** of conductor.
10. Crowded lines represent strong field while distant lines represent weak field. Equidistant parallel lines represent uniform field. Non-straight or non- parallel represent non-uniform field. In the diagram a is uniform while b, c, and d are non-uniform fields.

Field Lines due to some charge configurations.
1.Single positive or negative charge

Two equal and opposite charges :

Lines of force due to Two positive charges

Elect field lines due to straight line distribution :
And Electric field lines due to very large sheet of charge are shown in the previous page.

Electric dipole in electric field
When a dipole is placed in an electric field each charge experience a force (F=qe) . Positive, in the direction of field and negative, opposite to direction of field.
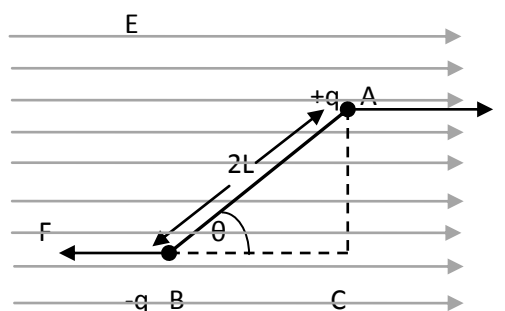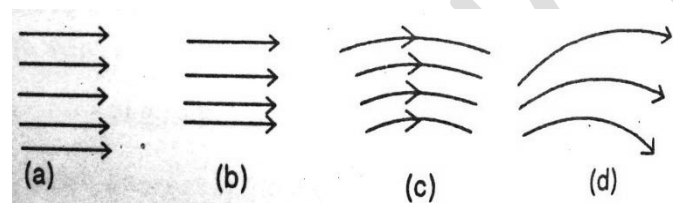

Net Force on dipole : F + (-F) = 0 zero
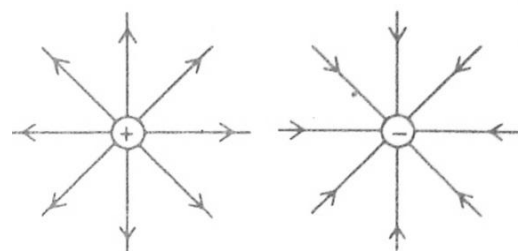
Hence dipole will not make any linear motion.

Torque on dipole: A couple of force is acting on the body of dipole system at different points, the forces are equal and opposite in uniform field. Hence they form a couple of forces which create a **torque.** Therefore dipole is capable of rotation in a uniform electric field. The moment of forces or Torque is

τ = F x AC   = qEx2Lsinθ = 2qL E Sinθ = **PESinθ**

or $\vec{\tau}$ = P x E

NOTE :

1.Direction of torque is normal to the plane containing dipole moment P and electric field E and is governed by right hand screw rule.

2. If Dipole is parallel to E the torque is **Zero**.

3. Torque is **maximum** when Dipole is perpendicular to E and that torque is PE

4. This equation gives the definition of dipole moment. If E is 1 N/C then P=T.

Therefore; **Dipole Moment of a dipole is equal to the Torque experience by that dipole when placed in an electric field of strength 1 N/C at right angle to it.**

5. If a dipole experiencing a torque in electric field is allowed to rotate, then it will rotate to align itself to the Electric field. But when it reach along the direction of E the torque become zero. But due to inertia it overshoots this equilibrium condition and then starts oscillating about this mean position.

6.*Dipole in Non-Uniform Electric field* :

In case Electric field is non-uniform, magnitude of force on +q and –q will be different, hence a net force will be acting on centre of mass of dipole and it will make a linear motion. At the same time due to couple of forces acting, a torque will also be acting on it.

Work done in rotating a dipole in a uniform Electric field:

1.If a dipole is placed in a uniform electric field experience a torque. If it is rotated from its equilibrium position, work has to be done on it. If an Electric dipole with moment P is placed in electric field E making an angle α, then torque acting on it at that instant is

τ = PESinα

2. If it is rotated further by a small angle dα then work done   dw = (PESinα).dα
Then work done for rotating it through an angle θ from equilibrium position of angle 0 is :-

W = $\int_0^\theta$(PEsinα). dα = PE$[-Cos\,\alpha]\,_0^\theta$

Or, **W =** PE $[-cos\theta + cos\,0]$ = **pE** $[\mathbf{1 - Cos\,\theta}]$

3.If a dipole is **rotated through 90⁰** from the direction of the field, then work done will be

W = pE $[1 - Cos\,90]$ = pE

4. If the dipole is **rotated through 180⁰** from the direction of the field, then work done will be :

W = pE $[1 - Cos\,180]$ = 2 pE

Potential Energy of a dipole kept in Electric field :
1. dipole in Equilibrium (P along E):-
A dipole is kept in Electric field in equilibrium condition, dipole moment P is along E
To calculate Potential Energy of dipole we calculate work done in bringing +q from zero potential i.e.∞ to location B, and add to the work done in bringing –q from ∞ to position A.
1.The work done on –q from ∞ up to A
    = -(*Work done up to B* + Work done from B to A)
2.Work done on +q = +(*Work done up to B*)
Adding the two
Total work done = Work done on –q from B to A
            = Force x displacement
            = -qE x 2L = - 2qLE
            =- P.E
This work done convert into Potential Energy of dipole
        U = -$\vec{P}$. $\vec{E}$
If P and E are inclined at angle θ to each other then magnitude of this Potential Energy is
        U = - P E Cos θ

Electric – Potential

(1)   Electric Potential is characteristic of a location in the electric field. If a unit charge is placed at that location it has potential energy (due to work done on its placement at that location). This potential energy or work done on unit charge in bringing it from infinity is called potential at that point.

(2)   Potential – Difference (i) is the work done on unit charge for carrying it from one location to other location A.

A   B

V_A ←——————————— q V∞

Potential at A ------------------------V_A

Energy with q at A   is   q V_A

Energy with Q at   B   is   q V_B

Difference of Energy $U_A - U_B = q (V_A - V_B)$

Using work energy theorem . $W = q((V_A - V_B)$

Or, $V_A - V_B = W / q$ & $U_A - U_B = W.$

If $V_B = 0$ { At ∞ Potential V = 0 , Inside Earth $V_E = 0$}

Then $V_A = W / q$

This equation gives definition of potential V at point A as under :-

"Potential of a point in electric field is the work done in bringing a unit charge from infinity (Zero potential) to that point, without any acceleration."

Expression of potential at a point due to source charge Q :-

Let there be a charge Q which creates electric field around it. Point P is at distance 'r" from it.   Let's calculate potential at this point.

P

Q ←————————— r ————————→

dr
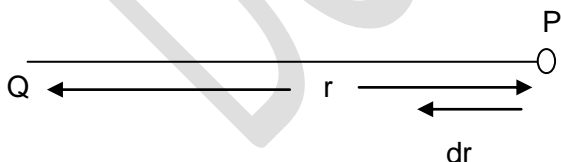
A test charge 'q' is moved for a small displacement dr towards Q.

Electric field due to Q at P, $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$

To move it against this electrical force we have to apply force in opposite direction

Hence applied force $F = -\dfrac{Qq}{4\pi\epsilon_0 r^2}$

Work   done   in   moving   distance   dr   is
$dw = -\dfrac{Qq}{4\pi\epsilon_0 r^2} dr$

Total work done in bringing the charge from distance ∞ to distance r is

$W = -\int_\infty^r \dfrac{Qq}{4\pi\epsilon_0 r^2} dr$

$= -\dfrac{Qq}{4\pi\epsilon_0} \int_\infty^r \dfrac{dr}{r^2}$

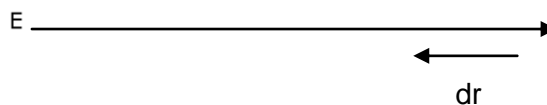$= -\dfrac{Qq}{4\pi\epsilon_0}\left[\dfrac{r^{-1}}{-1}\right]_\infty^r = \dfrac{Qq}{4\pi\epsilon_0 r}$

$W/q = \dfrac{Q}{4\pi\epsilon_0 r}$ OR $\mathbf{V = \dfrac{Q}{4\pi\epsilon_0 r}}$

Where Q is source charge, r is distance & V_r is potential at that point.

Basically V_r is also a "potential difference" between potential of this point P and Potential at ∞ (i.e., 0).

★★★★★★★★★★

Relation between E & V

E ————————————————————→

←———————
dr

A test charge q is moved against E for a small distance dr. then work done dw by applied force -qE is   dw = - qE dr

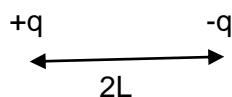Or, dw / q = - E dr
Or,   **dv = - E.dr**
Or,   **E = - dv / dr**

Electric field is derivative of potential difference. – ve sign show that direction of E is opposite to direction of dv.   I.e., dv decrease along the direction of E

E ————————————————————→

V_A V_B

V_A > v_B

This also show that an electric charge experience force from high potential towards low potential if allowed to move, it will do so in this direction only.

If E and are not collinear and make angle θ between them, then according to relation of work & force

dv = - E dr Cos θ

Or, - dv / dr = E Cos θ

Or, dv = - E . dr

$\vec{\rightarrow}$ $\vec{\rightarrow}$

Or V = E . dr

Or   { Potential difference is a scalar quantity (work) given by dot product of two vector

$\vec{E}$ & $\vec{dr}$.

Principle of super position:-

1)   Potential at a point due to different charges is Algebric sum of potentials due to all individual charges.

V = V$_1$ + V$_2$ + V$_3$

2)   Potential due to +ve charge is +ve
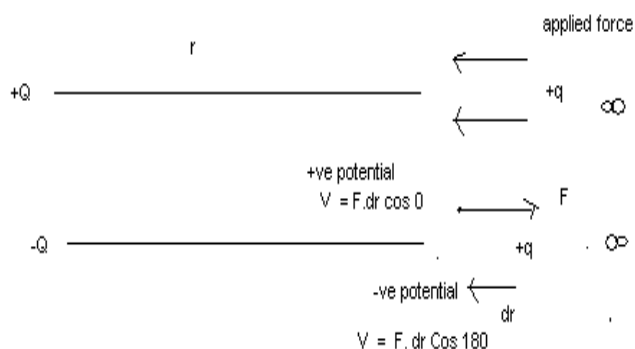
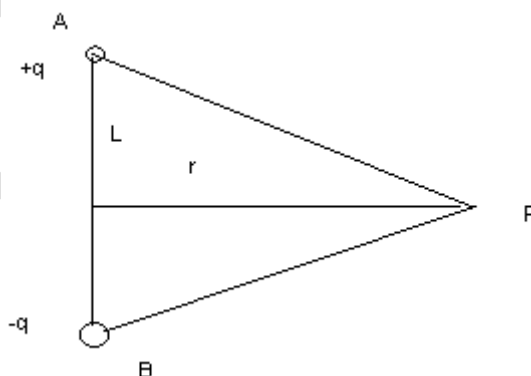Potential due to –ve charge is –ve

Potential due to a dipole

1)   At a point on axial line:-

+q   -q

2L

At P – V_{+q} = $\dfrac{Q}{4\Pi \in_0 (r-l)}$

V_{-q} = $\dfrac{Q}{4\Pi \in_0 (r+l)}$

Total V = – V_{+q} + V_{-q} = $\dfrac{\boldsymbol{Q}}{\boldsymbol{4\pi\epsilon_0}} \left(\dfrac{1}{r-l} - \dfrac{1}{r+l} \right)$

= $\dfrac{2Ql}{4\Pi \in_0 (r^2 - l^2)}$ = $\dfrac{P}{4\Pi \in_0 (r^2 - l^2)}$

If r >> L   Then V = $\dfrac{P}{4\Pi \in_0 r^2}$
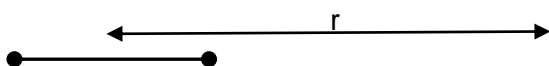
2)   At a point on equatorial line

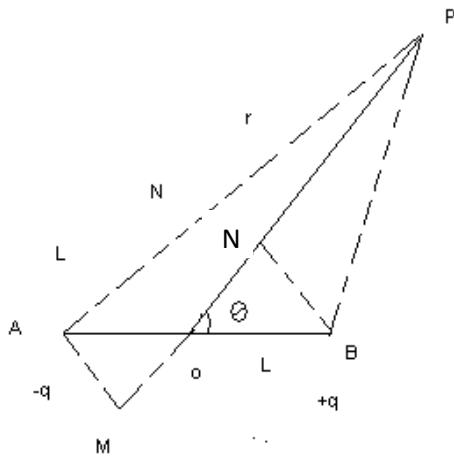- q & + q are placed at A & B. Point P is on equatorial

line

Every point on equatorial line is equidistant from +q & -q. Therefore +ve & -ve potential are equal **Hence net potential is zero.**

"Potential at every point on equatorial line of dipole is zero."

iii) **Potential due dipole at any general point.**

Draw normal from A & B on PO

PB $\approx$ PN = PO – ON = r – L Cos θ ------------ (i)

PA $\approx$ PM = PO + OM = r + L Cos θ ----------- (ii)

$$V_{+q} = \frac{Q}{4\Pi \in_0 PB} = \frac{Q}{4\Pi \in_0 (r - L\cos\theta)}$$

$$V_{-q} = \frac{-Q}{4\Pi \in_0 PA} = \frac{-Q}{4\Pi \in_0 (r + L\cos\theta)}$$
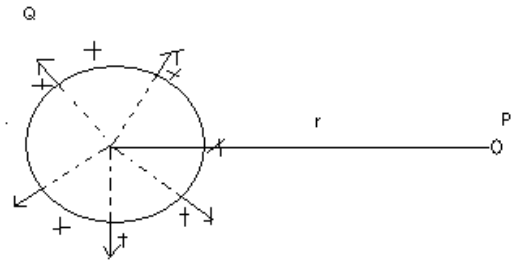
Total   V   =   V$_{+q}$ +   V$_{-q}$ =

$$\frac{Q}{4\Pi \in_0}\left(\frac{1}{r - LCos\theta} - \frac{1}{r - LCos\theta}\right)$$

$$= \frac{Q}{4\Pi \in_0}\left(\frac{r + L\cos\theta - r + LCos\theta}{r^2 - L^2Cos^2\theta}\right)$$

$$= \frac{QX\,2LCos\theta}{4\Pi \in_0 (r^2 - L^2Cos\theta)}$$

Or   V = $\dfrac{PCos\theta}{4\Pi \in_0 (r^2 - L^2Cos\theta)}$

If r > > L

Then,   Or,   V = $\dfrac{PCos\theta}{4\Pi \in_0 r^2}$

Potential due to spherical shell

A spherical shell is given change Q. The electric field is directed normal to surface i.e., Radially outward. "Hence charge on the surface of a shell behaves as if all the charge is concentrated at centre.

Hence potential at distance r is V $= \dfrac{Q}{4\Pi \in_0 r}$

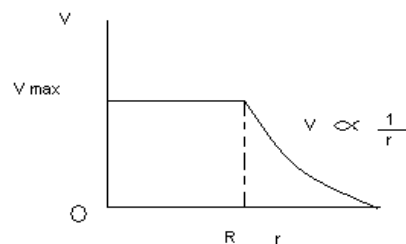**Potential on the surface** of shell V $= \dfrac{Q}{4\Pi \in_0 R}$

Inside shell Electric field is Zero.
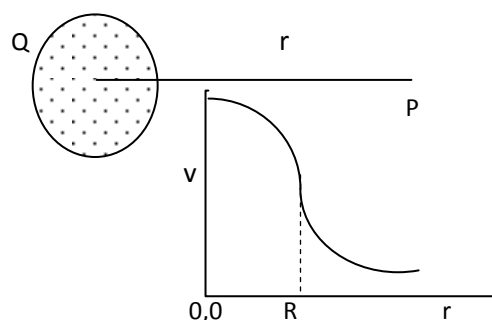Therefore change in potential dv = Zero X dr = 0 i.e., No change in potential.   Hence potential inside a spherical shell is same as on the surface and it is same at every point.

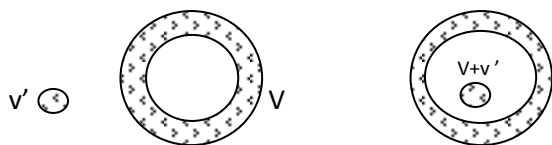It is V $= \dfrac{Q}{4\Pi \in_0 R}$ Where R is radius of shell.

Relation of V & r for spherical shell

In case of non-conducting sphere of charge. potential keeps on increasing up to centre as per diagram.

A body of potential v' is placed inside cavity of shell with potential V then potential of the body become V+v'
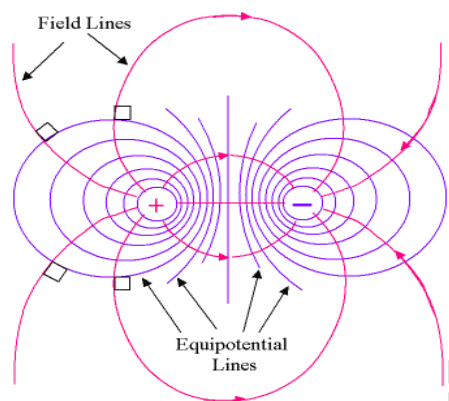

v' ⊙

.Equipotential Surface

A real or imaginary surface in an electric field which has same potential at very point is an equipotential surface or simply, an equipotential.
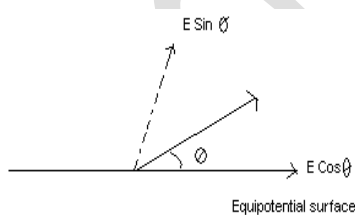
Ex:- A shell having electric charge at its centre, makes an equipotential surface as it has same potential

$$\frac{Q}{4\Pi \in_0 R}$$

at every point of the surface.

Electric lines of force and equipotential surface are at right angle to each other.

Proof:- Suppose E is not at right angle to equipotential surface, and makes angle θ with it. Then it has two components, E Cos θ along surface and E Sin θ normal to surface due to component E Cos θ, force q E Cos θ should be created on surface and it should move the charge. But we find that charges are in equilibrium. i.e.

E Cos θ = 0 ;

since E = 0, therefore Cos θ = 0 or ∠ θ = 90⁰

Hence E is always at right angle to equip. surface.

ii) V₂ _ V₁ = dv = - E Cos θ . dr
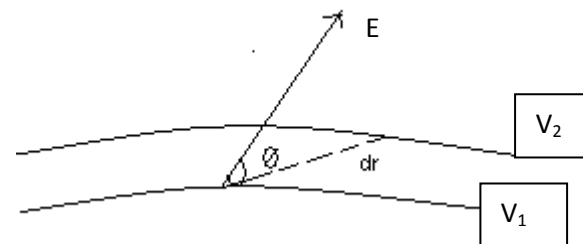
iii) No work is done in carrying an electric charge from one point of E.P. Surface to other point (Whatever is the path)
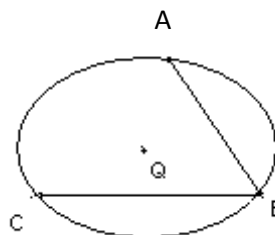
Net work done in carrying change from A to B is Zero, B to C is Zero, because W = qV and V is same on this equipotential Surface

iv) Surface of a conductor in electrostatic field is always an equipotential surface.

Distribution of charge on uneven surface: - charge density is more on the surface which is pointed, or has smaller radius. Therefore if a conductor is brought near pointed charged surface, due to high density of charge induction will be more. Electric field set up will be very strong. This leads to construction of use of lightning arrester used on the buildings.

Gauss's Law
Electric Flux
Think of air blowing in through a window. How much air comes through the window depends upon the **speed** of the air, the **direction** of the air, and the **area** of the window. We might call this air that comes through the window the "**air flux**."

We will define the **electric flux Φ** for an electric field that is perpendicular to an area as

Φ= E A
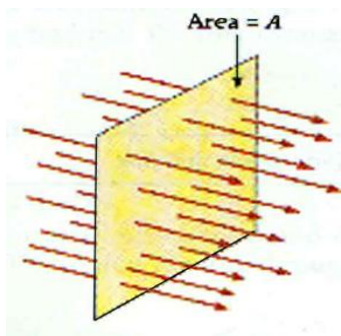
If the electric field **E** is **not** perpendicular to the area, we will have to modify this to account for that.
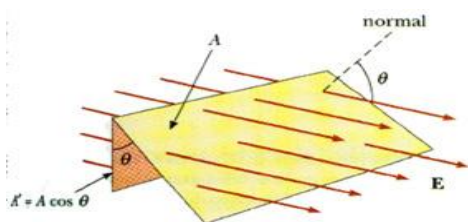
Think about the "air flux" of air passing through a window **at an angle θ**. The "effective area" is A cos θ or the component of the velocity perpendicular to the window is v cos θ. With this in mind, we will make a general definition of the electric flux as

Φ = E A cos θ

Φ You can also think of the electric flux as the **number** of electric field lines that cross the surface.

Remembering the "dot product" or the "scalar product", we can also write this as

$$\Phi = \vec{E} \cdot \vec{A}$$

where **E** is the electric field and **A** is a vector equal to the area A and in a direction **perpendicular** to that area. Sometimes this same information is given as

A = A **n**

where **n** is a **unit vector** pointing **perpendicular** to the area. In that case, we could also write the electric flux across an area as

Φ = **E** •**n** A

Both forms say the same thing. For this to make any sense, we must be talking about an area where the **direction** of **A** or **n** is constant.

For a curved surface, that will not be the case. For that case, we can apply this definition of the electric flux over a small area ΔA or $\vec{\Delta A}$ or $\Delta A \mathbf{n}$.
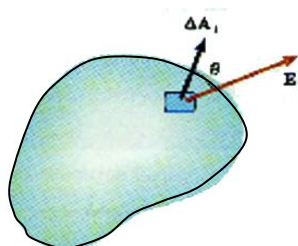

Then the electric flux through that small area is $\Delta\Phi$ and

$\Delta\Phi$ = E ΔA cos θ or

$\Delta\Phi$ = **E** •$\vec{\Delta A}$

To find the flux through all of a closed surface, we need to sum up all these contributions of $\Delta\Phi$ over the entire surface,

$$\Phi_c = \oint \mathbf{E} \cdot \vec{dA} = \oint E_n\, dA \text{ Cos}\theta$$

We will consider flux as **positive** if the electric field **E** goes from the inside to the outside of the surface and we will consider flux as **negative** if the electric field **E** goes from the outside to the inside of the surface. This is important for we will soon be interested in the **net** flux passing through a surface.
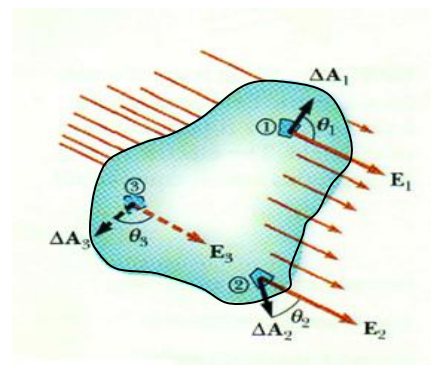
Gauss's Law : Total electric flux though a closed surface is $1/\varepsilon_0$ times the charge enclosed in the surface.
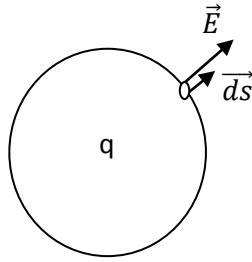
$$\Phi_E = q / \varepsilon_0$$

But we know that Electrical flux through a closed surface is $\oint \vec{E} . \vec{ds}$

$\therefore \oint \vec{E} . \vec{ds}$ = q / ε_0

This is Gauss's theorem.

PROOF : Let's consider an hypothetical spherical surface having charge q placed at its centre. At every point of sphere the electrical field is radial, hence making angle 0 degree with area vector.

At the small area flux $d\phi = \oint \vec{E}.\vec{ds}$

$= \oint E.ds.Cos0°$

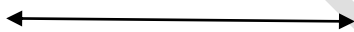$= \oint \dfrac{q}{4\pi\epsilon_0 r^2}$ ds   (E=$\dfrac{q}{4\pi\epsilon_0 r^2}$, Cos0=1)

$= \dfrac{q}{4\pi\epsilon_0 r^2} \oint ds$

For a sphere $\oint ds$ is $4\pi r^2$.

$\therefore \Phi = \dfrac{q}{4\pi\epsilon_0 r^2}$ X $4\pi r^2$.

Or, $\Phi = q / \epsilon_0$

This is Guass Theorem. (Hence proved)

<----------------->

Application of Gauss's Law

To calculate Electric Field due to different charge distributions.

For this purpose we consider construction of a **Guassian surface**.

Guassian Surface : It is an imaginary surface in the electric field which is
1.closed from all sides
2. Surface is Symmetrical about the charges in it
3. Electric field $\vec{E}$ on the surface is symmetrical

Electric field due to line charge :
Electric charge is distributed on an infinite long straight conductor with linear charge density λ. We have to find Electric field on a point *P* at normal distance r.

Consider a Gaussian Surface in the shape of a cylinder having axis along conductor. It has radius r so that point P lies on the surface. Let its length be *l.*
The electric field is normal to conductor, hence it is symmetrical to the surfaces of these cylinder.
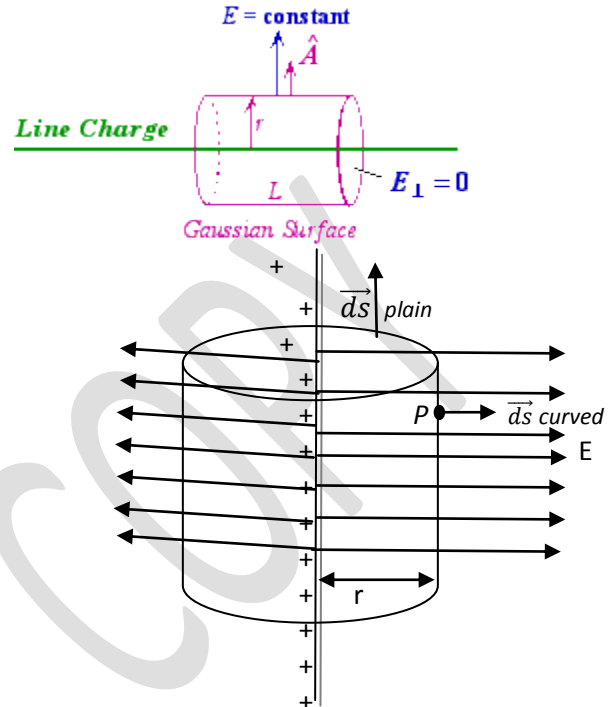
Now $\oint \vec{E}.\vec{ds} = \int \vec{E}.\vec{ds}$ for curved surface + $\int \vec{E}.\vec{ds}$ for 2 plane surfaces.
$= \int E.ds\, cos0 + \int E.ds\, cos90$
$= E \int ds$ for curved surface (E is uniform)
$= E2\pi rl$ ($\int ds = 2\pi rl$, for cylindrical curved surface)

The charge enclosed within Guassian surface $= \lambda l$

According to Gauss theorem : $\oint \vec{E}.\vec{ds} = q / \epsilon_0$
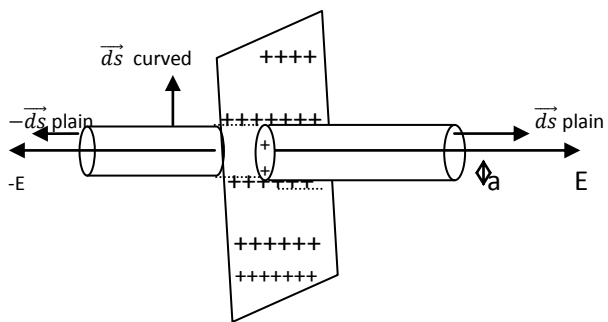
Putting values : $E2\pi rl = \lambda l / \epsilon_0$

Or, $\boxed{E = \dfrac{\lambda}{2\pi\varepsilon_0 r}}$

Electric field due to a plain surface : -

There is a very large plain surface having sueface density σ . There is a point P at normal distance r .

Let's consider a Gaussian surface, in shape of a cylinder which has axis normal to the sheet of charge and containing point P at its plain surface (radius a).

Electric field E is normal to the surface containing charge hence it is normal to the plain surface of cylinder and parallel to curved surface.

Now $\oint \vec{E}.\vec{ds}$ = $\int \vec{E}.\vec{ds}$ for curved surface + $\int \vec{E}.\vec{ds}$ for 2 plane surfaces.

= $\int E.ds\, cos90 + \int E.ds\, cos0 + \int -E.(-ds\, cos0)$

= *for plain surfaces* $2E \int ds$ (E is uniform)

= $2E\pi a^2$

The charge enclosed inside Gaussian surface q = σ.A

Or, q = $\sigma \pi a^2$

Applying Gauss's Law : $\oint \vec{E}.\vec{ds}$ = q / ε₀

Putting values $2E\pi a^2 = \dfrac{\sigma \pi a^2}{\varepsilon_0}$

Or $E = \dfrac{\sigma}{2\varepsilon_0}$

Electric Field due to charge distributed over a spherical shell :-
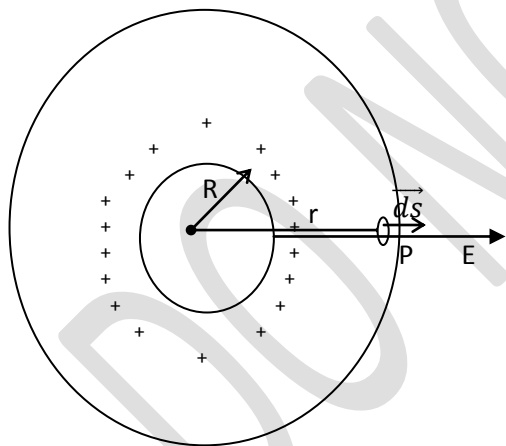
The spherical shell or spherical conductor has total charge q, surface charge density σ , radius R. We have to find Electric Field E at a point P at distance 'r '.

Case 1. If P is outside shell.

Let's assume a Gaussian surface, which is a concentric sphere of radius r and P lies on its surface.

Electric field is normal to surface carrying charge. Hence it is radially outward. Therefore for a small area on the Gaussian surface ds E is normal to surface i.e. angle between $\vec{ds}$ and $\vec{E}$ is 0.

Now $\oint \vec{E}.\vec{ds}$ = $\int \vec{E}.\vec{ds}$ for complete area of Gaussian surface

= $\int E.ds.Cos0$ = E$\int ds$ (E is uniform)

= E X $4\pi r^2$.   (for spherical shell $\int ds = 4\pi r^2$)

Charge within Gaussian surface = q

Applying Gauss's Law : $\oint \vec{E}.\vec{ds}$ = q / ε₀

Putting values   E X $4\pi r^2$ = q / ε₀

Or $E = \dfrac{q}{4\pi \epsilon_0 r^2}$

This expression is same as electric field due to a point charge q placed at distance r from P. i.e. In this case if complete charge q is placed at the centre of shell the electric field is same.
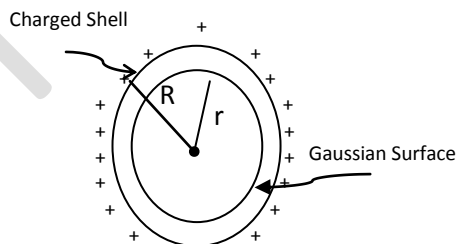
Case 2. If P is on the surface.

In above formula when r decrease to R the electric field increase.

On the surface *(replace r with R)* $E = \dfrac{q}{4\pi \epsilon_0 R^2}$

Hence this is electric field on the surface of a shell and its value is maximum compared to any other point.

Case 3. If P is within the surface. Or 'r'< R

Charged Shell

Gaussian Surface

Let's consider a Gaussian surface, a concentric spherical shell of radius r passing through P.

Then charge contained inside Gaussian surface is Zero.

According to Gauss's Theorem $\oint \vec{E}.\vec{ds}$ = q / ε₀
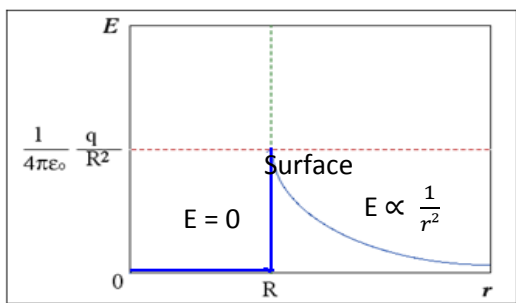
If q is zero then $\oint \vec{E}.\vec{ds}$ = 0.

As *ds* is not zero then   E = 0

It is very important conclusion reached by Gauss's Law that Electric field inside a charged shell is zero.

The electric field inside conductor is Zero. This phenomenon is called ***electrostatic shielding***.

Variation of E with r (distance from centre)

Electric Field due to (filled-up) sphere of charge

(Volumetric distribution of charge) :

$$E \propto \frac{1}{r^2}$$

Gaussian Surface

Charged Sphere

Case I. When P is Out side sphere. Same as in the case of charged shell $E = \dfrac{q}{4\pi\epsilon_0 r^2}$

Case 2. When point P is on the surface of shell: Same as in case of shell . $E = \dfrac{q}{4\pi\epsilon_0 R^2}$

Case 3 If point P is inside the charged sphere.

Consider Gaussian surface, a concentric spherical shell of radius r, such that point P lies on the surface.
Electric field is normal to the surface.

Now $\oint \vec{E}.\vec{ds} = \int \vec{E}.\vec{ds}$ for complete area of Gaussian surface

$= \int E.ds.Cos0 = E\int ds$ (E is uniform)

$= E \times 4\pi r^2$. (for spherical shell $\int ds = 4\pi r^2$)

Charge within Gaussian surface = charge density x volume.

$= \rho\frac{4}{3}\pi r^3$ (where ρ is the charge per unit volume.)

Applying Gauss's Law $\oint \vec{E}.\vec{ds} = q / \varepsilon_0$

Putting values $E \times 4\pi r^2 = \rho\frac{4}{3}\pi r / \varepsilon_0$

$\therefore \quad \boxed{E = \dfrac{\rho r}{3\varepsilon_0}}$

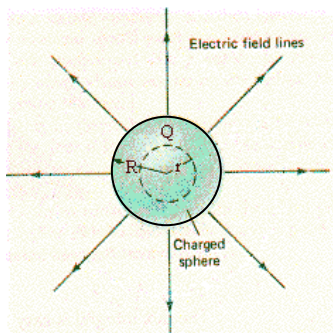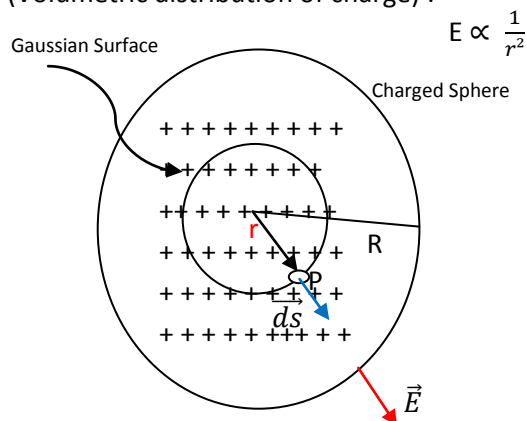It shows that inside a sphere of charge, the electric field is directly proportional to distance from centre.
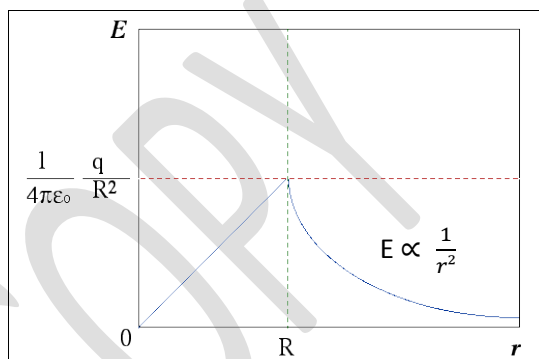
At centre r=0 $\quad \therefore \ E = 0$

On the surface $\ E = \dfrac{\rho R}{3\varepsilon_0} = \dfrac{q}{4\pi\epsilon_0 R^2} \qquad (\rho = q / \frac{4}{3}\pi r^3)$

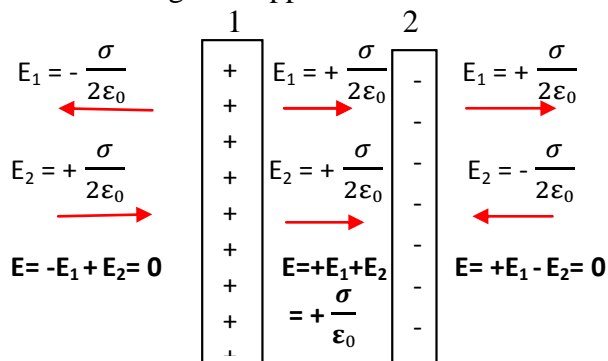Variation of E with r (distance from centre)

Electric field due to two charged parallel surface

Charges of similar nature
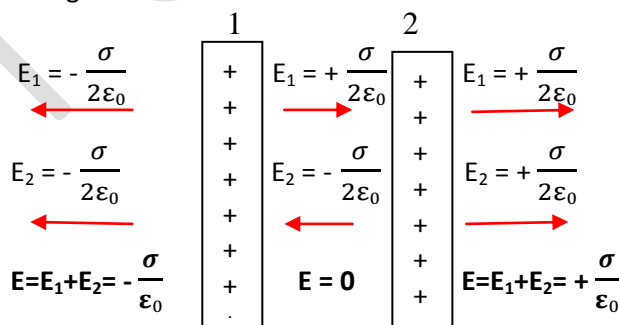
a. Charges of opposite nature :-

Equipotential Surface :

Energy of a charged particle in terms of potential:-

Work required to bring a charge q at a point of potential V is   W = qV. This work done on the charged particle converts to its potential energy.

Potential energy of charge q at potential V is **U = qV**

Electron-Volt : By relation Work/energy = qV,   *the smallest unit of work/energy is Electron Volt.*

One electron volt is the work done by/on one electron for moving between two points having potential difference of one Volt.

1 eV = 1.6 x 10^{-19} Joules

Potential Energy of system of charges

   (i)   System of Two charges :
A   B
q$_1$ -------- r ----------------q$_2$
Potential due to q1 at B is potential at distance r :
V = $\frac{q_1}{4\pi\varepsilon_0 r}$ ∴ Potential Energy of system U = $\frac{q_1 q_2}{4\pi\varepsilon_0 r}$

   (ii)   System of three charges
We make different pairs and calculate energy as under
U = $\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$ + $\frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$ + $\frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}}$

   (iii)   System of Four charges
Four charges make six pairs : Potential Energy U=
$\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$ + $\frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$ + $\frac{q_1 q_4}{4\pi\varepsilon_0 r_{14}}$ + $\frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}}$ + $\frac{q_2 q_4}{4\pi\varepsilon^0 r_{24}}$ + $\frac{q_3 q_4}{4\pi\varepsilon_0 r_{34}}$

The energy is contained in the system and not by any one member. But it can be used by one or more members.

Distribution of charge on irregular shaped conductors :

Potential at each point is equal.

*Electric field* is always normal to surface.

Charge is distributed **unevenly**. Charge per unit area is more at the surface which has smaller radius. Therefore charge density is always more on the corners.
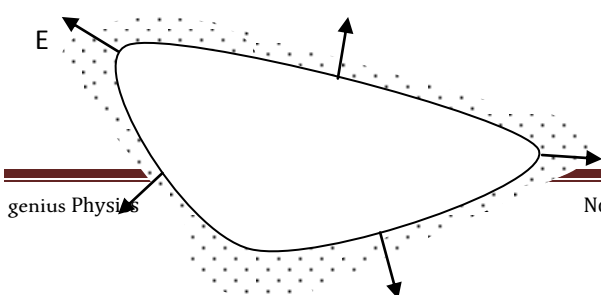
Corona discharge : when an uncharged body is brought near a charged body having sharp corners there is large number of charges at the corners. Due to induction, they induce large number of opposite charges. This creates a very strong Electric field between them. Finally the dielectric strength breaks-down and there is fast flow of charges. This Spray of charges by spiked object is called Corona discharge.

The lightening arrester work on the principles of Corona discharge where the charge pass through conductor of arrester, and the buildings are saved
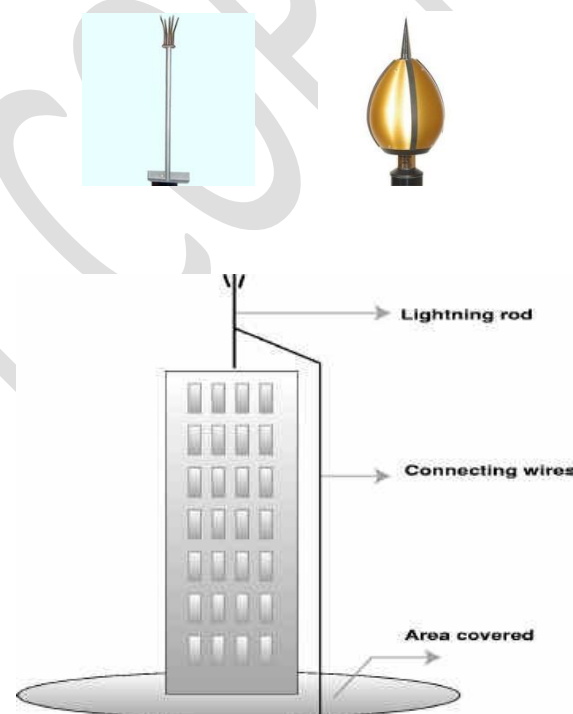
Van-de-Graff generator

Introduction : It's a device used to create very high potential which is used for experiments of nuclear physics in which a charged particle with very high energy is required to hit the nucleus as target.

Principles : The following principles are involved in the device.
1.Charge on a conductor always move to and stay on the outer surface.
2.Pointed Corners conduct charges very effectively. (corona discharge)

3.If charge q is given to a body, its potential increases by relation V=$\frac{q}{4\pi\varepsilon_0 r}$

4. If a body of small potential v' is placed inside a shell having potential V, then the body acquires potential V+v'

Description : There is a large spherical conducting shell of diameter of few meters placed on a non-conducting concrete structure few meters above the ground.

A long belt of insulating material like silk rubber or rayon moves around two pulleys, driven by a motor.

Two combs with pointed heads near belt are fitted. Lower one is spray comb and the upper Collecting Comb. The spray comb is connected with a high tension source.

There is a discharge tube. One end having source of ion to be accelerated is inside the shell. Target is placed at the other end connected to earth.

The whole system is enclosed in a steel chamber filled with nitrogen or methane at high pressure.

Working : The spray comb is given a positive potential ($\approx 10^4$Volt) w.r.t. earth by the source of high Tension. Due to sharp points there is spray of charge on belt. The belt moves up with power of motor. When the charges reach near upper comb, due to induction and corona discharge the charge on belt is transferred to comb. From comb it moves to inner layer of shell. Since charge always stay at the outer surface, it moves to outer surface and the inner surface again become without any charge, ready to receive fresh charge again. As shell receive charge it Potential increase according to relation V=$\frac{q}{4\pi\varepsilon_0 r}$. This potential is distributed all over and inside the shell.

The new charged particles which are coming having small potential v' from lower comb, acquire potential V+v' due to their position inside the shell. There new potential is slightly higher than shell, therefore charges move from belt to comb to shell. This increases V further. This process keeps on repeating and V increase to a very high value, that is break-down voltage of compressed nitrogen $\approx 10^7$volt.
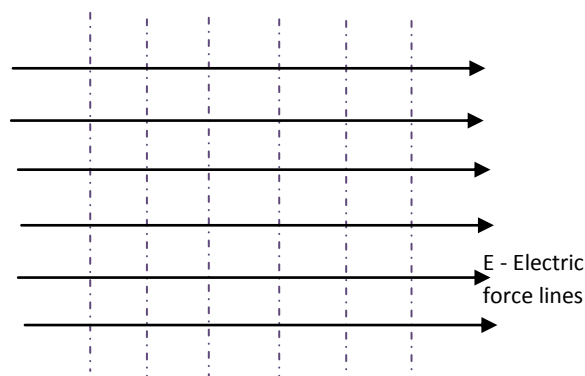
The ion inside discharged plate also acquires this potential due to its location inside the shell. Its energy increases by relation U = qV. The target is connected to earth at zero potential. Hence this ion gets accelerated and hits the target with very high energy.
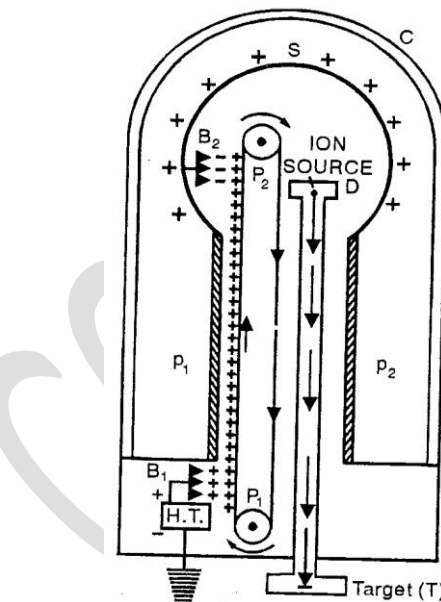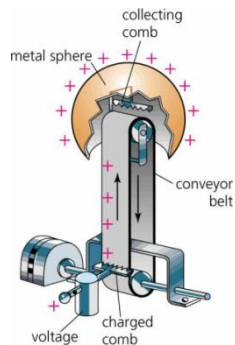
Relation between Equipotential surfaces and E-Lines
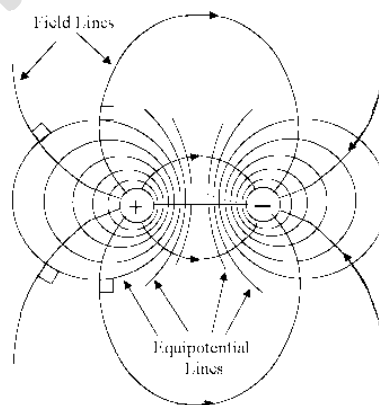
Equi potential lines

E - Electric force lines

CAPACITOR

It is a device to store charge and in turn store the electrical energy.

Any conductor can store charge to some extent. But we cannot give infinite charge to a conductor. When charge is given to a conductor its potential increases. But charge cannot escape the conductor because air, or medium around conductor is di-electric.

When due to increasing charge the potential increase to such extent that air touching the conductor starts getting ionized and hence charge gets leaked. No more charge can be stored and no more potential increase. This is limit of charging a conductor.

The electric field which can ionize air is 3×10^9 vm^{-1}.

CAPACITANCE OF A CONDUCTOR

Term capacitance of a conductor is the ratio of charge to it by rise in its Potential

$$C = \frac{q}{V}$$

In this relation if V=1 then C= q. Therefore ,

Capacitance of a conductor is equal to the charge which can change its potential by one volt.

Unit of capacitance : Unit of capacitance is farad, (symbol F).

One farad is capacitance of such a conductor whose potential increase by one volt when charge of one coulomb is given to it.

One coulomb is a very large unit. The practical smaller units are
i. Micro farad (μF) = 10^{-6}F.(used in electrical circuits)
Ii Pieco farad (pF) = 10^{-12} used in electronics circuits

Expression for capacitance of a spherical conductor :
If charge q is given to a spherical conductor of radius r, its potential rise by $V = \frac{q}{4\pi\epsilon_0 r}$
Therefore capacitance $C = \frac{q}{V}$ = q/$\frac{q}{4\pi\epsilon_0 r}$ = $4\pi\epsilon_0 r$

Or for a sphere $\boxed{C = 4\pi\epsilon_0 r}$

The capacitor depends only on the radius or size of the conductor.

The capacitance of earth (radius 6400 km) is calculated to be 711 x 10^{-6} coulomb.

PARALLEL PLATE CAPACITOR : -

Since single conductor capacitor do not have large capacitance , parallel plate capacitors are constructed.

Principle : Principle of a parallel plate capacitor is that an uncharged plate brought bear a charged plate decrease the potential of charged plate and hence its capacitance (C = $\frac{q}{V}$) increase. Now it can take more charge.   Now if uncharged conductor is earthed, the potential of charged plate further decreases and capacitance further increases. This arrangement of two parallel plates is called parallel plate capacitor.

Expression for capacitance :

Charge q is given to a plate

Of area 'A'. Another plate

is kept at a distance 'd'.

After induction an

Electric field E is set-up

Between the plates. Here
q = σA   and E= $\frac{\sigma}{\varepsilon_0}$
The Potential difference between plates is given by
V = Ed = $\frac{\sigma}{\varepsilon_0}$ d
Now C = $\frac{q}{V}$ = $\frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d}$ = $\frac{\varepsilon_0 A}{d}$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

If a dielectric of dielectric constant K is inserted between the plates, then capacitance increase by factor K and become
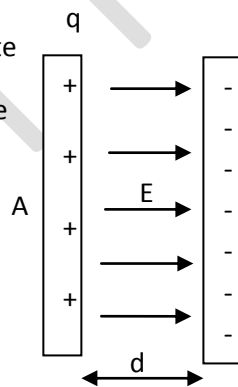
$$\boxed{C = \frac{\varepsilon_0 K A}{d}}$$

Note : The capacitance depends only on its configuration i.e. plate area and distance, and on the medium between them.
The other examples of parallel plate capacitors is

Cylindrical capacitor $C = \frac{4\pi\varepsilon_0 K L}{\log r^2/r_1}$

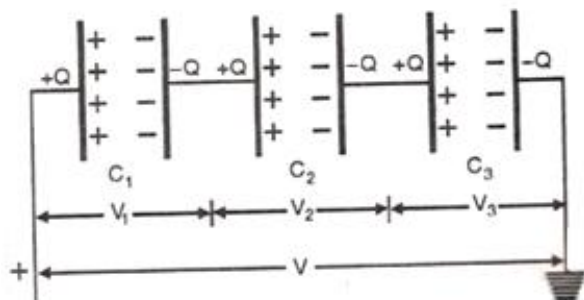and Spherical capacitor. $C = \frac{4\pi\varepsilon_0 K r_2 r_1}{\log r_2 - r_1}$

Combination of capacitors

Capacitors can be combined in two ways. 1. Series and 2. Parallel.

Series Combination :

If capacitors are connected in such a way that we can proceed from one point to other by only one path passing through all capacitors then all these capacitors are said to be in series.

Here three capacitors are connected in series and are connected across a battery of P.D. 'V'.

The charge q given by battery deposits at first plate of first capacitor. Due to induction it attract –q on the opposite plate. The pairing +ve q charges are repelled to first plate of Second capacitor which in turn induce -q on the opposite plate. Same action is repeated to all the capacitors and in this way all capacitors get q charge. As a result ; the charge given by battery q, every capacitor gets charge q.

The Potential Difference V of battery is sum of potentials across all capacitors. Therefore

 V = v_1 + v_2 + v_3

$$v_1 = \frac{q_1}{c_1} \ , \ v_2 = \frac{q_2}{c_2} \ , \ v_3 = \frac{q_3}{c_3}$$

Equivalent Capacitance : The equivalent capacitance across the combination can be calculated as C_e = q/V

Or   1/ C_e = V/q

   = (v_1 + v_2 + v_3) / q

   = v_1/q + v_2 /q + v_3/q

Or   **1/C_e = 1/C_1 + 1/C_2 + 1/C_3**

The equivalent capacitance in series decrease and become smaller then smallest member.

In series q is same. Therefore by q=cv, we have

$c_1 v_1 = c_2 v_2 = c_3 v_3$

or $v \propto \frac{1}{c}$ i.e. larger c has smaller v, and smaller c has larger v across it.

For 2 capacitor system C = $\frac{c_1 c_2}{c_1 + c_2}$, and $v_1 = \frac{c_2}{c_1 + c_2}$ .v

If n capacitor of capacitance c are joint in series then equivalent capacitance $C_e = \frac{c}{n}$

Parallel combination :

If capacitors are connected in such a way that there are many paths to go from one point to other. All these paths are parallel and capacitance of each path is said to be connected in parallel.
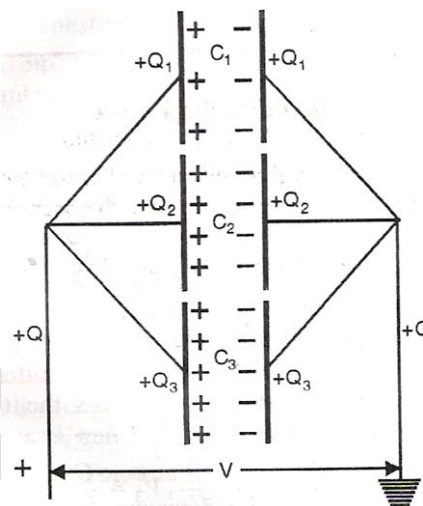
Here three capacitors are connected in parallel and are connected across a battery of P.D. 'V'.

 The potential difference across each capacitor is equal and it is same as P.D. across Battery.

The charge given by source is divided and each capacitor gets some charge. The total charge

   q = q_1 + q_2 + q_3

Each capacitor has charge

 $q_1=c_1 v_1$, $q_2=c_2 v_2$, $q_3=c_3 v_3$

Equivalent Capacitance : We know that

q = q_1 + q_2 + q_3

divide by v $\frac{q}{v} = \frac{q_1}{v} + \frac{q_2}{v} + \frac{q_3}{v}$

   or, **C = c_1+c_2+c_3**

The equivalent capacitance in parallel increases, and it is more than largest in parallel.

In parallel combination V is same therefore

   (v =) $\frac{q_1}{c_1} = \frac{q_2}{c_2} = \frac{q_3}{c_3}$

In parallel combination q $\propto$ c . Larger capacitance larger is charge.

Charge distribution : $q_1=c_1 v$, $q_2=c_2 v$, $q_3=c_3 v$.

In 2 capacitor system charge on one capacitor

   $q_1 = \frac{c_1}{c_1+c_2+..}$.q

   n capacitors in parallel give   C = nc

Energy stored in a capacitor: When charge is added to a capacitor then charge already present on the plate repel any new incoming charge. Hence a new charge has to be sent by applying force and doing work on it. All this work done on charges become energy stored in the capacitor.

At any instant work done $dw = V.dq$, or $dw = \frac{q}{c}.dq$

Therefore work done in charging

the capacitor from charge 0 to q $W = \int_0^q \frac{q}{c}.dq$

$$= \frac{1}{c}\int_0^q q.dq = \frac{1}{c}\frac{q^2}{2} = \frac{q^2}{2c}$$

This work done convert into electrical Potential

Energy stored in the capacitor $\boxed{U = \frac{1}{2}\frac{q^2}{c} = \frac{1}{2}qv = \frac{1}{2}cv^2}$

This energy is stored in the form of Electric field between the plates.

Energy per unit volume $u = \frac{1}{2}cv^2/V = \frac{1}{2}\frac{\varepsilon_0 A E^2 d^2}{dAd}$

Or, energy density $\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$

Connecting two charged capacitors :- When two conductors are connected the charges flow from higher potential plate to lower potential plate till they reach a common potential.

Common Potential : A capacitor of capacitance c_1 and potential v_1 is connected to another capacitor of capacitance c_2 and potential v_2. The charge flow from higher potential to lower potential and it reach an in between value V such that

$V= \frac{total\ charge}{Total\ capacitance}$ or $V = \frac{c_1v_1+c_2v_2}{c_1+c_2}$

Loss of Energy on connecting two conductors :
A capacitor of capacitance c_1 and potential v_1 is connected to another capacitor of capacitance c_2 and potential v_2. The charge flow from higher potential to lower potential and in this process it looses some energy as charge has to do some work while passing through connecting wire. The energy is lost in form of heat of connecting wire.

Expression for energy lost : In the above two capacitors the energy contained in the two before connection, $E_1 = \frac{1}{2}c_1v_1^2 + \frac{1}{2}c_2v_2^2$ .... (i)

Common Potential after connection, $V = \frac{c_1v_1+c_2v_2}{c_1+c_2}$

Combined capacitance $c_1 + c_2$

Energy in combination : $\frac{1}{2}(c_1+c_2)(\frac{c_1v_1+c_2v_2}{c_1+c_2})^2$

Hence Loss in energy : $E_1 - E_2$

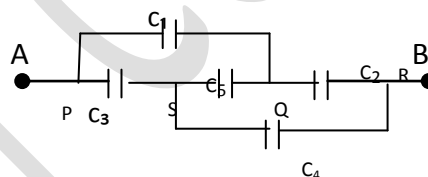$= \{\frac{1}{2}c_1v_1^2 + \frac{1}{2}c_2v_2^2\} - \{\frac{1}{2}(c_1+c_2)(\frac{c_1v_1+c_2v_2}{c_1+c_2})^2\}$

$= \frac{1}{2}(\frac{c_1c_2}{c_1+c_2})(v_1-v_2)^2$

It is a positive number which confirm that there is loss of energy in transfer of charges. Hence

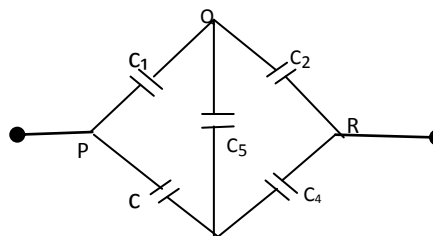$\boxed{\text{loss of energy} = \frac{1}{2}(\frac{c_1c_2}{c_1+c_2})(v_1-v_2)^2}$

Wheatstone bridge in combination of capacitors :

Five capacitors joined in following manner is called wheatstone bridge connection.

Or, it is redrawn as under :

In the above arrangement, if ratio $c_1/c_2 = c_3/c_4$ then the bridge is said to be *balanced*. In such case the potential at point Q and S are equal.
The potential across c_5 is zero hence it does not carry any charge. In this way it is not participating in storage of charges. Then it can be omitted for further calculations. Calculations are done for c_1 c_2, c_3 and c_4 only.

Dielectrics: are non conducting materials. They do not have free charged particles like conductors have. They are two types.
i.   Polar : The centre of +ve and –ve charges do not coincide. Example HCl, H_2O, They have their own dipole moment.

ii.   Non-Polar : The centers of +ve and –ve charges coincide. Example CO_2 , C_6H_6 . They do not have their own dipole moment.

In both cases, when a dielectric slab is exposed to an electric field, the two charges experience force in opposite directions. The molecules get elongated and develops i. surface charge density σ_p and not the volumetric charge density. This leads to development of an induced electric field E_p , which is in opposition direction of external electric field E_o. Then net electric field E is given by E = E_o - E_p . This indicates that net electric field is decreased when dielectric is introduced.

The ratio $\frac{Eo}{E}$ = K is called **dielectric constant** of the dielectric.

Clearly electric field inside a dielectric is   E= $\frac{Eo}{K}$.

Dielectric polarization : when external electric field E_0 is applied , molecules get polarized and this induced dipole moment of an atom or molecule is proportionate to applied electric field. i.e.   p $\propto$ E_o

or p = $\alpha\varepsilon_o E_o$

here α is a constant called atomic / molecular polarizability. It has dimensions of volume (L^3) it has the order of 10^{-29} to 10^{-30} m^3.

This **polarization** is a vector quantity and is related to resultant electric field E as under :

$$\vec{p}= \chi_e\varepsilon E$$

Where χ_e is a constant called electric susceptibility of the dielectric.

The induced charge σ_p is due to this polarization, hence
$$\sigma_p = \vec{p}.\hat{n}$$

When this dielectric is introduced between the two plates having charge density σ then resultant electric field can be related as

$E.\hat{n}$ = E - E_p = $\frac{\sigma-\sigma p}{\varepsilon_0}$ = $\frac{\sigma-\vec{p}.\hat{n}}{\varepsilon_0}$

or $(\varepsilon^0 E + p).\hat{n}$ = σ

or $\vec{D}.\hat{n}$ = σ

The quantity $\vec{D}$ is called **electric displacement** in dielectric.

We can prove that K = 1+ χ_e