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TEST

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1.) $f(x,y) = 2x - y + 2x^2 + 2xy + y^2$

Starting point $= (4, 1)$

formula:- $x_1 - [J]^{-1} \Delta f(x_1)$

$$J = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\Delta f(x) = \begin{bmatrix} 2 + 4x + 2y \\ -1 + 2y + 2x \end{bmatrix}_{x_1}$$

$$\Delta f(x) = \begin{bmatrix} 20 \\ 9 \end{bmatrix}$$

$$= (4,1) - \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 9 \end{bmatrix}$$

$$= (4,1) - (5.5, -1)$$

$$= (-1.5, 2)$$

2) $f(x,y) = 4x^2 - 4xy + 2y^2$

Sol: Given

$$X_1 = (2,3)$$

We Compute steepest decent direction from;

$$\nabla f(x,y) = (8x-4y,\ 4y-4x)$$

$$\nabla f(x_0) = \nabla f(2,3)$$

$$= (4,4)$$

minimize the function;

$$\phi(t) = f((2,3) - t(4,4))$$

$$= f(2-4t,\ 3-4t)$$

Compute;

$$\phi'(t) = -\nabla f(2-4t,\ 3-4t)\cdot(4,4)$$

$$= -(8(2-4t) - 4(3-4t),\ 4(3-4t) - 4(2-4t))\cdot(4,4)$$

$$= -(16-32t-12+16t,\ 12-16t-8+16t)\cdot(4,4)$$

$$= -(-16t+4,\ 4)\cdot(4,4)$$

$$= 64t-32.$$

This strictly convex has global max,

$$\phi'(t) = 64t-32$$

$$t = \frac{1}{2}\quad \text{as}\quad \phi''(t) = 64 > 0$$

$$x_1 = x_0 - \frac{1}{2}\nabla f(x_0) = (2,3) - \frac{1}{2}(4,4) = \underline{(0,1)}$$

3.) $f(x,y) = x - y + 2x^2 + 2xy + y^2$

Sol:

let $X = (0,0)$

$S_1 = (-1,0)$

$f_1 = f(x) = 0$

to find the optimum step length α_1^*,

we minimize

$$f(X + \alpha_1 S_1) = f(0 + \alpha(-1),\ 0 + \alpha(0))$$

$$= f(-\alpha, 0)$$

$$f(-\alpha, 0) = (-\alpha) - 0 + 2(-\alpha)^2 + 2(-\alpha)(0) + 0^2$$

$$= -\alpha + 2\alpha^2$$

$$= 2\alpha^2 - \alpha$$

as $\frac{df}{d\alpha} = 0$ at $\alpha = \frac{1}{4}$; we have ; $\alpha_1^* = \frac{1}{4}$

$$X_2 = X_1 + \alpha_1^* S_1$$

$$= \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} + \frac{1}{4} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -\frac{1}{4} \\ 0 \end{Bmatrix} = \begin{Bmatrix} -0.25 \\ 0 \end{Bmatrix}$$

$$= (-0.25, 0)$$