26/11/2020

Re-class Test-2

CB.EN.U4CSE19453

1) $f(x_1, x_2) = 10 + x_1^2 - 5x_1x_2 + 9x_2^2 + x_2$

$$\frac{\partial f}{\partial x_1} = 2x_1 - 5x_2$$

$$\frac{\partial f}{\partial x_2} = -5x_1 + 18x_2 + 1$$

$2x_1 = 5x_2$

$x_1 = \frac{5}{2}x_2$

$-5 \times \frac{5}{2} \times 2 + 18x_2 + 1 = 0$

$-25x_2 + 36x_2 + 2 = 0$

$11x_2 = -2$

$x_2 = \frac{-2}{11}$

$x_1 = \frac{5}{\cancel{2}} \times \frac{\cancel{2}}{11}$

$x_1 = \frac{-5}{11}$

2) $f(x_1, x_2) = 4x_1 x_2 - x_1^4 - x_2^4$

$4x^3$

$12x^2 + 1 = \begin{bmatrix} -12x^2 & 4 \\ 4 & -12x^2 \end{bmatrix}$ at $(1,1)$

$A_1 = [-12x^2] \longrightarrow = -12(1)^2 = -12$

$A_2 = \begin{vmatrix} 12(-x^2) & 4 \\ 4 & -12x^2 \end{vmatrix}$

$= (-12)(-12) - 16$

$= 144 - 16$

$=$ +ve

$= 128$

$A_1 = (-1)^1 =$ -ve val $= (-1)$

$A_2 = (-1)^2 =$ +ve $= 1$

So negative definite

hessian matrix is indefinite at $(1,1)$ & $(1,1)$

is a saddle point

3) Conceptual

$\nabla f(x^*)$

4.) $f(x_1, x_2, x_3) = (x_1-4)^4 + (x_2-3)^2 + 4(x_3+5)^4$

$$d^{(t)} = (1,1,0)^T \longrightarrow [4,2,-1]^T$$

$f'(x_1, x_2, x_3) = 4(x_1-4)^3 + 2(x_2-3) + 16(x_3+5)^3$

$\Rightarrow 4(0)^3 + 2(2-3) + 16(4)^3$

$\Rightarrow \cancel{64(16)+2(-1)}$

$\Rightarrow$ 10[illegible]

$$\begin{array}{r} 64 \\ 16 \\ \hline 384 \\ 64 \\ \hline 1024 \end{array}$$

$4(x_1-4)^3 = 0$

$x_1 = 4$

$x_2 = 3$

$x_3 = -5$

$(-4, -3, 5)$

True

5) $f(x,y) = 25x^2 + y^2$ $\qquad X_1 = (0,1)$

$$\nabla f = \begin{pmatrix} 50x \\ 2y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\nabla f_1' = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \qquad S = -\nabla f(x_1) = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$$f\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 0 \\ 2 \end{pmatrix}\right) = f(0, 1-2\lambda_1)$$

$$= f(0, 1-2\lambda)$$

$$= 0^2 + (1-2\lambda)^2$$

$$= 1^2 + 4\lambda^2 - 2(1)(2\lambda)$$

$$= 1 + 4\lambda^2 - 4\lambda$$

$$\frac{\partial f}{\partial \lambda_1} = 8\lambda - 4 = 0$$

$$\lambda = \frac{1}{2}$$

$$X_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -\cancel{\frac{2}{2}}\ -1 \end{pmatrix}$$

$$= (0, 0)$$

6) $f(x, y) = 2x^2 + y^2$ $\qquad x_1 = (1, 1)$

$$\nabla f = \begin{pmatrix} 4x \\ 2y \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$S_1 = -\nabla f(x_1) = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

$$f\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} -4 \\ -2 \end{pmatrix}\right) = f(1 - 4\lambda_1, 1 - 2\lambda_1)$$

$$f = 2(1 - 4\lambda_1)^2 + (1 - 2\lambda_1)^2$$

$$= 2(1^2 + 16\lambda_1^2 - \lambda_1) + (1 + 4\lambda^2 - 4\lambda)$$

$$= 2(1 + 16\lambda^2 - 8\lambda) + (1 + 4\lambda^2 - 4\lambda)$$

$$= 2 + 32\lambda^2 - 16\lambda + 1 + 4\lambda^2 - 4\lambda$$

$$f = 36\lambda^2 - 20\lambda + 3$$

$$\frac{\partial f}{\partial \lambda} = 0 \quad \Rightarrow \quad 36 \times 2\lambda - 20 = 0$$

$$72\lambda = 20$$

$$\lambda = \frac{20}{72}$$

$$\underline{\lambda = \frac{5}{18}}$$

$$x_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{5}{18}\begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{-20}{18} \\ \frac{-10}{18} \end{pmatrix} = \begin{pmatrix} 1 - \frac{20}{18} \\ 1 - \frac{10}{18} \end{pmatrix} = \begin{pmatrix} \frac{-1}{9} \\ \frac{4}{9} \end{pmatrix}$$

7.) $f(x,y) = 4x - y + 2x^2 + 2xy + y^2$ $\qquad X_1 = (1,0)$

$$X_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - (H^{-1}) \nabla f(x_1)$$

$$\nabla f = \begin{bmatrix} 4 + 4x + 2y \\ -1 + 2x + 2y \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 \\ -1+2 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix} \Rightarrow H^{-1} = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -2 & 4 \end{bmatrix}$$

$$H^{-1} = \begin{bmatrix} \frac{2}{4} & \frac{-2}{4} \\ \frac{-2}{4} & \frac{4}{4} \end{bmatrix}$$

$$H^{-1} \nabla f(x_1) = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{2} & \frac{-1}{2} \\ \frac{-8}{2} & +1 \end{bmatrix}$$

$$= \begin{bmatrix} 7/2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 7/2 \\ -3 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 3 \end{bmatrix}$$

$1 - \frac{7}{2} = 1 - 3.5$
$= -2.5,$ $\qquad (-2.5, 3)$

8) $f(x,y) = 2x - y + 2x^2 + 2xy + y^2$ $\quad (1,1)$

$$x_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = H^{-1} \nabla f(x_1)$$

$$\nabla f(x_1) = \begin{bmatrix} 2 + 4x + 2y \\ -1 + 2x + 2y \end{bmatrix} - \begin{bmatrix} 2+4+2 \\ -1+2+2 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

$$H = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix} \qquad H^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix}$$

$$H^{-1} \nabla f(x_1) = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{2} - \frac{3}{2} \\ -\frac{8}{2} + 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{2} \\ -1 \end{bmatrix}$$

$$x_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{bmatrix} \frac{5}{2} \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 2.5 \\ 1 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1.5 \\ 2 \end{bmatrix}$$

9) $f(x_1, x_2) = x_1 x_2$

$x^0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \lambda^{(0)} = 100$

$10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

$\begin{bmatrix} \frac{1}{10} & 0 \\ 0 & \frac{1}{10} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} \\ \frac{1}{10} \end{bmatrix} = 10 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

10) $f(x) = \frac{1}{2} \sum_{i=1}^{m} [f_i(x)]^2 = \frac{1}{2} F(x)^T F(x) \qquad \nabla^2 f(x)$

$\nabla^2 f(x) = \nabla F(x)^T \nabla F(x) + \sum_{i=1}^{m} f_i(x) \nabla^2 f_i(x)$