

24/4/21

HW-6- AVL, 2-4 Trees, Hashing

19453

1) @ Order: 15, 20, 24, 10, 13, 7, 30, 36, 25

Insert = 15;

(15)⁰

Insert = 20;

(20 > 15)

(15)¹
└ (20)⁰

Insert = 24

(24 > 20)

Inorder = 15, 20, 24
a b c

here b = y;
single rotation

~~z~~ (15)² a
└ (20)¹ b
└ (24)⁰ c

(20)⁰
└ (15)⁰ (24)⁰

Insert = 10;

(20)¹
└ (15)¹ (24)⁰
└ (10)⁰

Insert = 13;

(20)²
└ (15)² (24)⁰
└ (20)¹ (13)⁰

Inorder = 10, 13, 15

Double rotation

(20)¹
└ (13)⁰ (24)⁰
└ (10)⁰ (15)⁰

(20)²
└ (15)¹ (24)⁰
└ (10)⁰ (13)⁰

Insert = 7;

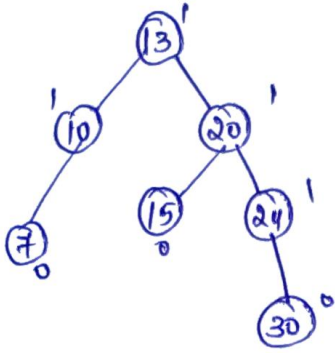
(20)²
└ (13)² (24)⁰
└ (10)¹ (15)⁰
└ (7)⁰

inorder:
7 10 30 20
a b c

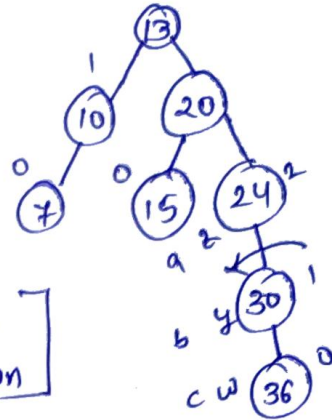
b = y; Single rotation;

(13)⁰
└ (10)¹ (20)⁰
└ (7)⁰ (15)⁰ (24)⁰

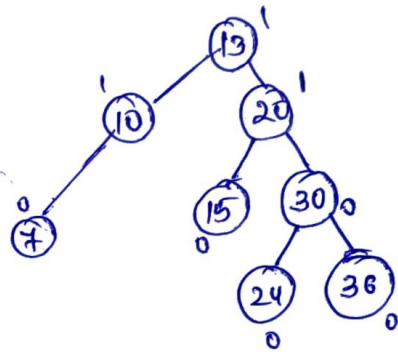
Insert = 30;



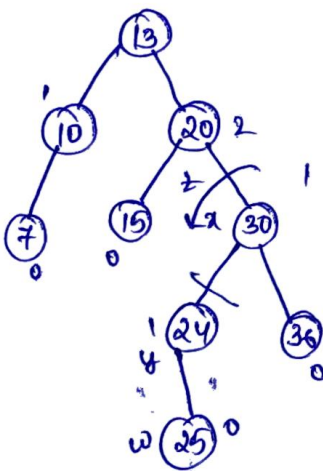
Insert = 36;



[Single rotation]



Insert = 25;

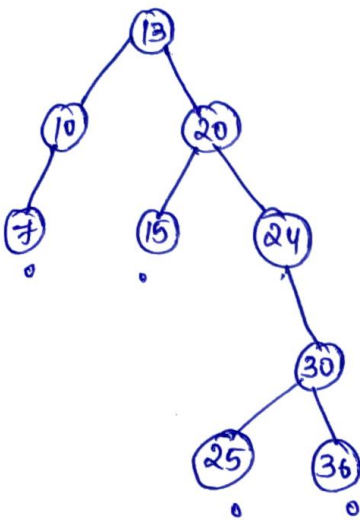


Inorder:

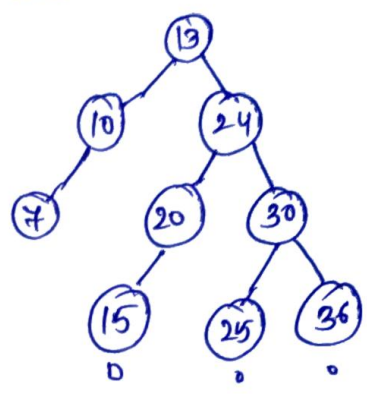
20 24 25 30
a b c

[Double rotation]



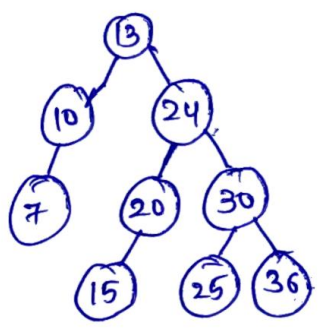


Final:-

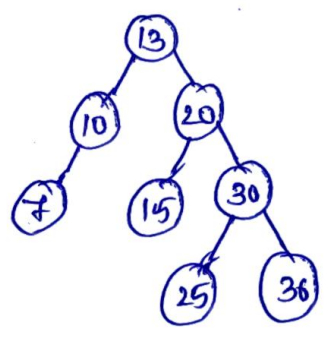


1(b)

AVL tree:-

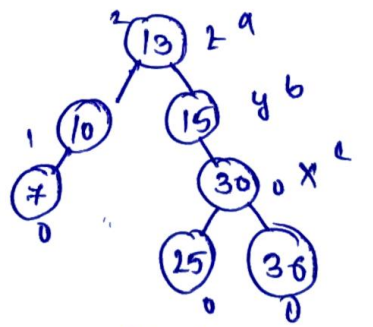


delete 24:-



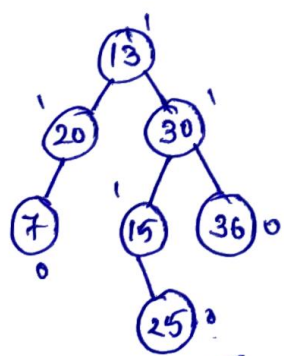
delete = 20:-

Inorder: 13, 15, 30
a b c



(single rotation)

final

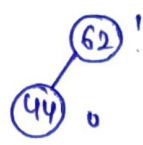


2) Insert: 62, 44, 78, 17, 50, 88, 48, 54 after delete key(62)

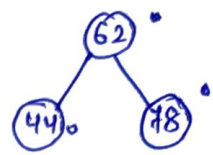
insert = 62;



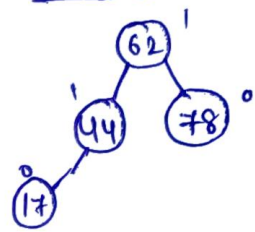
insert = 44;



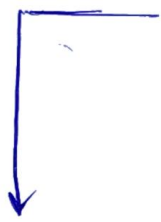
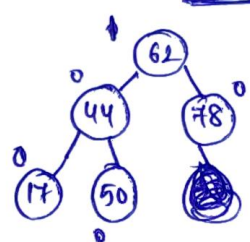
insert = 78;



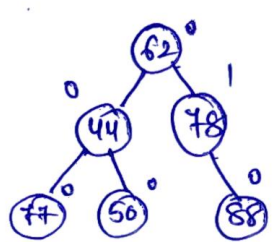
insert = 17;



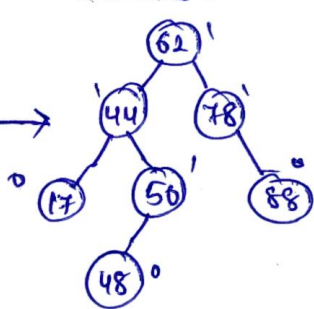
insert = 50;



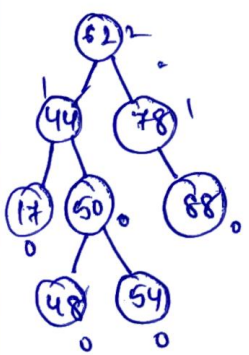
insert = 88;



insert = 48;

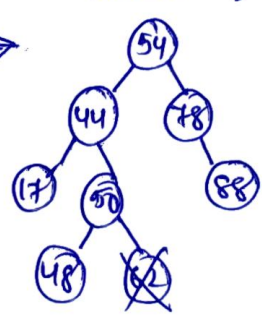


insert = 54;

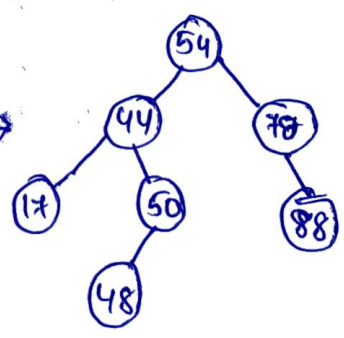


delete 62;

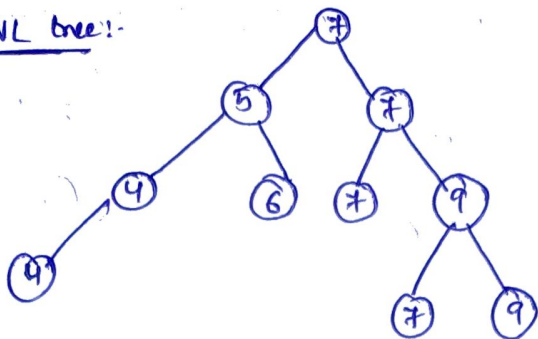
- (1) replace 62 with 54;
- (2) now, delete 62;



Final:



3) AVL tree:-

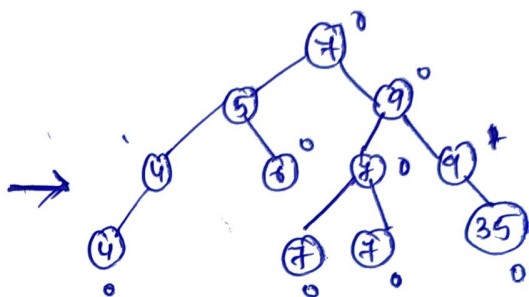
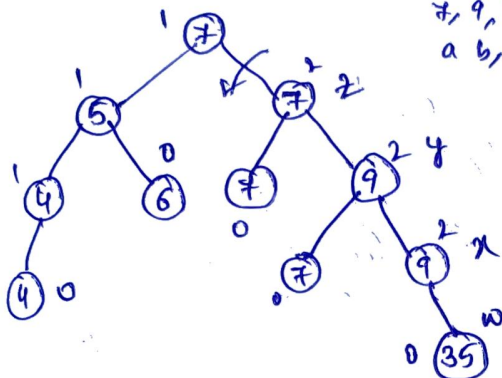


insertion of 35, 69, 100 :-

insert = 35:

Inorder:
7, 9, 35
a, b, c

(Single rotation)

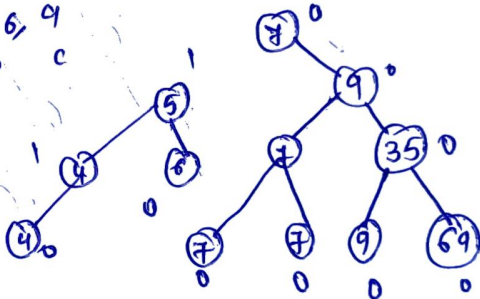
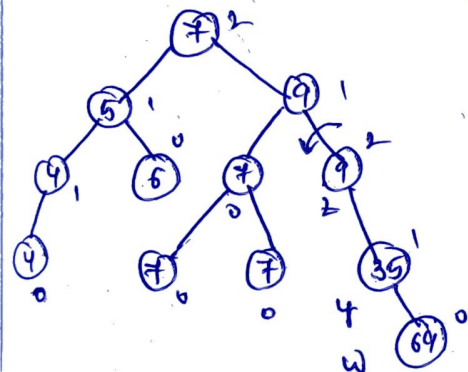


insert = 69:

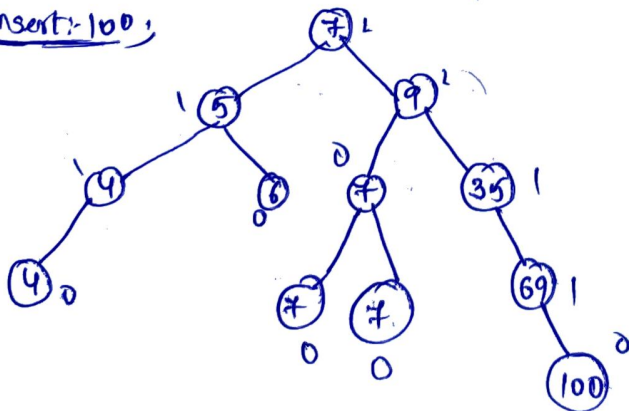
Inorder:

(Single rotation)

9, 35, 69
a, b, c



Insert: 100:



Final

4.) Insertion of 10, 20, 15, 25, 30, 16, 18, 19

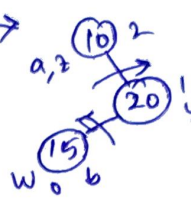
Insert = 10:



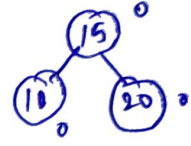
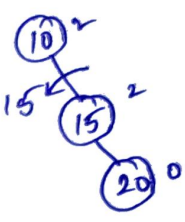
Insert = 20



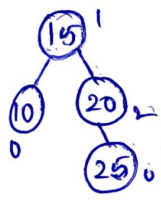
Insert = 15



(double-rotation)

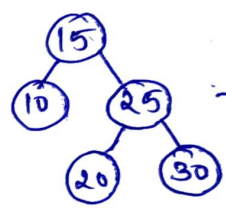
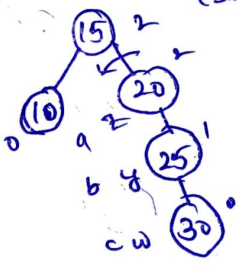


Insert = 25;

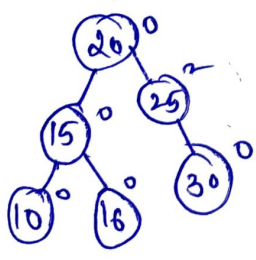
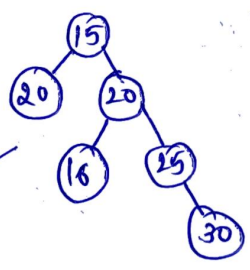
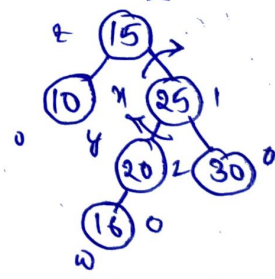


Insert = 30;

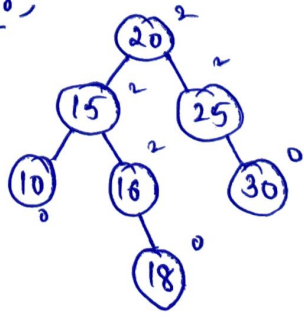
(single rotation)



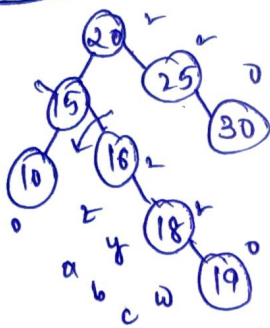
insert = 16;



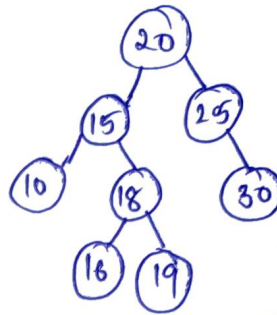
insert = 18;



Insert=19;



Single-rotation
→



final

5) $K = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p\}$

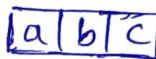
insert=a;

①

insert=b;

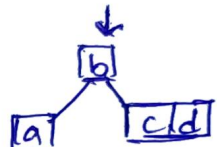


insert=c;



insert=d;

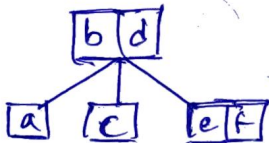
Overflow condition;
hence split.



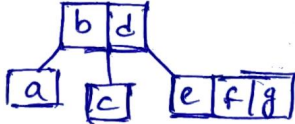
(second element
splitting)

insert=f;

Overflow;
hence split.

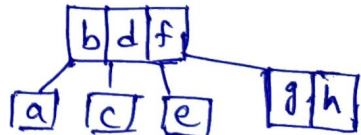


insert g;

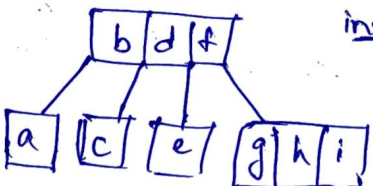


insert h;

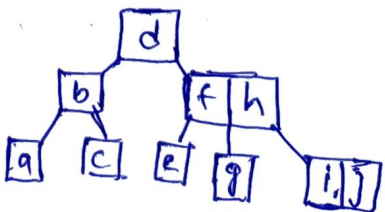
overflow
hence split



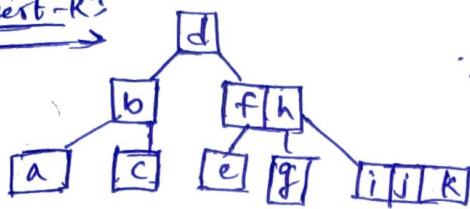
insert-i;



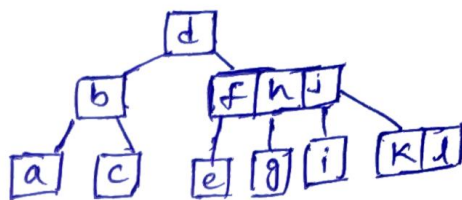
insert-j; overflow; hence split.



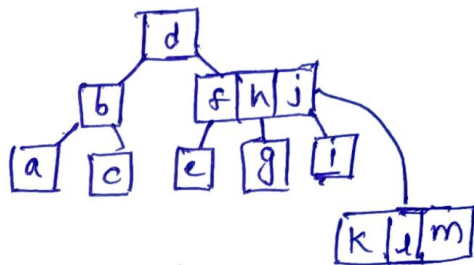
insert-k;



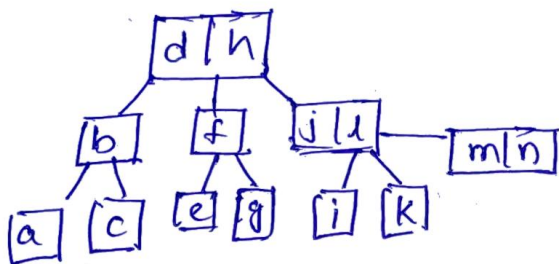
insert-l; overflow hence split



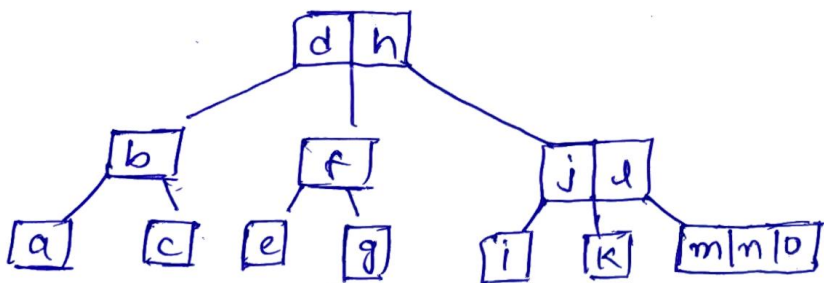
insert-m;



insert n;
overflow
so, split.



insert o;



6) $K = \{3, 1, 5, 4, 2, 9, 10, 8, 7, 6\}$

insert = 3;



→

insert = 1;



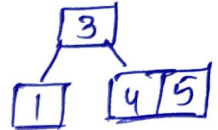
→

insert = 5;

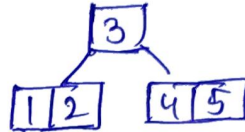


→

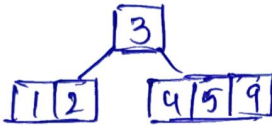
insert = 4;
Overflow
So, split;



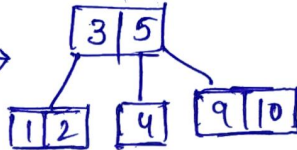
insert = 2;



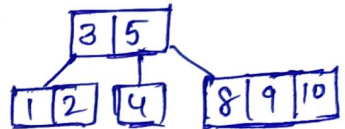
insert = 9;



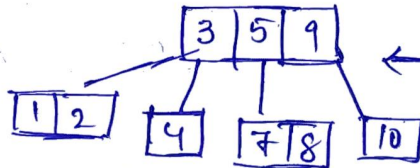
insert = 10;
Overflow,
hence split.



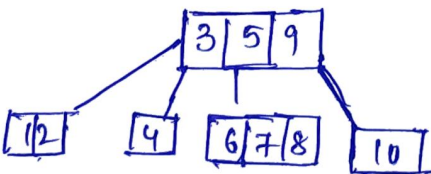
insert = 8;



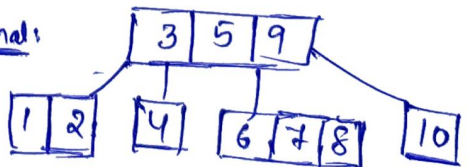
insert = 7;
Overflow;
hence split



insert = 6;



≡ Final:

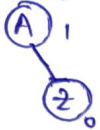


7) @ $K = \{A, z, B, Y, C, X, D, W, E, V, F, L\}$

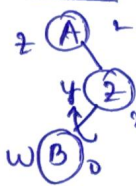
Insert = A;



Insert = z;



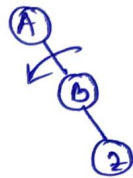
Insert = B;



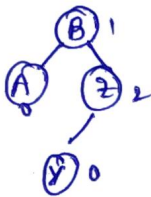
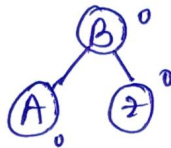
Inorder

A, B, z
a b c

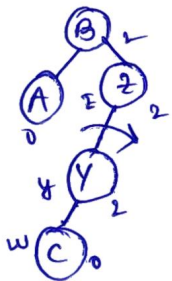
(double rotation)



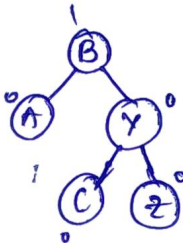
Insert = X;



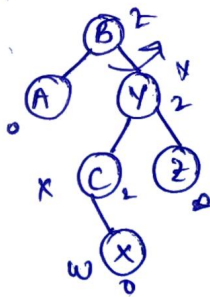
insert = c



C, Y, z
a b c
Single rotation



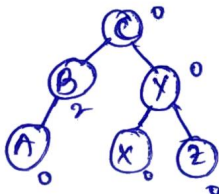
insert X;



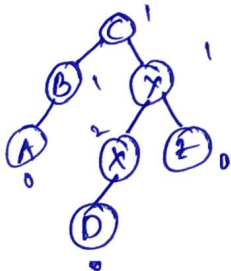
Inorder

BC X Y
a b c

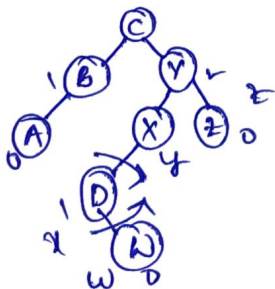
(double rotation)



insert D;



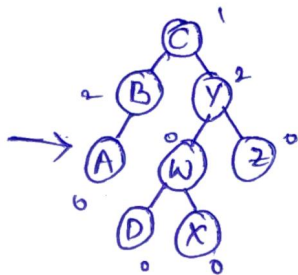
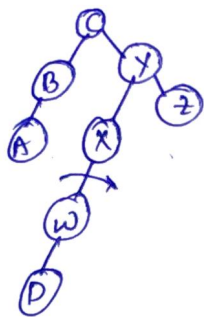
insert W;



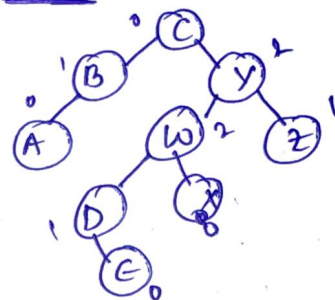
double rotation,



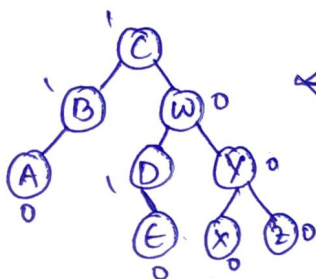
DW X Y
a b c



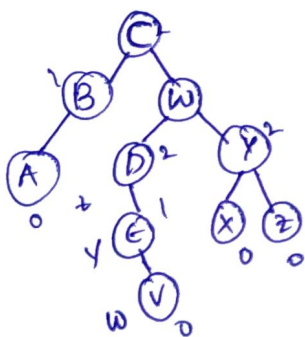
insert E;



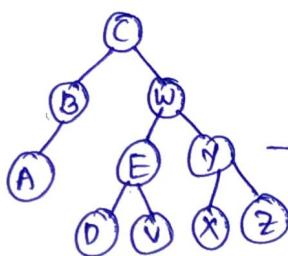
$\begin{pmatrix} D & E & W & X \\ a & b & c \end{pmatrix}$ Single rotation.



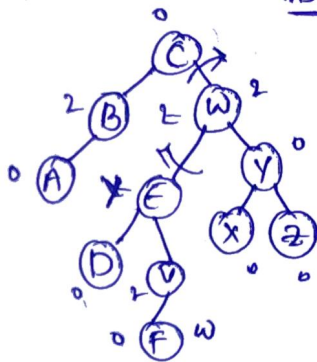
insert V;



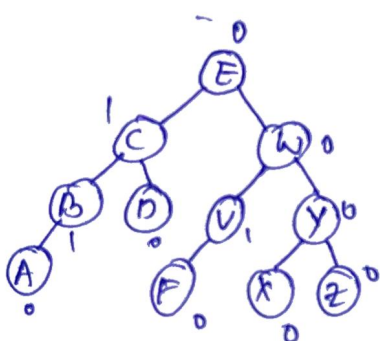
(Single rotation)



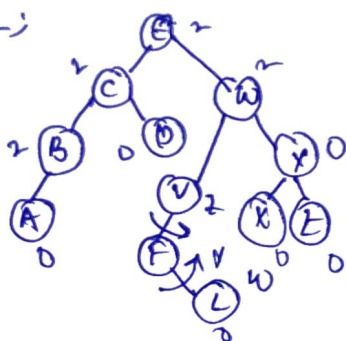
insert F;



double rotation,



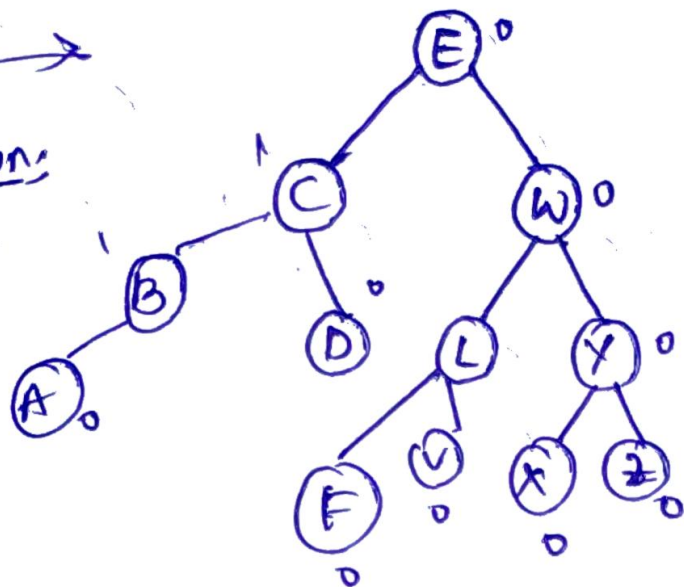
insert -L;



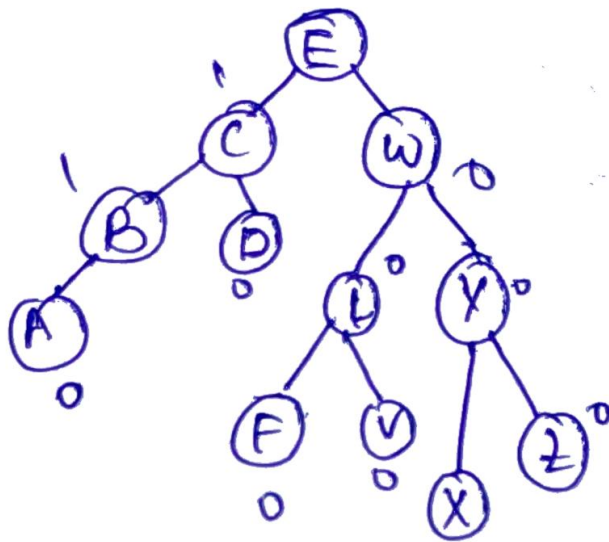
double rotation



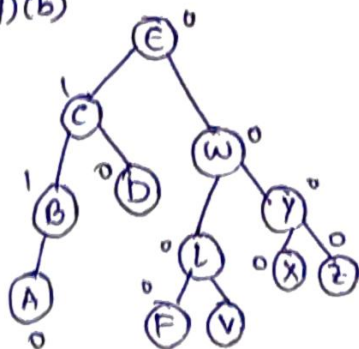
double
rotation



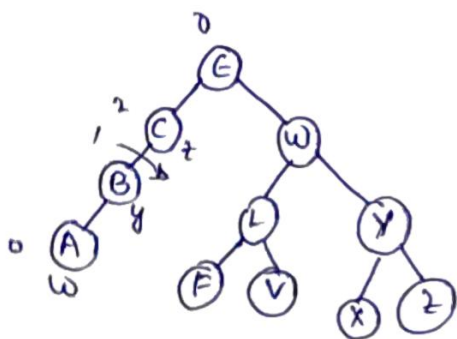
Final :-



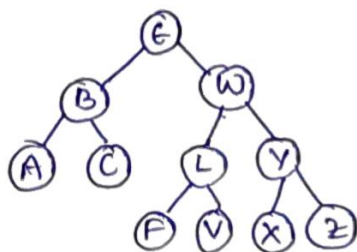
7)(b)



delete D
→



A B C
a b c
(Single rotation)



7(c) :-

• AVL trees are strictly balanced.

• Here, we can find the maximum value by starting from root, and constantly travelling down the right-most branch till end.

Hence,

the time complexity for finding the max. value in an AVL tree is $O(\log N)$

where N - no. of elements.

T. Findmax() $\rightarrow O(\log N)$

8) (a) $h(x) = x \% 7$

Size = 7

$I = \{2341, 4234, 2839, 430, 22, 397, 3920\}$

(a) Linear probing:

$2341 \% 7 = 3$

$4234 \% 7 = 6$

$2839 \% 7 = 4$

$430 \% 7 = 3$ — Collision

$(430+1) \% 7 = 4$

$(430+1) \% 7 = 5$

$22 \% 7 = 1$

$397 \% 7 = 5$ — Collision

$(397+1) \% 7 = 6$

$(397+2) \% 7 = 0$

$3920 \% 7 = 0$ — Collision

$(3920+1) \% 7 = 1$ — Collision

$(3920+2) \% 7 = 2$

6	4234
5	430
4	2839
3	2341
2	3920
1	22
0	397

(b) Separate Chaining:

0	3920
1	22
2	
3	2341 → 430
4	2839
5	397
6	4234

→ insert 2341

→ insert 4234

→ insert 2839

→ insert 430

→ Collision occurs;
add to chain

→ insert 22

→ insert 397

→ insert 3920

8(c) double hashing:

$$h(x) = (2x-1) \% 7$$

$$2341 \% 7 = 9$$

$$4234 \% 7 = 6$$

$$2839 \% 7 = 4$$

$$430 \% 7 = 3 \rightarrow \text{collision}$$

$$(430 \% 7) + 1 (7 - (2 \times 430 - 1) \% 7) \% 7 = 5$$

$$22 \% 7 = 1$$

$$397 \% 7 = 5 \rightarrow \text{collision};$$

$$(397 \% 7) + 1 (7 - (2 \times 430 - 1) \% 7) \% 7 = 3$$

$$(397 \% 7) + 2 (7 - (2 \times 430 - 1) \% 7) \% 7 = 1$$

$$(397 \% 7) + 3 (7 - (2 \times 430 - 1) \% 7) \% 7 = 6$$

$$(397 \% 7) + 4 (7 - (2 \times 430 - 1) \% 7) \% 7 = 4$$

$$(397 \% 7) + 5 (7 - (2 \times 430 - 1) \% 7) \% 7 = 2$$

$$3920 \% 7 = 0$$

$$h_1 = (h(x) + i \cdot h'(x)) \% \text{size}$$

$$h'(x) = x \% 7$$

6	4234
5	430
4	2839
3	2341
2	397
1	22
0	3920

(a)
q) Linear probing :-

$$I = \{8, 33, 15, 26, 22\}$$

$$\text{size} = 7$$

$$h(K) = K \bmod 7$$

$$8 \bmod 7 = 1$$

$$33 \bmod 7 = 5$$

$$15 \bmod 7 = 1 \text{ --- Collision}$$

$$(15 + 1) \bmod 7 = 2$$

$$26 \bmod 7 = 5 \text{ --- Collision}$$

$$(26 + 1) \bmod 7 = 6$$

$$22 \bmod 7 = 1 \text{ --- Collision}$$

$$(22 + 1) \bmod 7 = 2 \text{ --- Collision}$$

$$(22 + 2) \bmod 7 = 3$$

6	26
5	33
4	
3	22
2	15
1	8
0	

q) (b)

When there is collision; 1 (1 to size) will be added to data to find a slot empty, the process would continue till a unique key is calculated;

$$\text{Load factor} = \left(\frac{\text{no. of elements}}{\text{table size}} \right)$$

$$= \frac{5}{7}$$

$$= \boxed{0.7}$$

$$10) (a) n(x) = x \% 10$$

$$\text{Size} = 10$$

$$I = \{51, 23, 73, 99, 44, 79, 89, 38\}$$

$$51 \% 10 = 1$$

$$23 \% 10 = 3$$

$$73 \% 10 = 3 \rightarrow \text{Collision}$$

$$(73 + 1^2) \% 7 = 4$$

$$99 \% 10 = 9$$

$$44 \% 10 = 4 \rightarrow \text{Collision}$$

$$(44 + 1^2) \% 7 = 5$$

$$79 \% 10 = 9 \rightarrow \text{Collision}$$

$$(79 + 1^2) \% 10 = 0$$

$$89 \% 10 = 9 \rightarrow \text{Collision}$$

$$(89 + 1^2) \% 10 = 0 \rightarrow \text{Collision}$$

$$(89 + 2^2) \% 10 = 3 \rightarrow \text{Collision}$$

$$(89 + 3^2) \% 10 = 8$$

$$38 \% 10 = 8 \rightarrow \text{Collision}$$

$$(38 + 1^2) \% 7 = 9 \rightarrow \text{Collision}$$

$$(38 + 2^2) \% 7 = 2$$

$$\text{quad ratio} = \frac{(1 + 2^2)}{\text{Size}}$$

9	99
8	39
7	
6	
5	44
4	73
3	23
2	38
1	51
0	79

(b) when there is collision, i^2 (1 to size) will be added to key, and the process would continue till a unique key is calculated.

$$\text{Load factor} = \frac{\text{no. of elements}}{\text{Size of table}}$$

$$= \frac{8}{10}$$

$$= 0.8$$

11) Chaining :-

$$h(K) = K \% 9$$

$$I = \{5, 28, 19, 15, 20, 33, 12, 17, 10\}$$

$$\text{Size} = 9$$

$$5 \% 9 = 5$$

$$28 \% 9 = 1$$

$$19 \% 9 = 1 \text{ --- Chaining}$$

$$15 \% 9 = 6$$

$$20 \% 9 = 2$$

$$33 \% 9 = 6 \text{ --- Chaining}$$

$$12 \% 9 = 3$$

$$17 \% 9 = 8$$

$$10 \% 9 = 1 \text{ --- Chaining}$$

