COMPSCI 250 Discussion #1: What is a Proof?
Individual Handout

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In Excursion 1.3 of the textbook Dave tells a story about how mathematical proof works in the real world, or at least the real world of professional mathematics. We then look at a piece of Java code, and talk about how to prove it **partially correct** using **preconditions**, **postconditions**, and **loop invariants**.

What we want you to do in discussion today is to prove some statements. We know we don't have all the formal tools for writing proofs yet, but we think it's worthwhile for you to practice making arguments as convincing as you can, **with reference to definitions**. Check your first proofs with your discussion leader, to be sure that they find them convincing, before you go on.

You'll be working in groups of three or four. You might find it useful to put one person in charge of moderating discussion, and another in charge of writing down the group's answers on the single response sheet.

Here are the definitions of the terms involved in the statements:

- A **natural** is a non-negative integer like 0, 1, 2, 3, and so forth.
- The **successor** of a natural is the natural that comes after it – there always is one.
- The **predecessor** of a natural is the one before – every natural except 0 has one.
- The **addition** and **multiplication** operations work the way you already understand. (But don't assume the same for division!)
- A natural is an **even number** if it is 2 times some natural.
- A natural is an **odd number** if it is 2 times some natural, plus one.
- Naturals obey the **Least Number Axiom** – if any natural has a particular property, there must be a *smallest* natural with that property. (We often use this to get a contradiction.)

Writing Exercises:

1. Prove that if x is an even natural, x's predecessor (if it has one) is odd.
2. Prove that if x is an odd natural, x's predecessor is even.
3. Prove that if x's predecessor is odd, then x is even.
4. Prove that if x's predecessor is even, then x is odd.
5. Prove that every natural is either odd or even. (**Hint:** By the Least Number Axiom, if any natural is *neither* odd *nor even*, there's a least such natural. Could it be 0? If not, what about its predecessor? Use the results of (1) – (4) to get a contradiction.)
6. Prove that no natural is both odd and even. (Similar to (5) – get a contradiction by assuming some natural is both.)