Names: ____________________

COMPSCI 250 Discussion #6: More Induction Problems Group Response Sheet

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Today's exercise is to solve some more varied problems using mathematical induction, and write careful proofs of each of the four statements you are given. Remember that each proof needs a base case, a clear statement of the inductive hypothesis, and a clear argument for the inductive step.

- (a) Recall that the **Fibonacci numbers** are defined recursively by the rules $F(0) = 0$, $F(1) = 1$, and for any $n \geq 1$, $F(n+1) = F(n) + F(n-1)$. Prove that for any positive integer n, the numbers $F(n)$ and $F(n+1)$ are relatively prime.

- (b) In this problem we consider tiling a large equilateral triangle made up of 4^k smaller triangles, as described on the Individual Handout, with trapezoidal figures made of three of the smaller triangles. Your task is to prove that for any k, if we divide a big triangle into 4^k small ones and remove *one* of the small ones at one of the points, we can tile the remainder of the figure with these trapezoids.

- (c) Begin with a rectangle in the Euclidean plane and divide it into regions by drawing any number of straight lines or circles. Show that the resulting figure may be **two-colored**, meaning that we can paint each region either red or blue such that no two red regions touch at more than a point, and that no blue regions touch at more than a point.

- (d) We are told that a particular county has n cities, with $n > 1$, and that from each city x to each other city y there is a road that may traversed in only one direction – we are not told which. We want to prove that for any such county, there is a path starting at one city, visiting each city exactly once, and ending (if $n > 1$) at a different city. This path uses $n - 1$ of the roads and traverses each road in the correct direction.