Names: _____

CMPSCI 250 Discussion #5: Practicing Induction Proofs
Group Response Sheet

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Today's exercise is to write careful proofs, using mathematical induction, of the four statements on the Individual Handout. Remember that each proof needs a base case, a clear statement of the inductive hypothesis, and a clear argument for the inductive step.

- (a) Let S(n) be the sum for i from 1 to n of i^2. Prove that for any natural number n, $S(n) = n(n+1)(2n+1)/6$.

- (b) Let S(n) be the sum for i from 1 to n of $(-1)^i i$, so that $S(3) = -1 + 2 - 3 = -2$. Prove that for any natural number n, $S(n) = n/2$ if n is even and $S(n) = -(n+1)/2$ if n is odd.

- (c) Let L be a line segment in the plane. Prove that for any natural number n, if we place n distinct points on L (none of them at the endpoints of L), then we divide it into exactly $n + 1$ line segments.

- (d) Consider a solid (a rectangular parallelepiped) made by attaching n sugar cubes in a line, where each cube has a side of 1 centimeter. Prove **by induction** that if n is any *positive integer*, the surface area of this solid is $4n + 2$ square centimeters. (An induction for positive integers has a base case of 1 instead of 0.)