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## COMPSCI 250 Discussion #3: Practicing Predicate Proofs Group Response Sheet

David Mix Barrington and Ghazaleh Parvini  
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Here are four practice statements to prove. For this exercise, you should go slightly overboard in justifying your steps. In the first example you should use variables  $x$  of type  $X$ ,  $y$  of type  $Y$ , and  $z$  of type  $Z$ . In the other three examples all the variables are of type  $A$ .

1. Let  $X$ ,  $Y$ , and  $Z$  be three sets, and let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be functions (both total and well-defined). Assume that  $f$  and  $g$  are each onto functions, and let  $h : X \rightarrow Z$  be defined by the rule  $h(x) = g(f(x))$ . **Prove that  $h$  is onto.**
2. Let  $R$  be a binary relation on  $A$ . Assume that  $R$  is transitive and symmetric. Also assume  $\forall x : \exists y : R(x, y)$ . **Prove that  $R$  is reflexive.**

3. Let  $R$  and  $S$  be two binary relations on the same set  $A$ . Assume that both  $R$  and  $S$  are antisymmetric. Define  $T$  to be the intersection of  $R$  and  $S$ , so that  $\forall x : \forall y : T(x, y) \leftrightarrow (R(x, y) \wedge S(x, y))$ . **Prove that  $T$  is antisymmetric.**