# COMPSCI 250 Discussion #5: Practicing Induction Proofs
## Individual Handout

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Today's exercise is to write careful proofs, using mathematical induction, of the four statements below. Remember that each proof needs a base case, a clear statement of the inductive hypothesis, and a clear argument for the inductive step.

- (a) Let S(n) be the sum for $i$ from 1 to $n$ of $i^2$. Prove that for any natural number $n$, $S(n) = n(n+1)(2n+1)/6$.

  *This is a summation similar to the first one we saw in lecture. Remember to maintain the distinction between $S(n)$, the sum itself, and $P(n)$, the statement that the sum has the alleged value. The key identity about the sum is that $S(n+1)$ (the sum of the first $n+1$ terms) is equal to $S(n)$ (the sum of the first $n$ terms) plus $(n+1)^2$ (the $n+1$'st term).*

- (b) Let S(n) be the sum for $i$ from 1 to $n$ of $(-1)^i i$, so that $S(3) = -1 + 2 - 3 = -2$. Prove that for any natural number $n$, $S(n) = n/2$ if $n$ is even and $S(n) = -(n+1)/2$ if $n$ is odd.

  *Here you are going to need to use a Proof By Cases within the inductive step. If $n$ is even, $P(n)$ says what $S(n)$ is, and you have that $S(n+1) = S(n) - (n+1)$ because $n+1$ is odd. If $n$ is odd, you have $S(n+1) = S(n) + (n+1)$ because $n+1$ is even.*

- (c) Let $L$ be a line segment in the plane. Prove that for any natural number $n$, if we place $n$ distinct points on $L$ (none of them at the endpoints of $L$), then we divide it into exactly $n+1$ line segments.

  *You can use common facts from geometry – such as the fact that any point on a line segment divides it into two smaller segments. The point of the proof is to explain the global situation (the number of segments) in terms of what happens locally when one new point is added.*

- (d) Consider a solid (a rectangular parallelepiped) made by attaching $n$ sugar cubes in a line, where each cube has a side of 1 centimeter. Prove **by induction** that if $n$ is any *positive integer*, the surface area of this solid is $4n + 2$ square centimeters. (An induction for positive integers has a base case of 1 instead of 0.)

  *Again we explain the global situation in terms of the local effect of adding one more cube to the solid. Of course it's easy to determine the area of the solid all at once, by adding the areas of all of its sides, but that's not what we want. Use $P(n)$ to prove $P(n+1)$ by reasoning about what happens when the $n+1$'st cube is added.*