Simulation of neutrino oscillation with considering both the mass-less and massive representation

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1 PROPOSAL

Neutrino oscillations in vacuum and with matter effects will be investigated. Both the mass-less and massive representation of neutrino will be considered. It is expected that mass-less equations will demonstrate oscillation only with matter effects while massive characterization of neutrino exhibits oscillation in both vacuum and in matter.

2 THEORETICAL APPROACH

2.1 VACUUM OSCILLATION (MASSIVE REPRESENTATION)

If the neutrino mass eigenstates differ from the flavor eigenstates, the flavor states may be expressed as a linear combination of the mass eigenstates. The traditional mixing angle formalism provides a convenient parametrization of the mixing

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{2.1}$$

Putting this into wave equation gives us

$$\frac{i}{c}\frac{d}{dt} = \begin{pmatrix} \psi_e(t) \\ \psi_\mu(t) \end{pmatrix} = \frac{\delta m^2}{4E} \begin{pmatrix} -cos(2\theta) & sin(2\theta) \\ sin(2\theta) & cos(2\theta) \end{pmatrix} \begin{pmatrix} \psi_e(t) \\ \psi_\mu(t) \end{pmatrix} \tag{2.2}$$

2.2 Matter Effects (Massive Representation)

The neutrino flavour equation with matter effects taken into account is

$$i\frac{d}{dx} = \begin{pmatrix}\psi_e \\ \psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}A - \delta m^2 cos(2\theta) & \delta m^2 sin(2\theta) \\ \delta m^2 sin(2\theta) & \delta m^2 cos(2\theta) - A\end{pmatrix}\begin{pmatrix}\psi_e \\ \psi_\mu\end{pmatrix} \tag{2.3}$$

Where, $A(x) = 2\sqrt{2}G_f N_e(x)E(x)$ is space depended cross term and G_f is the Fermi constant

2.3 Mass-less neutrino in vacuum

The equation for mass-less spin-$\frac{1}{2}$ dirac fermions can be obtained from dirac equation

$$\gamma_\mu\partial_\mu\psi(x) - iL\gamma_4\partial_t\gamma_i\partial_i\psi(x) = 0 \tag{2.4}$$

2.4 Mass-less neutrino in matter

Eq. (2.4) can be modified with the consideration of matter effect

$$\left[\gamma_\mu\partial_\mu + m + \frac{f_4}{2}\gamma_4\left(1+\gamma_5\right) - iL\gamma_4\partial_t\gamma_i\partial_i\right]\psi(x) = 0 \tag{2.5}$$

where, $f_4 = \frac{G_f}{sqrt2}\rho_n$, G_f is the Fermi constant and ρ_n is the neutron number density

3 Computational Approach

There are two variations of problem in terms of numerical modeling. One is time dependent evolutionary problem and other being time independent stationary problem.

1. Time dependent problems can be solved with monte carlo integration method.
2. Time independent stationary problems can be solved using either of the following ways
    - Metroplois algorithm with introducing probability transition function
    - Monte carlo integration with introducing S-matrix (Scattering matrix)

4 Further Goal

1. Analyzing entropy in the phase transition during neutrino oscillation
2. Spontaneous symmetry breaking in electro-weak interaction in muon decay