On the relativistic dynamics of electromagnetic two body problem

Shakir Ahmed[1] and Mahiyath K. Chowdhury[1]
Department of Physics, Shahjalal University of Science & Technology

(Dated: 4 December 2018)

A system of two classical point charges interacting through their electromagnetic field is considered with special relativistic rigor. We approach to solve the equation of motion for the charges numerically and end up with several peculiar results which are at variance with the usual concept of classical electrodynamics, let alone the natural picture. The reason of the abnormalities and the necessary analytical consideration that should have taken into account to obtain physically allowable solution of the problem, are discussed.

I. INTRODUCTION

A system of two point charge particles is considered. Each particle interacts with the electromagnetic field originated from the other particle according to Maxwell's theory for electrodynamics and thus gets accelerated due to Lorentz force acting upon them. The fields (E, B) created by the charges are determined with *Lienard-Wiechert* retarded potentials derived from the equations of electromagnetic four-potential. Assuming the conservation of relativistic momentum with Lorentz force being the cause for the change of four-momentum, we get the following equation of motion

$$\ddot{\mathbf{r}}(\dot{\mathbf{r}}, \mathbf{r}, t) = \frac{1}{m_0 \gamma(\dot{\mathbf{r}})} \left(\mathbf{F}_L - \frac{(\dot{\mathbf{r}} \cdot \mathbf{F}_L)\dot{\mathbf{r}}}{c^2} \right) \tag{1}$$

$$\mathbf{F}_L = e\big[\mathbf{E} + \mathbf{v} \times \mathbf{B}\big] \tag{2}$$

Since, the fields evaluated from Lienard-Wiechert retarded potential consists of the usual $\frac{1}{R^2}$ field as well as a radiation-like $\frac{1}{R}$ field, it should be evident that the equation of motion given by them should predict a damping due to electromagnetic radiation from moving charges as the fields suggest.

II. NUMERICAL ANALYSIS

Using the RK4 scheme we solved Eq. (1) for an electron and a proton being the two body system. All units are taken into the Hartree atomic units. The numerical model has been analyzed for several different initial conditions for electron with the position of proton being at the origin and the velocity being null.
It is found that for whatever initial condition is being chosen, the electron never falls into the proton. In some of the cases Fig. [1][2][3][4] while the electron is being approached to the proton, it gets repelled away from it and never comes back. As the electron moves towards proton its velocity is gradually increased until it reaches the point Fig. [2], from where it begins to move away from the proton, as if there exists a repulsive interaction between them. The velocity of electron afterward remains somewhat constant with respect to time indicating a comparatively much smaller Lorentz force between the charges.
We also have found some rotating orbit for electron Fig. [5][6]. It appears that the orbit is shifted away from the centre (proton) with time, becoming larger in radius.
With the classical picture of an electron revolving around a proton, if one takes into account the effect of radiation damping, then one would assume that, owing to loss of energy by radiation, the electron would approach the proton and eventually falls into it which is exactly opposite description of our found results from the governing equation of motion.

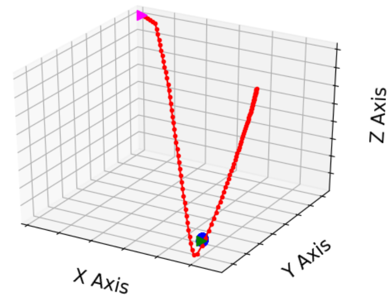

FIG. 1. 3-D trajectory of an electron and proton with initial electron velocity, $v_e = (-0.08c, -0.0008c, 0.00001c)$ and position, $(r, \theta, \phi) = (0.005, \frac{\pi}{4}, \frac{\pi}{4})$.

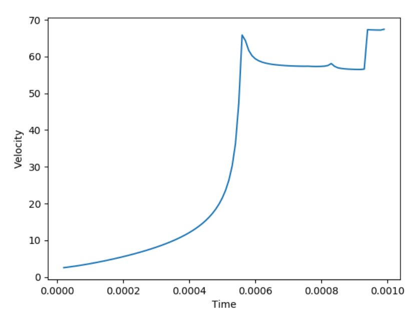

FIG. 2. Velocity vs. Time diagram of electron with initial velocity, $v_e = (-0.08c, -0.0008c, 0.00001c)$ and position, $(r, \theta, \phi) = (0.005, \frac{\pi}{4}, \frac{\pi}{4})$. [With corresponding Hartree atomic unit for time and velocity]

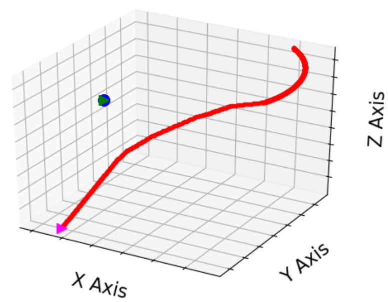

FIG. 3. 3-D trajectory of an electron and proton with initial electron velocity, $v_e = (-0.08c, -0.0008c, 0.00001c)$ and position, $(r, \theta, \phi) = (0.05, \frac{\pi}{4}, \frac{\pi}{2})$.

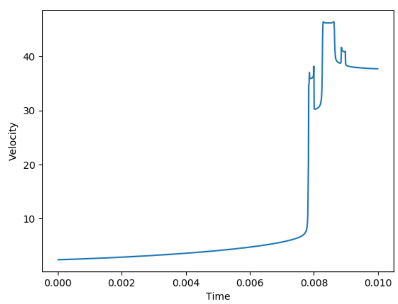

FIG. 4. Velocity vs. Time diagram of electron with initial velocity, $v_e = (-0.08c, -0.0008c, 0.00001c)$ and position, $(r, \theta, \phi) = (0.005, \frac{\pi}{4}, \frac{\pi}{2})$. [With corresponding Hartree atomic unit for time and velocity]

III. DISCUSSION

The results we have found so far have several features which are at variance with our familiar ideas of classical physics. Further analysis of the model confirms that, even though the retarded field includes a radiation-like

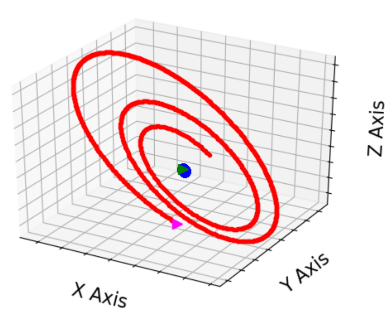

FIG. 5. 3-D trajectory of an electron and proton with initial electron velocity, $v_e = (0.01c, -0.01c, -0.01c)$ and position, $(r, \theta, \phi) = (0.05, \frac{\pi}{4}, \frac{\pi}{4})$.

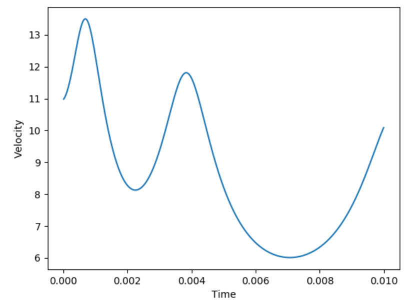

FIG. 6. Velocity vs. Time diagram of electron with initial velocity, $v_e = (0.01c, -0.01c, -0.01c)$ and position, $(r, \theta, \phi) = (0.005, \frac{\pi}{4}, \frac{\pi}{4})$. [With corresponding Hartree atomic unit for time and velocity]

component, it does not contribute any damping term to the equation of motion. Furthermore, if the damping by radiation is explicitly implied onto the system, self contradiction in energy arises. For the principle of conservation of energy to be hold true, it is necessary to add an extra term to the equation of motion as the radiation reaction force. Although this correction reserves energy in the system, it breaks the time-symmetry of Maxwell's theory.

We also considered the radiation reaction term in our numerical model. But nothing good has come out of it. Every initial condition that we tried collapsed into undefined results with the velocity of electron eventually being greater than the speed of light. It is possible to analytically demonstrate that no condition exists for this system to provide with any physically allowable solutions.

Dirac has worked on this problem. He re-evaluated the radiation reaction term with relativistic rigor and found out that it is possible to subtract the infinities in a Lorentz invariant way. However, his model was also inconsistent with the classical notion of physics as well. The same result concerning the inability of the electron in a Hydrogen atom to spiral inward and fall into the nucleus has been predicted by Dirac's theory as well.

This problem stays unless the effect of advanced field is taken into account which was first ignored because it breaks the principle of causality. The advanced solution describing fields converging from infinity and future along the future cone of world line onto the source charge is manifestly unrealistic. It is somewhat surprising that in a theory which uses advanced potentials, and where, therefore, the physical mechanism is by no means clear, the equations derived from the theory lead to physically understandable results, whereas when the theory uses retarded potentials alone, the corresponding equations lead to many unexpected nonphysical results.

IV. CONCLUSION

Pretty much every phenomena described from Maxwell's electromagnetic field theory remains consistent with itself and nature as well until it is composed with Lorentz theory. And, it is a surprising fact that the usual radiation phenomenons can be described with only the retarded effect while the motion of charges can not. Which indicates that the phenomenon of radiation by the accelerated electric charge is a unidirectional one in terms of time (whereas the basic Maxwell equations are time symmetric). So, in case of radiation phenomena the question is therefore, *why do we have an electrodynamics arrow of time?* Field theory does not offer any answer here. It also stops at providing a scenario consistent with causality.

These comments therefore underscores the fact that there are conceptual problems with the classical field theory, and thus provide further motivation for looking at the alternative offered by action at a distance.

[1] C. J. Eliezer, Proc. R. Soc. Lond. A **194**, 543 (1948).
[2] C. J. Eliezer, in *Mathematical Proceedings of the Cambridge Philosophical Society*, Vol. 39 (Cambridge University Press, 1943) pp. 173–180.
[3] C. J. Eliezer, Reviews of Modern Physics **19**, 147 (1947).
[4] R. O'Connell, Physics Letters A **313**, 491 (2003).
[5] F. Rohrlich, Annals of Physics **13**, 93 (1961).
[6] J. M. Jauch and F. Rohrlich, *The theory of photons and electrons: the relativistic quantum field theory of charged particles with spin one-half* (Springer Science & Business Media, 2012).
[7] A. Schild, Physical Review **131**, 2762 (1963).
[8] J. L. Synge, Proc. R. Soc. Lond. A **177**, 118 (1940).
[9] A. Schild and J. Schlosser, Journal of Mathematical Physics **9**, 913 (1968).
[10] J. Cook, Australian Journal of Physics **25**, 141 (1972).
[11] S. Persides and J. Pascalis, Annals of Physics **87**, 161 (1974).
[12] R. Partridge, Nature **244**, 263 (1973).
[13] J. A. Wheeler and R. P. Feynman, Reviews of Modern Physics **17**, 157 (1945).
[14] J. A. Wheeler and R. P. Feynman, Reviews of modern physics **21**, 425 (1949).
[15] A. M. Steane, American Journal of Physics **83**, 256 (2015).
[16] A. CARAT and L. GALGANI, New Perspectives in the Physics of Mesoscopic Systems , 24.
[17] J. D. Jackson, *Classical electrodynamics* (John Wiley & Sons, 2012).
[18] P. Lorrain and D. Corson, (1970).
[19] M. Zahn, (1979).
[20] J. Narlikar, Annual Review of Astronomy and Astrophysics **41**, 169 (2003).
[21] E. H. Kerner, Journal of Mathematical Physics **3**, 35 (1962).
[22] B. Nigam, Physical Review **145**, 1026 (1966).
[23] M. Frisch, *Inconsistency, asymmetry, and non-locality: A philosophical investigation of classical electrodynamics* (Oxford University Press, 2005).
[24] G. Belot, Canadian Journal of Philosophy **37**, 263 (2007).
[25] F. A. Muller, Philosophy of Science **74**, 253 (2007).
[26] M. FA, (2006).
[27] S. Errede, "A brief history of the development of classical electrodynamics," (2007).
[28] M. B. Valente, THEORIA. Revista de Teoría, Historia y Fundamentos de la Ciencia **26**, 51 (2011).