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Computational Physics Project

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On The Relativistic Dynamics of Electromagnetic Two Body Problem

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Abstract

A system of two point charged particles, is considered. Each particle interacts with the electromagnetic field originated from the other particle according to Maxwell's theory for electrodynamics and thus gets accelerated due to Lorentz force acting upon them.
We approach the problem with special relativistic rigor and apply to classical (as distinguished from quantum mechanics) electrodynamics ending up with Lienard-Wiechert potentials.
Investigating the numerical results from the proposed model, we first find some common known phenomena such as Bremsstrahlung radiation and Synchrotron radiation. On the way to brute forcing for Hopf bifurcation we come up with a relatively stable periodic solution. But peculiar results arise from the system, further analyzing the solution with varying parameters. We try to find out the analytical consideration that should have taken into account to obtain a physically allowable solution of the particular two body problem.

Contents

1 Theory

1.1 Relativistic Classical Electrodynamics

Let (x_0, x_1, x_2, x_3) be the time and space co-ordinates of any point in Minkowski space, and let the metric tensor $g_{\mu\nu}$ be given by[1]

$$g_{00} = 1, \qquad g_{\mu\nu} = -1 \qquad for \quad \mu = \nu$$

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The velocity of light is taken as unity. Assuming that the world-line of electron in space-time is given by the equations $z_\mu = z_\mu(s)$, where s is the proper time, the electromagnetic potentials A_μ of its field at the point x_μ satisfy the equations

$$\frac{\partial A_\mu}{\partial x_\mu} = 0, \qquad \Box A_\mu = 4\pi j_\mu \tag{1}$$

where

$$\Box \equiv \frac{\partial^2}{\partial x_0^2} - \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_3^2}$$

and j_μ is the charge-current density vector, given by

$$j_\mu = e \int \frac{dz_\mu}{ds} \delta(x_0 - z_0)\delta(x_1 - z_1) \times \delta(x_2 - z_2)\delta(x_3 - z_3) ds \tag{2}$$

The field equations, $F^{\mu\nu}$, are connected with the potentials by the relations

$$F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu} \tag{3}$$

1.2 Lienard-Wiechert Potential

Some particular solutions of Eqs. (3) were first considered by Lienard and Wiechert. These solutions are of the form

$$A^{\nu} = \int \frac{\rho(x_{\mu}, x_{\nu})}{\Delta x_{\mu}} d^{\mu} x_{\mu} \tag{4}$$

$$A^{\mu} = \int \frac{j(x_{\mu}, x_{\nu})}{\Delta x_{\mu}} d^{\mu} x_{\mu} \tag{5}$$

Where ρ and J are the charge and current density by a charge e moving in Minkowski space under world-line.
They can be expressed in (x, t) for Euclidean space in terms of delta function as follows[2]

$$\rho(\boldsymbol{x}, t) = e\delta(\boldsymbol{x} - \boldsymbol{x}(t)) \tag{6}$$

$$\boldsymbol{j}(\boldsymbol{x}, t) = e\boldsymbol{v}(t)\delta(\boldsymbol{x} - \boldsymbol{x}(t)) \tag{7}$$

$\boldsymbol{x}(t)$ is the position of the particle at time t and $\boldsymbol{v}(t) \equiv \frac{d\boldsymbol{x}(t)}{dt} \equiv \dot{\boldsymbol{x}}(t)$ is its velocity. The 4-potentials A_{ν} and A^{μ} are, in fact the scalar and vector potentials respectively of an electromagnetic field.

1.2.1 Retarded Potentials

With the help of Jacobian transformation and Dirac delta function, the volume integration can be carried out immediately in Eqs. (4) and (5) with the results in terms of retarded time[3]

$$\phi(\boldsymbol{r}, t) = e \int \frac{1}{|\boldsymbol{r} - \boldsymbol{r_s}(t')|} \delta[f(t')] dt' \tag{8}$$

$$A(\boldsymbol{r}, t) = e \int \frac{\boldsymbol{v}(t')}{|\boldsymbol{r} - \boldsymbol{r_s}(t')|} \delta[f(t')] dt' \tag{9}$$

$\boldsymbol{r}(t)$ is a point in the field and $\boldsymbol{r_s}(t)$ is the position of the source of the field at time t with t' being the retarded time, the time when the field generated to get to the point $\boldsymbol{r}$.
and $f(t')$ is defined by,

$$f(t') = t' - t + \frac{|\boldsymbol{r} - \boldsymbol{r_s}(t')|}{c} \tag{10}$$

After performing time integration, we finally obtain

$$\phi(\boldsymbol{r},t) = \frac{e}{\kappa(t')|\boldsymbol{r}-\boldsymbol{r_s}(t')|} \tag{11}$$

$$A(\boldsymbol{r},t) = \frac{e\boldsymbol{v}(t')}{\kappa(t')|\boldsymbol{r}-\boldsymbol{r_s}(t')|} \tag{12}$$

where

$$\kappa(t') = 1 - \boldsymbol{n}(t').\boldsymbol{\beta}(t')$$

$\boldsymbol{n}(t')$ is a unit vector along the relative distance $\boldsymbol{r}-\boldsymbol{r_s}(t')$ and $\boldsymbol{\beta} = \boldsymbol{v}/c = \dot{\boldsymbol{r}}_s/c$

$$\boldsymbol{n}(t') = \frac{\boldsymbol{r}-\boldsymbol{r_s}(t')}{|\boldsymbol{r}-\boldsymbol{r_s}(t')|} = \frac{\boldsymbol{R}}{R}$$

1.2.2 Electric and Magnetic Fields

Having found the potentials, we are now ready to calculate the electromagnetic fields due to a moving charge. According to Eqs. (3) the electric and magnetic field can be evaluated from the following relations

$$\boldsymbol{E}(\boldsymbol{r},t) = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial t} \tag{13}$$

$$\boldsymbol{B}(\boldsymbol{r},t) = \boldsymbol{n}(t') \times \boldsymbol{E}(\boldsymbol{r},t) \tag{14}$$

Substituting Eqs. (11) and (12) into Eqs. (13) gives

$$\boldsymbol{E}(\boldsymbol{r},t) = e\left[\frac{1-\boldsymbol{\beta}^2}{\kappa^3 R^2}\big(\boldsymbol{n}-\boldsymbol{\beta}\big) + \frac{1}{c\kappa^3 R}\boldsymbol{n} \times \Big[\big(\boldsymbol{n}-\boldsymbol{\beta}\big) \times \dot{\boldsymbol{\beta}}\Big]\right]_{f(t')=0} \tag{15}$$

The solution for electric field from the retarded potentials requires that $f(t')$ be 0. So, the retarded time t' can be found from Eqs. (10) as

$$t' = t - \frac{R}{c}$$

It is notable that these retarded potentials take into account the relativistic effect of finite speed, stating that no field can propagate with a speed more than c setting the universal speed limit to the speed of light.

1.3 Equations of Motion

When a charge moves within the range of an electric and a magnetic field, it interacts with the fields and thus experiences a force (given by Lorentz force)

$$\boldsymbol{F}_L = e\big[\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\big] \tag{16}$$

This force accelerates the charge according to Newton's second law of motion which also holds true for relativistic mechanics, thus

$$\begin{aligned} \frac{d\boldsymbol{p}(\boldsymbol{r},t)}{dt} &= \boldsymbol{F}_L \\ \Rightarrow \frac{d}{dt} m_0 \gamma(\boldsymbol{v})\boldsymbol{v} &= \boldsymbol{F}_L \end{aligned}$$

It can be shown that from the equation above, that the acceleration

$$\ddot{\boldsymbol{r}}(\dot{\boldsymbol{r}}, \boldsymbol{r}, t) = \frac{1}{m_0\gamma(\dot{\boldsymbol{r}})} \left(\boldsymbol{F}_L - \frac{(\dot{\boldsymbol{r}} \cdot \boldsymbol{F}_L)\dot{\boldsymbol{r}}}{c^2} \right) \tag{17}$$

For numerical convenience it is necessary to express this 2nd order ordinary differential equation into a bunch of 1st order differential equations.

$$\begin{aligned} \boldsymbol{y}_1 &= \boldsymbol{r} \\ \boldsymbol{y}_2 &= \frac{d}{dt}\boldsymbol{y}_1 \\ \frac{d}{dt}\boldsymbol{y}_2 &= \frac{1}{m_0\gamma(\dot{\boldsymbol{r}})} \left(\boldsymbol{F}_L - \frac{(\dot{\boldsymbol{r}} \cdot \boldsymbol{F}_L)\dot{\boldsymbol{r}}}{c^2} \right) \end{aligned} \tag{18}$$

2 Numerical Analysis

2.1 Numerical Modeling

To get the picture of the electromagnetic two body problem we need to solve the differential equations of motion found in the previous section. We are interested in the motion or trajectory of the two body system. Therefore, our problem reduces to a initial value problem.
There exists a couple of methods to obtain solution from a differential equation numerically. The Runge-Kutta family of methods are the most popular for both their numerical accuracy and computational efficiency. We will use the 4^{th} order Runge-Kutta method for our purpose.
Runge-Kutta method is basically a modified predictor-corrector method that utilizes several predictor and corrector steps adding them together (multiplied by some coefficients) with the last known value of solution function. In the case of vectors these corrector values can be interpreted as velocities with which the function approaches to gain the next value of solution. Even though we are working with retarded potentials, as a precaution, we have introduced the relativistic velocity addition formula to these corrector velocities.

2.1.1 Computational Stages

1. Provide the charges and masses of the bodies and initiate the system with their position in our coordinate system and velocities at initial time.
2. Calculate the retarded time, t' from Eqs. (10) for $f(t') = 0$ with secant mehthod.
3. Calculate the Lorentz force from Eqs. (16)
4. Solve the differential eqs. (18) with $4th$ order Runge-Kutta method
5. Repeat from stage 2 for both particles

We assumed the potentials as being the simple Coulomb-like for first 3 time steps since retarded time might have no meaning there.

2.2 Numerical Results

We now present some of the cases we achieved by this modeling.
We have taken a proton and an electron as our two body system. In all the cases the proton is at the origin with zero velocity. The initial position of the electron is given in polar co-ordinates which is transformed to our numerically discretized cartesian co-ordinate. We have used Hertree atomic units[4] to define the physical constants such as light velocity and to calculate all the quantity in our model.
Electron is indicated by red dots and proton, by blue dots.

2.2.1 Synchrotron Radiation

- With initial electron velocity, $v_e = (-0.08c, -0.0008c, 0.00001c)$ and position, $(r, \theta, \phi) = (0.005, \frac{\pi}{4}, \frac{\pi}{4})$

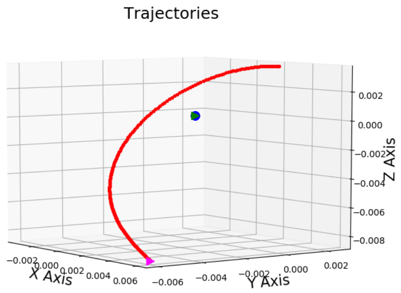

Figure 1: Electron approaches in a circular orbit around proton

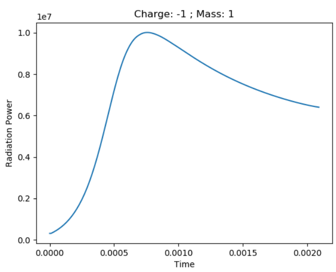

Figure 2: The radiated power by electron during this motion

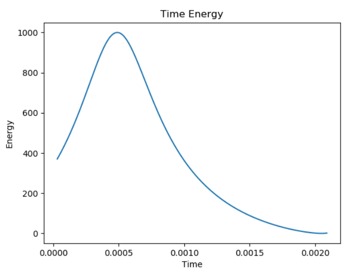

Figure 3: The relativistic energy of the system over time

- With initial electron velocity, $v_e = (-0.08c, -0.0008c, 0.00001c)$ and position, $(r, \theta, \phi) = (0.05, \frac{\pi}{4}, \frac{\pi}{4})$

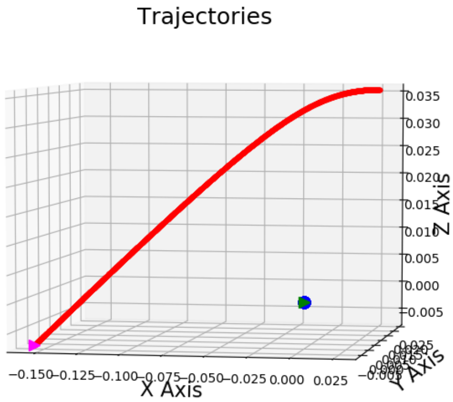

Figure 4: Electron approaches in almost a circular orbit around proton

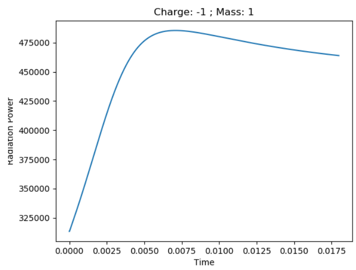

Figure 5: The radiated power by electron during this motion

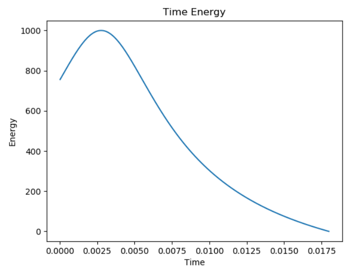

Figure 6: The relativistic energy of the system over time

In both the cases the radiation curve is very similar to synchrotron radiation[5][3][6]. Charged particles in a magnetic field radiate because they experience an acceleration perpendicular to the field. If the particles are non-relativistic, the radiation is referred to as cyclotron emission. When the particles are relativistic, however, the radiation is referred to as synchrotron emission[5].

2.2.2 Bremsstrahlung Radiation

With initial electron velocity, $v_e = (0.001c, -0.008c, -0.001c)$ and position, $(r, \theta, \phi) = (0.005, \frac{\pi}{4}, \frac{\pi}{4})$

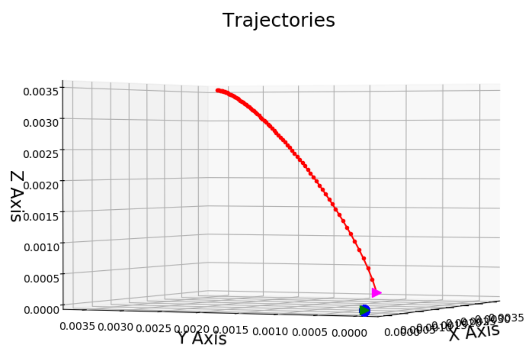

Figure 7: Electron directly approaches to proton

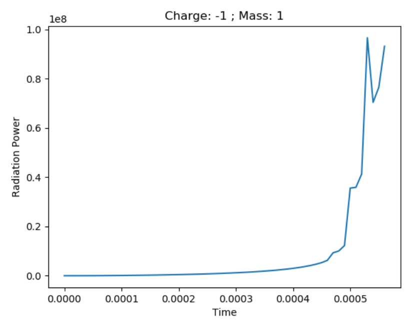

Figure 8: The radiated power by electron during this motion

When a high speed electron encounters the Coulomb field of another charge, it emits bremsstrahlung radiation, also known as free-free emission. The word bremsstrahlung means braking radiation because the electron rapidly decelerates when the other charge is a massive ion. We can relate this fact to our numerical result.

2.2.3 Periodic Orbit

With initial electron velocity, $v_e = (-0.08c, -0.0008c, 0.00001c)$ and position, $(r, \theta, \phi) = (0.004, \frac{\pi}{4}, \frac{\pi}{2})$

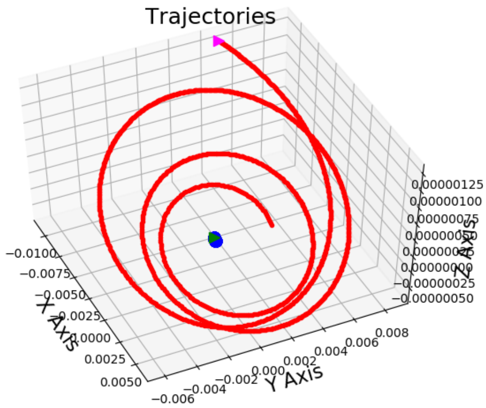

Figure 9: Electron in a periodic orbit around proton with outward shift.

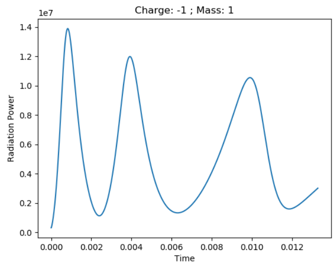

Figure 10: The radiated power by electron during this motion

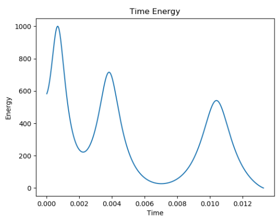

Figure 11: The relativistic energy of the system over time

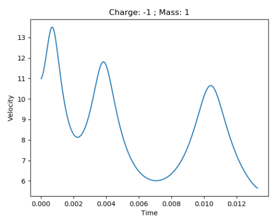

Figure 12: Velocity of electron over time

2.2.4 Electron being repelled away from proton

- With initial electron velocity, $v_e = (0.01c, -0.01c, -0.01c)$ and position, $(r, \theta, \phi) = (0.05, \frac{\pi}{4}, \frac{\pi}{4})$

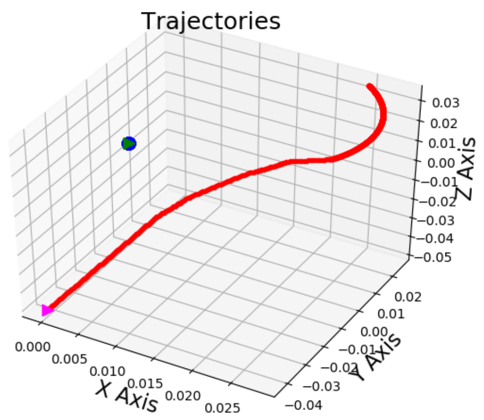

Figure 13: Electron being repelled away from proton

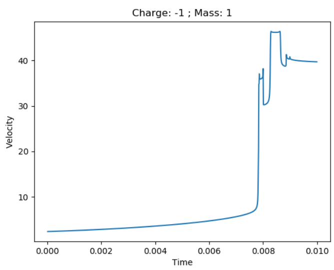

Figure 14: Velocity of electron over time

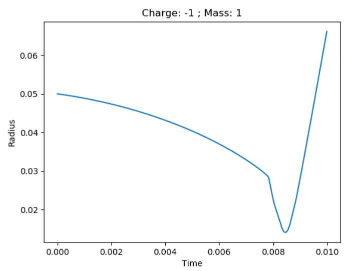

Figure 15: Distance between proton and electron over time

Fig. 15 shows the distance between two bodies over time. It is clearly noticed that as the electron approaches the proton, distance gets shorter, and all of a sudden electron achieves an acceleration normal to the surface of effective radius, shooting the electron away from proton.

- With initial electron velocity, $v_e = (0.01c, -0.01c, -0.01c)$ and position, $(r, \theta, \phi) = (0.005, \frac{\pi}{4}, \frac{\pi}{4})$

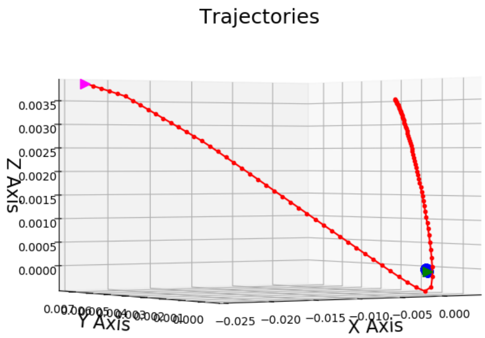

Figure 16: Electron being repelled away from proton

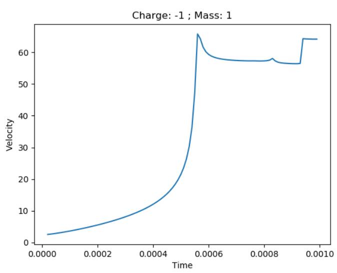

Figure 17: Velocity of electron over time

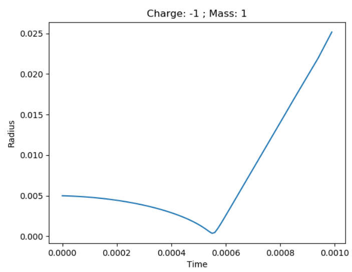

Figure 18: Distance between proton and electron over time

This case also shows the same repulsion behaviour similar to the previous one. Upon reaching a critical point, velocity of electron increases rapidly, moving it away from proton.

3 Discussion

Our numerical model briefed in section 2.1 has been applied to a two body system consisting of electron and proton, thus can be considered as a Hydrogen atom.
With the classical picture of the Hydrogen atom, that is, an electron revolving in an orbit around a proton, if one takes into account the effect of radiation damping, then one would assume that, owing to loss of energy by radiation, the electron would approach the nucleus and eventually falls into it. Our model did not exhibit any such case at all. Moreover, the results reveal several features which appear to be at variance with familiar ideas of physics. To investigate the reason for these abnormality we first go on to explore the existing mathematical models used for classical electrodynamics, which can be classified in three different groups.[7]

1. In the simplest case each particle is influenced solely by the retarded fields produced by other particles.
2. Another model has each particle moving under the influence of the retarded fields from the other particles plus its own "radiation reaction" field. Within this category, various different formulations for radiation reaction have been proposed.
3. The third alternative theory does not include any explicit radiation reaction terms. Instead it assumes that each particle is influenced by both retarded and advanced fields from the others.

To understand these models we first ought to perceive the concepts of "radiation reaction" and "advanced fields".

3.1 Radiation Reaction

If an uncharged body is accelerated by an external force, then an inertial force of same strength but opposite direction is opposing the acceleration. If the accelerated body is electrically charged, then an additional force is opposing the external accelerating force. This additional force is called radiation back-reaction. The external accelerating force must do additional work, because the accelerated charged body is immediately radiating a part of the acquired energy in form of electromagnetic waves[8]. Abraham and Lorentz computed the (non-relativistic) radiation back-reaction force

$$\mathbf{F}_{rad} = \frac{2e^2\ddot{\mathbf{v}}}{3c^3 4\pi\epsilon_0} \quad if \quad v \ll c \tag{19}$$

They approached this result by two different methods: One method was based on considerations with regard to energy conservation. Their second computation, in which they considered in particular an accelerated electron, was based on a classical model of the electron: They assumed the electron to be a sphere with radius of about $3 \times 10^{-15}m$, and they demonstrated that the retarded force, which the electron is exerting onto itself by means of the fields which it is radiating, just has the value (19). Both derivations are presented in very detail in [9].

3.1.1 Maxwell-Lorentz Theory

Lorentz first considered this radiation back reaction force and came up with the equation of motion for electron[10]

$$m\dot{\mathbf{V}} - \frac{2e^2}{3c^3}\ddot{\mathbf{V}} = e\Big[\mathbf{E} + \frac{\mathbf{V}}{c} \times \mathbf{H}\Big] \tag{20}$$

This method, has been only partially successful and has encountered difficulties in the process of second quantization, thus arises infinities[1]. It is a non-relativistic theory and can not be made relativistic in a starightforward manner, since 'size' is not a relativistically invarient concept and a sphere in one frame of reference wll not be a sphere in another Lorentz frame. Furthermore, the theory does not give a stable model of electron, since any finite charge distribution would explode if acted upon by purely electromagnetic forces, the different parts of the electron repelling each other according to Coulomb law.

3.1.2 Dirac's Theory

Dirac later found another way out to deal with the infinities in Maxwell-Lorentz theory and proposed a theory that agrees with well established principles, such as the principle of relativity and the principles of conservation of energy and momentum. The reaction of the radiation field on the motion of the electron is effectively taken into account.
The equation of motion for a single electron according to Dirac's theory[1]

$$m\dot{\nu}_\mu - \frac{2}{3}e^2(2k+1)\Big[\frac{d\dot{\nu}_\mu}{ds} + \dot{\nu}^2\nu_\mu\Big] = e\nu_\sigma F^{\sigma}_{\mu\, ext} \quad \nu^2 \equiv \nu_\mu\nu^\mu = 1 \tag{21}$$

The sign of radiation reaction terms depends on that of $2k + 1$. Consequently, by considering the various cases $2k+1 > 0$, $2k+1 = 0$, $2k+1 < 0$ separately, we would have different types of radiation fields.
But when Dirac's theory has been applied to various problems[11], it appears that several of them are in conflict with the familiar ideas of physics. The self-accelerating motions of a free electron, the artificial nature of the only physically allowable solution of the problem of an electron that is disturbed by a pulse of electromagnetic radiation, the inability of the electron in the Hydrogen atom to spiral inwards and fall into the nucleus and the absence of a physically allowable solution of the problem of an electron moving in the field of a thin infinite-charged plate, all appear to suggest that in some respects the Lorentz-Dirac equations are unsatisfactory.
Moreover, it has been shown that with whatever initial velocity the electron may be projected towards the proton, the electron would be brought to rest before it could reach the proton. The electron then turns back and moves away from the proton with velocity and acceleration which keep on increasing. Ultimately, the electron escapes to infinity. **Point to be noted that, our discussed numerical model based on Lienard-Wiechert retarded potential has a feature similar to this scenario**.

3.2 Advanced Field

Recall the Eqs. (3), (1) and (10)

$$\frac{\partial A_\mu}{\partial x_\mu} = 0, \qquad \Box A_\mu = 4\pi j_\mu$$

$$F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$$

$$f(t') = t' - t + \frac{|\boldsymbol{r} - \boldsymbol{r_s}(t')|}{c}$$

All of these equations are symmetric in time which indicates that they would not loose integrity and physical meaning in a universe where time went backwards in conventional sense. If we swap t and t' in Eqs. (10), all of the field equations still make sense and provide information about the particles in motion in which case we would get another set of equations of motion in terms of the following f'

$$f'(t') = t' - t - \frac{|\boldsymbol{r} - \boldsymbol{r_s}(t')|}{c} \tag{22}$$

and the electric field in terms of $f'(t')$

$$\boldsymbol{E}'(\boldsymbol{r},t) = e\left[\frac{1-\boldsymbol{\beta}^2}{\kappa^3 R^2}(\boldsymbol{n}+\boldsymbol{\beta}) + \frac{1}{c\kappa^3 R}\boldsymbol{n}\times\left[(\boldsymbol{n}+\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right]_{f'(t')=0} \tag{23}$$

where,

$$\kappa(t') = 1 + \boldsymbol{n}(t')\cdot\boldsymbol{\beta}(t')$$

It has been discussed earlier in sec. 1.2.2 that the solution of the fields require that $f'(t') = 0$ providing us with $t' = t + \frac{|\boldsymbol{r}-\boldsymbol{r_s}(t')|}{c}$. Which is clearly an advanced (future) time relative to the corresponding event in space-sime and the physical meaning for E' would be the field originated from future motion of the particle. The potentials for this advanced field identified as advanced potentials are another solution set for Eqs. (4) and (5).

The implication of the advanced field can be derived from Dirac's relativistically generalized equations of motion described in sec. 3.1.2. Eqs. (21) shows that the effective field on an electron is the $f_{\mu\sigma}$ field. It is then safe to assume that the electric field of a moving electron consists of two parts:

$$\frac{1}{2}\Big(E + E'\Big) \tag{24}$$

$$(k + \frac{1}{2})\Big(E - E'\Big) \tag{25}$$

If this assumption is considered and the Lorentz-Dirac equations is obtained by taking the field of an electron to be a combination of retarded and advanced fields with the term $2k+1$ being negative, the the application of it[1] show that there are many respects in which these equations give results in harmony with our usual notion of physics. What corresponds to self-acceleration in the Lorentz-Dirac case would now correspond to self-retardation and what corresponds to emission of radiation would now be absorption of radiation.

It is somewhat surprising that a theory which uses advanced potentials and therefore the physical mechanism is by no means clear, the equations derived from the theory lead to physically understandable results, whereas when the theory uses retarded potentials alone, the corresponding equations lead to many unexpected results. **We, however, when introduced advanced field in our numerical model did not get any different result than of the case where only retarded field is considered. The non-physical abnormality shown previously still remains. This might be because of the fact that future motion corresponds to the motion, calculated from only the retarded fields, in our model.**

3.3 Wheeler-Feynman Absorber Theory

Considering the case when $2k+1=0$ or, $k=-\frac{1}{2}$ the equation of motion for a single charge becomes, following Eqs. (21)

$$m\dot{\nu}_{\mu} = e\nu_{\sigma}F^{\sigma}_{\mu\ ext} \tag{26}$$

It can be noted that this equation does not contain the usual radiation reaction terms. Thus for a single electron problem this equation is the same as that in a theory which neglects radiation damping altogether. Hence this theory would allow the existence of stationary states unlike the other theories in which it is impossible for an electron in the presence of an electromagnetic field to visit the same orbit continually.
Wheeler and Feynman considered this case for $k=-\frac{1}{2}$ to get rid of the usual, third order time derivative radiation back reaction term and introduced a new approach[8] (which they named Absorber theory) to allow for radiation damping to come into appropriate place. Moreover, the Abraham-Lorentz derivation of radiation back-reaction force assumed the electron to be a sphere which is in violation with our modern day concept of elementary particles. Feynman and Wheeler corrected this, accepting electron as a zero-radius point-charged-particle. Their approach is explained below[8].

Let a particle with charge q being accelerated by $\dot{\mathbf{v}}$ at the retarded time $t' = t - \frac{R}{c}$ at position $\mathbf{r}$. The accelerated particle is emitting the retarded fields E and B as indicated in Eq. (15). Let another particle with charge q_k being in interaction with E and B, thus experiencing a Lorentz force originated from the retarded fields.
Now Wheeler and Feynman assume, that the accelerated point charge q_k will emit not only a retarded field, but also an advanced field of same strength. Thereby, the retarded and advanced fields shall have only half the amplitude of the fields. The advanced fields radiated by q_k move backwards through time and arrive at the position $\mathbf{r}$ of the primary source q just at the time t', at which this source is radiating the retarded fields.
If the radiation emitted by q is absorbed by many particles q_k at different distances from q, and if the particles q_k radiate advanced fields, then these fields will superpose to a total field at the position $\mathbf{r}$ of q. It is the central idea of absorber theory, that the retarded field radiated by q will sooner or later be completely absorbed by other particles q_k. Each absorber particle q_k thereupon emits retarded fields and advanced fields of same strength. The superposition of all advanced fields arriving at the primary source q at time t' then shall bring about the radiation-reaction force acting upon q.

4 Conclusion

In the sense of classical (as distinguished from quantum mechanics) electrodynamics, when an electron and a proton is taken as a two body system (much like a Hydrogen atom), the electron should approach the proton closer ans closer until it falls into it for the loss of energy due to radiation damping. Apparently this does not occur in nature because electron is not allowed to have a continuous energy spectrum, so it stays on a stable orbit where it has an allowable energy.

However, even without any quantum consideration, relativistically derived equation of motion with retarded Lienard-Wiechert potentials fails to demonstrate the classical picture of Hydrogen atom. Taking a combination of retarded and advanced fields seems to be in conformity with our present day physical ideas according to Elizer and C.J's work on this very same problem[1]. In which case we would need a radiation reaction force, for radiation damping of moving charge interacting with an external field, because it is not possible to get an accurate description of a non-independent (independent as in sense of external influence) moving charge by taking only Lienard-Wiechert into consideration which is demonstrated from our numerical model. These retarded potentials can safely be applied to predict a large (non-quantum mechanical), free charge. In our model, we calculated the radiation terms independently for a single charge, therefore have achieved some results resembling the physical radiation such as Synchrotron radiation.

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