

Tiny :3
(Note: for the Amplitude is the absolute value of the coefficient)
(note: x + c can be rewritten as x-(c) with -c being the phase shift)
(Note: if the Amplitude is negative flip it upside down)
(note: if the graph goes downward at the start its negative, if it goes up its positive)
If I have to find y(1.3) or something then plug 1.3 into x in the equation to get the answer
Set size = tick and phase shift = start

Transforming points

how to get all 5 transformation points
For first point use C as X and the and put the Y as the starting point
then add 1/4 of the period to C every time I go to the next point and go one amplitude up or down

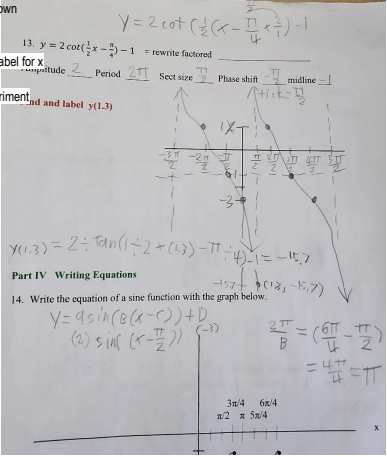
graph to equation
Find

- A: find out how many units the graph goes up from the bottom and divide it in half
 - B: How long does one cycle of the graph go for then set that equal to $\frac{2\pi}{B}$
 - C: phase shift = horizontal distance from origin to middle
 - D: how many units does the midline of the graph move up or down
- To find the frequency from a period you just $\frac{1}{\text{period}}$

to graph sec and csc functions you can
if graphing csc first graph sin
if graphing sec first graph cos
if A is negative just flip it upside down
draw U shaped lines that have their center at the max and min points and the lines go extend outward until the nearest inflection points
and when writing ranges you do $(-\infty, x] \cup [x, \infty)$
with x being the usual ranges
Note: a problem like $y=2+4\sec(2x)$ the +2 is D not A and A = 4
If A is negative draw it upside down

LABEL GRAPH

Use the phase shift as the first label for x
Then divide the period by 4 and
add 1/4 of the period every increment
To convert the period (p)
you can do $B = \frac{2\pi}{p}$



	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
SIN	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{4}}{2} = 1$
COS	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	0
Tan	0	$\frac{\sqrt{3}}{3}$	1	undefined

Degrees	Radians	Coordinates (x, y)
30°	$\frac{\pi}{6}$	$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
45°	$\frac{\pi}{4}$	$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
60°	$\frac{\pi}{3}$	$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
90°	$\frac{\pi}{2}$	(0, 1)
120°	$\frac{2\pi}{3}$	$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$
135°	$\frac{3\pi}{4}$	$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
150°	$\frac{5\pi}{6}$	$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$
180°	π	(-1, 0)
210°	$\frac{7\pi}{6}$	$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$
225°	$\frac{5\pi}{4}$	$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
240°	$\frac{4\pi}{3}$	$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$
270°	$\frac{3\pi}{2}$	(0, -1)
300°	$\frac{5\pi}{3}$	$(\frac{1}{2}, -\frac{\sqrt{3}}{2})$
315°	$\frac{7\pi}{4}$	$(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
330°	$\frac{11\pi}{6}$	$(\frac{\sqrt{3}}{2}, -\frac{1}{2})$
360°	2π	(1, 0)

- A = 2 (if nothing is there it is 1)
- B = 2π (if nothing is there it is 1)
- C = $\frac{1}{2}$ (if nothing is there it is 0)
- D = -1 (if nothing is there it is 0)

To get the **Amplitude** you take the number before the trig function (coefficient) with it being the 2 in $y = 2\cos$
To get the **period** you put $\frac{2\pi}{B}$ and in this case be being 2π resulting in 1 as the period
To get the **Sect Size (tick)** you divide the **period** by 4 so its $\frac{1}{4}$
to get the **Phase shift (start)** you can take C which in this case is $\frac{1}{2}$
to get the **midline** you take D which in this case is -1
to get the **range** is $[D - A, D + A]$ so $[-1 - 2, -1 + 2] = [-3, 1]$
to get the **frequency** is $\frac{B}{2\pi}$ so the reciprocal of the period which in this case is still 1

Property	Symbol	Value	Explanation
Amplitude	A	2	Height from midline to peak
Period	$\frac{2\pi}{B}$	1	Length of one full cycle
Section Size		$\frac{1}{4}$	Period ÷ 4
Phase Shift	C	0.5	Shift right by 0.5
Midline	y=D	y = -1	Vertical shift
Range		[-3, 1]	From minimum to maximum y-value
Frequency	$\frac{B}{2\pi}$	1	

finding asymptotes

how to find vertical asymptotes for sec and csc functions

- rewrite the function using reciprocal identity
 - $\sec x = \frac{1}{\cos x}$
 - $\csc x = \frac{1}{\sin x}$
- set the denominator = 0 (where the function is undefined)
 - For sec: $\cos x = 0$
 - For csc: $\sin x = 0$
- solve for x
 - $\cos x = 0 \rightarrow x = \frac{\pi}{2} + n\pi$
 - $\sin x = 0 \rightarrow x = n\pi$ (where n is any integer)
- if there's a B or C inside (like $y = \sec(Bx - C)$ or $y = \csc(Bx - C)$) solve
 - $\cos(Bx - C) = 0 \rightarrow \sin(Bx - C) = 0$
 - $\Rightarrow Bx - C = \frac{\pi}{2} + n\pi$ or $Bx - C = n\pi$
 - then $\Rightarrow x = \frac{C + \frac{\pi}{2} + n\pi}{B}$ or $x = \frac{C + n\pi}{B}$
- draw **dotted vertical lines** at those x-values — that's where asymptotes go

tips for sec and csc graphs

- For sec: first graph cosine, then draw U and n shapes centered on the max and min points of cosine
- For csc: first graph sine, then draw U and n shapes centered on the max and min points of sine
- the graph never crosses the asymptotes
- each "U" or "n" section repeats every **period**

example

for $y = 12\sec x$
 $\rightarrow \cos x = 0$ at $x = \frac{\pi}{2} + n\pi$
vertical asymptotes at $x = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$

You can do this !!!!

How to get frequency (cycles/sec) from $y = \sin(Bx + \dots)$ or $y = \cos(Bx + \dots)$

Key idea:

If the function is written as $y = \sin(Bx + \dots)$ or $y = \cos(Bx + \dots)$, the coefficient B is the angular frequency (in radians per second). The ordinary frequency (cycles per second Hz) is

$$f = \frac{\omega}{2\pi} = \frac{B}{2\pi}$$

Phase shifts, vertical shifts, and amplitude do not change the frequency.

Period T (seconds per cycle) is the reciprocal:

$$T = \frac{1}{f} = \frac{2\pi}{B}$$

Checklist (quick)

1. Identify the coefficient B multiplying x inside the trig argument (make sure it's in radians).
2. Compute $f = \frac{B}{2\pi}$.
3. Optionally compute period $T = 1/f = \frac{2\pi}{B}$.
4. Phase shift and amplitude don't affect frequency.

Worked examples

Example A - $y = \sin(200\pi x)$

1. Inside is $200\pi x$ so $B = 200\pi$ (radians per second).
2. Frequency:

$$f = \frac{B}{2\pi} = \frac{200\pi}{2\pi} = 100 \text{ Hz}$$

3. Period:

$$T = \frac{1}{f} = \frac{1}{100} = 0.01 \text{ seconds}$$

Answer: $f = 100 \text{ Hz}$, $T = 0.01 \text{ s}$

Example B - $y = 2 \cos(17x + 3) - 2$

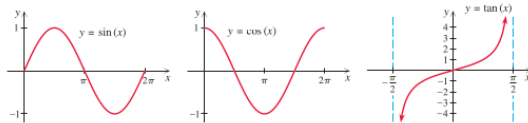
1. Inside is $17x + 3$. Factor: coefficient of x is $B = 17$ (radians per second). The "3" is a phase shift - it doesn't change frequency.
2. Frequency:

$$f = \frac{B}{2\pi} = \frac{17}{2\pi} \approx 2.69 \text{ Hz}$$

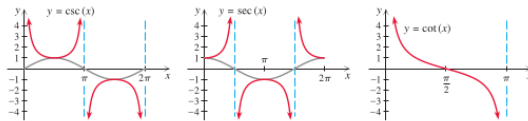
3. Period:

$$T = \frac{1}{f} = \frac{2\pi}{17} \approx 0.37 \text{ seconds}$$

Answer: $f \approx 2.69 \text{ Hz}$, $T \approx 0.37 \text{ s}$



Domain (k any integer)	$(-\infty, \infty)$	$(-\infty, \infty)$	$x \neq \frac{\pi}{2} + k\pi$
Range	$[-1, 1]$	$[-1, 1]$	$(-\infty, \infty)$
Period	2π	2π	π
Fundamental cycle	$[0, 2\pi]$	$[0, 2\pi]$	$[\frac{\pi}{2}, \frac{3\pi}{2}]$



Domain (k any integer)	$x \neq k\pi$	$x \neq \frac{\pi}{2} + k\pi$	$x \neq k\pi$
Range	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, \infty)$
Period	2π	2π	π
Fundamental cycle	$[0, 2\pi]$	$[0, 2\pi]$	$[0, \pi]$

$$1. y = 2 \sin(3x - \frac{\pi}{4}) + 1$$

- Factor: $3x - \frac{\pi}{4} = 3(x - \frac{\pi}{12}) \rightarrow C = \frac{\pi}{12}$.
- $A = 2$, $B = 3$, $D = 1$.
- Period = $\frac{2\pi}{3}$. Section = $\frac{\pi}{6}$.
- Phase shift: right $\frac{\pi}{12}$. Midline $y = 1$. Range $[-1, 3]$.

$$2. y = -3 \cos(\frac{1}{2}x + \frac{\pi}{6}) - 2$$

- Factor: $\frac{1}{2}x + \frac{\pi}{6} = \frac{1}{2}(x + \frac{\pi}{3}) \rightarrow C = -\frac{\pi}{3}$ (left $\frac{\pi}{3}$).
- $A = 3$ (negative \rightarrow reflection), $B = \frac{1}{2}$, $D = -2$.
- Period = $\frac{2\pi}{1/2} = 4\pi$. Section = π .

- Midline $y = -2$. Range $[-5, 1]$.

$$3. y = 0.5 \tan(2x - \frac{\pi}{6})$$

- Factor: $2x - \frac{\pi}{6} = 2(x - \frac{\pi}{12}) \rightarrow$ phase shift right $\frac{\pi}{12}$.
- $A = 0.5$ (vertical stretch factor), $B = 2$, $D = 0$.
- Period = $\frac{\pi}{2}$ (since $\tan: \pi/B$). Section (use period/4 = $\pi/8$ if you want quarter points).
- Vertical asymptotes: solve $2x - \frac{\pi}{6} = \frac{\pi}{2} + n\pi \rightarrow x = \frac{5\pi}{12} + \frac{n\pi}{2}$.
- No midline shift ($y=0$).

$$4. y = -2 \cot(\frac{1}{3}x + \frac{\pi}{4}) + 2$$

- Factor: $\frac{1}{3}x + \frac{\pi}{4} = \frac{1}{3}(x + \frac{3\pi}{4}) \rightarrow C = -\frac{3\pi}{4}$.
- $A = 2$ (negative \rightarrow reflection), $B = \frac{1}{3}$, $D = 2$.
- Period for $\cot: \pi/B = 3\pi$. Section = $3\pi/4$.
- Asymptotes: solve $\frac{1}{3}x + \frac{\pi}{4} = n\pi \rightarrow x = 3n\pi - \frac{3\pi}{4}$.
- Midline $y = 2$.

$$5. y = 4 \sec(\frac{\pi}{6}x - \frac{\pi}{3}) - 1$$

- Factor: $\frac{\pi}{6}x - \frac{\pi}{3} = \frac{\pi}{6}(x - 2) \rightarrow$ phase shift right 2.
- $A = 4$, $B = \frac{\pi}{6}$, $D = -1$.
- Period = $\frac{2\pi}{\pi/6} = 12$. Section = 3.
- Asymptotes ($\cos = 0$): $\frac{\pi}{6}(x - 2) = \frac{\pi}{2} + n\pi \rightarrow x - 2 = 3 + 6n \rightarrow x = 5 + 6n$.
- Range: $(-\infty, -5] \cup [3, \infty)$.

$$6. y = -\csc(\frac{2\pi}{5}x + \pi) + 1$$

- Factor: $\frac{2\pi}{5}x + \pi = \frac{2\pi}{5}(x + \frac{5}{2}) \rightarrow$ phase shift left 2.5 (i.e. $C = -2.5$).
- A is effectively 1 (csc amplitude), $B = \frac{2\pi}{5}$, $D = 1$.
- Period = $\frac{2\pi}{2\pi/5} = 5$. Section = 1.25.
- Asymptotes ($\sin = 0$): $\frac{2\pi}{5}(x + 2.5) = n\pi \rightarrow x = 2.5n - 2.5$. So asymptotes at $x = 2.5(n - 1)$ (or list: $\dots, -5, -2.5, 0, 2.5, 5, \dots$ depending on n).
- Range: $(-\infty, 0] \cup [2, \infty)$? Wait - compute $D \pm |A|$: midline 1, amplitude 1 \rightarrow critical y -values for csc are $1 \pm 1 = 0$ and 2 . Because leading minus flips sign, the actual finite extrema will be at $y = 0$ and $y = 2$, but csc never takes values strictly between 0 and 2. So range: $(-\infty, 0] \cup [2, \infty)$.

