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Equally likely events:

The outcomes are said to be equally likely if none of them is expected to occur in preference to other.

Exa:

Independent events: Events are said to be independent, if the occurrence of one does not affect the other event, then they are said to be dependent events.

Exa:- ~~probability of drawing~~ The result of the first toss of a coin does not affect the result of successive tosses at all.

Dependent events:-

If the occurrence of one event affects the happening of the other event, then they are said to be dependent events.

Exa: the probability of drawing an ace from a pack of 52 cards is 4/52. If this card is not replaced before the second draw, the probability of getting an ace again is 3/51 as there are now only 51 cards left & they contain only 3 aces.

1

Probability

Unit V

Statistics is the science of decision making with calculated risks in the face of uncertainty. The theory of probability enables us to take decision under conditions of uncertainty with a calculated risk.

The theory of probability has its origin in the games of chance related to gambling like drawing cards from a pack of cards or throwing of dice, ______ etc.

Terminology

① Random experiment :→

~~Random~~ An experiment E is called a random experiment if when conducted repeatedly under essentially homogeneous conditions, the result is not unique, i.e. it does not give the same result. The result may be any one of the various possible outcomes.

~~Trial & Event~~ :

Exa: ① measuring blood pressure of a group of individuals
② checking an automobile's petrol mileage
③ Tossing a coin & observing the face that appears.
④ Testing a product to determine whether it is defective or an acceptable product.
⑤ measuring daily rainfall, & so on

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Sample space:

The set of all possible distinct outcomes (events) for a random ~~exper~~ experiment is called the sample space (event space) provided.

① no two or more of these outcomes can occur simultaneously.

② exactly one of the outcomes must occur, whenever the ~~exper~~ experiment is performed.

Exa:- ①

Consider the experiments of recording a person's blood type. The four possible outcomes are the following simple events.

E_1 = Blood type A , E_2 : Blood type B

E_3 = " type AB, E_4 : " type O

The sample space is $S = \{E_1, E_2, E_3, E_4\}$.

② Tossing two coin simultaneously

$E_1 = HH$, $E_2 = HT$, $E_3 = TH$. $E_4 = TT$

Complementary Events

If E is any subset of the sample space, then its complement denoted by $\bar{E}$. Contain all the element of the sample space that are not part of E.

$\bar{E} = S - E = \{$all sample elements not in E$\}$

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Theorem of probability

(1) The addition theorem
(2) The multiplication theorem

(1) **Addition theorem:→**

if two events A & B are mutually exclusive (so that if one happens the other cannot happen) the

$$\boxed{\text{probability (A or B)} = \text{probability (A)} + \text{probability (B)}}$$

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

Q:- In a bag there are 4 red balls, 3 white balls, 2 yellow & 1 green ball. if we take out a ball without seeing it, find the probability of coming out of either a red or a white ball.

Sol: probability of a red ball coming out $= \frac{4}{4+3+2+1} = \frac{4}{10}$

probability of a white ball coming out $= \frac{3}{4+3+2+1} = \frac{3}{10}$

probability of coming out of either a red or a white ball $= \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

Q:→ A card is drawn at random from an ordinary pack of playing cards. find the probability that a card drawn is either a spade or the ace of diamond

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Note:- 52 cards

13 cards in each of the four suits.

(I) 13 clubs (चिड़ी)
(II) 13 diamonds (ईंट)
(III) 13 hearts (पान)
(IV) 13 spades (हुकुम)

4 king (राजा / बादशाह)
4 Jack जोकर
4 Ace इक्के
4 Queen (बेगम / रानी)

Sol:- probability of drawing a spade $= \frac{13}{52}$

probability of drawing ace of diamonds $= \frac{1}{52}$

$\therefore$ probability of drawing either a spade or ace of diamonds $= \frac{13}{52} + \frac{1}{52} = \frac{14}{52}$ (or) $\frac{7}{26}$.

Q:- If A, B, C are mutually exclusive & exhaustive events, find $p(B)$ if $\frac{1}{3}p(C) = \frac{1}{2}p(A) = p(B)$

Sol:- Since the events are mutually exclusive & exhaustive

$p(A) + p(B) + p(C) = 1$

Let $p(B) = k$

$\Rightarrow \frac{1}{3}p(C) = k$ & $\frac{1}{2}p(A) = k$

$p(C) = 3k$, $p(A) = 2k$, $p(B) = k$

Hence, $3k + 2k + k = 1 \Rightarrow 6k = 1$, $k = 1/6$.

$\boxed{p(B) = 1/6}$

~~A bag contains~~

multiplication theorem:

when we examine the probability of two or more dependent events occurring successively, the multiplication theorem or law of multiplication is used.

$$\boxed{P(AB) = P(A) \times P(B)}$$

Q:- If we toss a coin twice, what is the probability that both time it will be head upward?

Sol. probability of head upward in the first toss $= 1/2$

probability of head upward in the 2nd toss $= \frac{1}{2}$

proba " " of head upward in both tosses $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Q:- what is the probability of throwing two "fours" in two throws of a dice?

Sol. probability of a four in 1st throw $= 1/6$

probability of a four in 2nd throw $= 1/6$

Hence, probability of throwings two fours in two throws $= 1/6 \times 1/6 = \frac{1}{36}$

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(i) $n(E) = {}^6C_1 = 6.$

probability $= \frac{{}^6C_1}{6^6} = \frac{1}{6^5}$

(i) Combination $= {}^nC_r = \frac{n!}{r!(n-r)!}$

(ii) Permutation $= {}^nP_r = \frac{n!}{(n-r)!}$

(2) $n(E) = 6!$, = The total no. of ways in which all dice show different faces is same as the the No. of arrangements of 6 No.'s 1, 2, 3, 4, 5, 6 by taking all at time $= 6!$

$P = \frac{6!}{6^6}$

~~(iii)~~

Q:- A bag contains 6 red & 8 green balls

(1) if one ball is drawn at random, then what is the probability of the ball being green.?

(2) if two balls are drawn at random, then what is the probability that one is red & the other green?

Sol: (1) Since the bag contains 6 red & 8 green balls, Therefore it contains $6+8 = 14$ equally likely outcomes

i.e. $n(S) = {}^{14}C_1 = \frac{14!}{1!(14-1)!} = 14$ ways.

$n(E) = {}^8C_1 = \frac{8!}{1!(8-1)!} = 8$ ways.

Hence $P(A) = \frac{n(E)}{n(S)} = \frac{\cancel{8}}{\cancel{14}} = \frac{4}{7}$

(2) one red can be drawn $= {}^6C_1 = 6$ ways

one green balls can be drawn $= {}^8C_1 = 8$ ways

$n(S) = {}^{14}C_2 = 91$

$n(E) = {}^6C_1 \times {}^8C_1 = 48.$

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probability

let S be the sample space

& A be a subset of S representing an event.

Then the probability of the event A is defined as

$$P(A) = \frac{\text{Number of elements in } A}{\text{No. of elements in } S} = \frac{n(A)}{n(S)}$$

$$(\text{or})\ P(A) = \frac{n}{m+n} = \frac{\text{No. of favourable outcomes}}{\text{Total No. of possible outcomes}} = \frac{n(A)}{n(S)}$$

Note: (1) $P(\phi) = 0,\ P(S) = 1$ & $0 \le P(A) \le 1$

(2) $$P(\bar{A}) = \frac{\text{No. of elements in } \bar{A}}{\text{No. of elements in } S} = \frac{n(S) - n(A)}{n(S)}$$

$$= 1 - \frac{n(A)}{n(S)} = 1 - P(A).$$

$$\Rightarrow \boxed{P(A) + P(\bar{A}) = 1}$$

Q:→ Six dice are thrown simultaneously. find the probability that

(i) all of them show the same face

(ii) all of them show different faces

(iii) exactly three of them show the same face & remaining three show different faces.

(iv) at least four of them show the same face.

1) The Total No. of element events in sample space S

$$n(S) = 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^6.$$

2

Trial & Event :

Performing of a random expriment is called a 'Trial' and outcome or outcomes are termed as 'events'. for instance, tossing of a coin would be called a Trail & the result (falling head or tail upward) an event.

Exhaustive case :

The total No. of possible outcomes of a random experiment is called the exhaustive cases for the expriment.

Exa: in toss of a single coin, we can get head or Tail. Hence exhaustive No. = 2

Favourable cases or events :

The No. of outcomes which result in the happening of a desired event are called favourable cases.

Exa:- In the drawing a card from a pack of cards, the cases favourable to getting a heart is 13 & getting a king is 4.

Mutually Exclusive events :- Two or more events are said to be mutually exclusive if the happening of any one of them excludes the happening of all others in the same experiment.

Exa: In toss of a coin the events 'head' and tail' are mutually exclusive because if head comes, we can't get tail & if tail comes we can't get head

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Three students A, B & C are given a problem in statistics. The probabilities of their solving the problem are $3/4$, $2/4$ & $1/4$ respectively. What is the probability that if all of them try, the problem would be solved?

Sol:. probability that A will fail to solve the problem

$$= 1 - \frac{3}{4} = \frac{1}{4}.$$

probability that B will fail to solve the problem

$$= 1 - \frac{2}{4} = \frac{2}{4}$$

probability that C will fail to solve the problem

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ probability that all the students A, B & C will fail to solve the problem

$$= \frac{1}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{6}{64}$$

Hence, the probability that if all of them try, the problem would be solved $= 1 - \frac{6}{64} = \frac{58}{64}$ or $\frac{29}{32}$