Unit - I

Measures of central Tendency

- Arithmetic mean or Average
  - Simple
  - Weighted
- Median
- mode
- Geometric mean
- Harmonic mean
- Quartiles, Deciles, percentiles.

Notations used :-

x_i = values of variables or measurments
m_i = mid-point for the i^{th} class in the data set
f_i = No. of observations (or frequency) in the i^{th} class ($i = 1, 2, 3, \dots N$)
N = total No. of observations in the population.
n = Number of observations in the sample (sample size)
l = lower limit of any class interval.
h = width of the class interval
cf = Cumulative frequency
$\sum$ = Summation (sigma) of all values of observations.

Arithmetic Mean of unclassified data

(i) if $x_1, x_2, x_3, \dots x_N$ → values of observations.

Arithmetic mean of the population of N observations $(\bar{x}) = \dfrac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \dfrac{\sum_{i=1}^{N} x_i}{N}$

(ii) for a sample containing n observations $x_1, x_2, \dots x_n$,
the sample AM $= \dfrac{x_1 + x_2 + x_3 \dots + x_n}{n} = \left(\dfrac{\sum_{i=1}^{n} x_i}{n}\right)$

(iv) when observations x_i $(i=1,2,\dots N)$ are grouped as a frequency distribution then A.M

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i f_i}{N}$$

where $f_i \rightarrow$ frequency (No. of observations) & $N = \sum f_i$

shortcut method (Unclassified Data)

- let $x_1, x_2, x_3, \dots x_N$ be observations.
- Arbitrary assumed mean $= A$

Deviation from assumed mean $= (d_i) = x_i - A$

Now $$A.M. = A + \frac{\sum_{i=1}^{n} d_i}{N}$$

for frequencies cases.

$$A.M. = A + \frac{\sum_{i=1}^{n} f_i d_i}{N}$$ where $N = \sum_{i=1}^{N} f_i$

Arithematic mean of classified data

A.M. $\bar{x} = \frac{\sum f_i m_i}{N}$

m_i = mid value of i^{th} class interval

f_i = frequency of i^{th} CI

$n = \sum f_i$ = sum of all freq.

AM $\bar{x} = A + \frac{\sum f_i d_i}{N} \cdot h$

where h = width of class Int.

$\bar{x} = A + \frac{\sum f_i d_i}{N}$

$d_i = \frac{m_i - A}{h}$ = deviation from the assumed mean.

(i) The class intervals must be closed

(ii) The width of each class Interval should be equal.

(iii) The values of observations in each class interval must be uniformaly distributed b/w its Lower and upper limits.

(iv) The mid value of each class interval must represent the average of all values in that class.

Measures of Central tendency

Q.1 : The following table gives the marks obtained by ten students in a class test

Roll No. –	1	2	3	4	5	6	7	8	9	10
marks –	2	8	3	10	9	4	3	7	2	6

Calculate average marks secured by students.

Q(2): In a survey of cement companies, the profit (in Rs. Lakh) earned during year was 15, 20, 10, 35 and 32. find the arithmetic mean of the profit earned.

Q-3: if A, B, C and D are four chemicals costing (in Rs) 15, 12, 8 and 5 per 100g respectively, and are contained in a given compound in the ratio of 1, 2, 3 and 4 parts respectively then what should be the price of the resultant Compound.

$f_i \rightarrow 1, 2, 3, 4.$

Q(4) The number of new orders received by a company over the last 25 working days were recorded as : 3, 0, 1, 4, 4, 4 2, 5, 3, 6, 4, 5, 1, 4, 2, 3, 0, 2, 0, 5, 4, 2, 3, 3, 1

Calculate the A.M. for the No. of orders received over all similar working days.

No. of orders	(By Frequency)

Q(5): from the following information on the No. of defective Components in 1000 boxes.

No. of defective Components :	0	1	2	3	4	5	6
No. of boxes :	25	306	402	200	51	10	6

Calculate the A.M. of defective Components for the whole of the production

Q:-(6) The following tables gives the marks obtained by students in an examination.

marks —	1	2	3	4	5	6	7
No. of stu. —	1	2	4	3	6	3	1

Calculate average marks secured by students.

Q(7) The daily earnings (in Rs) of employees working on a daily bases in a firm are.

Daily earnings (Rs) :	100	120	140	160	180	200	220
Number of employees :	3	6	10	15	24	42	75

Calculate the average daily earning for all employees.

Q:8: The following distribution gives the pattern of overtime work done by 100 employees of a company. Calculate the average overtime work done per employee

overtime (hrs)	10-15	15-20	20-25	25-30	30-35	35-40
Number of employees	11	20	35	20	8	6

Q:9: The average rainfall for a week, excluding Sunday was 10cm. Due to heavy rainfall on Sunday, the average for the week rose to 15 cm. How much rainfall was there on Sunday?

Q: Find out combined mean from the following data

AM	Series x	Series y
No. of items	12	20
	80	60

The mean monthly salary paid to all employees in certain company was Rs. 3000. The mean monthly salary paid to male and female employees were Rs. 3200 & Rs. 2200 respectively obtain the percentage of male and female employees in company.

Case①:. frequencies are given in Cumulative form (more than or less than)

Q①:. Length (in meters) (more than) : 1.0 1.5 2.0 2.5 3.0 3.5
preference of housewives : 50 48 42 40 10 5
find average length of slabs..?

Q②:→ In an examination of 675 candidates, the examiner supplied the following information:
marks obtained (%) (Less than): 10 20 30 40 50 60 70 80
No. of candidates : 7 39 95 201 381 545 631 675

Case②:- frequencies are not given but have to be calculated from the given data.

Q③:. 168 handloom factories have the following distribution of average No. of workers in various income groups.

Income groups :	800–1000	1000–1200	1200–1400	1400–1600	1600–1800
No. of firms :	40	32	26	28	42
Average No. of workers :	8	12	8	9	4

find the mean salary paid to the workers.

Case③:. Complete data are not given :-

Q④:. The pass result of 50 students who took a class test is given below:

marks :	40	50	60	70	80	90
No. of students :	8	10	9	6	4	3

If the average marks for all the students was 51.6. find out the average marks of the students who failed.

~~Problem~~

Solution of Problem

1 ⇒ ~~The following tables gives~~

Roll No.	marks (x_i)	(Deviation from assumed mean A = 5) (d_i)
1	2	−3
2	8	+3
3	3	−2
4	10	+5
5	9	+4
6	4	−1
7	3	−2
8	7	+2
9	2	−3
10	6	+1
Total	54	+4

By Direct Method,

$$AM = \frac{\sum x_i}{N} = \frac{54}{10} = 5.4 \text{ marks.}$$

By deviation method,

$$AM = 5 + \frac{\sum d_i}{10} = 5 + \frac{4}{10} = 5.4 \text{ marks.}$$

Q (5):

No. of Defective Components	Number of Boxes (f_i)	$f_i x_i$
0	25	0
1	306	306
2	402	804
3	200	600
4	51	204
5	10	50
6	6	36
Total.	1000	2000

$$AM = \frac{1}{n}\sum_{i=0}^{6} f_i x_i = \frac{1}{1000} \times 2000 = 2 \text{ defective Components.}$$

Q(6)..

marks (x_i)	No. of stu. (f_i)	Total marks ($f_i x_i$)	Dev. from ass. average ($x=4$) d_i	Product of Dev. & freq $f_i d_i$
1	1	1	−3	−3
2	2	4	−2	−4
3	4	12	−1	−4
4	3	12	0	0
5	6	30	+1	+6
6	3	18	+2	+6
7	1	7	+3	+3
Total	20	84		+4

By Direct method

$$AM = \frac{1}{n}\sum_{i=1}^{7} f_i x_i = \frac{84}{20} = 4.2 \text{ marks}$$

By deviation method.

$$AM = x + \frac{\Sigma f_i d_i}{n} = 4 + \left(\frac{+4}{20}\right) = 4.2 \text{ marks.}$$

Q(7): let us assumed mean A = 160.

Daily earnings (Rs)	No. of employees	$d_i = x_i - A = x_i - 160$	$f_i d_i$
100	3	−60	−180
120	6	−40	−240
140	10	−20	−200
160	15	0	0
180	24	20	480
200	42	40	1680
220	75	60	4500
	175		6040

$$Am = 160 + \frac{6040}{175} = Rs.\ 194.51$$

classified data:-

Q.8 :-

Overtime (hrs) x_i	No. of employees (f_i)	mid-value (m_i)	$d_i = m_i - 22.5$	$f_i d_i$
10-15	11	12.5	-10	-110
15-20	20	17.5	-5	-100
20-25	35	22.5	0	0
25-30	20	27.5	5	100
30-35	8	32.5	10	80
35-40	6	37.5	15	90
	100			60

$$AM. = 22.5 + \frac{1}{n}\sum f_i d_i = 22.5 + \frac{60}{100}$$

$$= 22.5 + 0.6$$

$$= 23.1 \text{ hrs.}$$

Q.9 ⇒ Total Rainfall of 7 days = $7 \times 15 = 105$ cm.

Total Rainfall of 6 days = $6 \times 10 = 60$ cm

Rainfal on Sunday = $105 - 60 = 45$ cm.

Q.10: Combined mean = $\frac{12 \times 80 + 20 \times 60}{140} = \frac{2160}{140} = 15.43$

Q.11: Calculate % of male & female employees,

Total employees = 100, Males = A, females = 100 - A

$$\text{Combined mean} = \frac{(a_1 \times n_1) + (a_2 \times n_2)}{n_1 + n_2}$$

$$3000 = \frac{3200 \times A + 2200(100 - A)}{A + (100 - A)} \Rightarrow$$

$$A = \frac{8000}{1000} = 80$$

Hence, % of male employees = 80% & female employees
= 100 − 80 = 20%

Case ①

① Length (in metors) (more than)	preference of housewives	class Interval	f_i'
1.0	50	1.0–1.5	50 − 48 = 2
1.5	48	1.5–2.0	48 − 42 = 6
2.0	42	2.0–2.5	42 − 40 = 2
2.5	40	2.5–3.0	40 − 10 = 30
3.0	10	3.0–3.5	10 − 5 = 5
3.5	5		

class Interval	mid-value (m_i')	Perference of housewives (f_i')	$d_i' = m_i' - 2.25$	$f_i'd_i'$
1.0 – 1.5	1.25	2	−1	−2
1.5 – 2.0	1.75	6	−0.50	−3
2.0 – 2.5	2.25 (A)	2	0	0
2.5 – 3.0	2.75	30	+0.5	15
3.0 – 3.5	3.25	5	+1	5.
		45		20

The mean length of the slab is

$$\bar{x} = A + \frac{\Sigma f_i d_i'}{n} = 2.25 + \frac{20}{45} = 2.58$$

②

marks obtained (%)	Cumulative frequency	class-Int.	Frequency	m_i'	d_i'	$f_i'd_i'$
less than 10	7	0–10	7	5	−3	−21
20	39	10–20	39 − 7 = 32	15	−2	−64
30	95	20–30	95 − 39 = 56	25	−1	−56
40	201	30–40	201 − 95 = 106	(35) A	0	0
50	381	40–50	381 − 201 = 180	45	1	180
60	545	50–60	545 − 381 = 164	55	2	328
70	631	60–70	631 − 545 = 86	65	3	258
80	675	70–80	675 − 631 = 44	75	4	176
			675			801

Mean ($\bar{x}$) of marks obtained =

$$\bar{x} = A + \frac{\Sigma f_i d_i}{n} \times h = 35 + \frac{801}{675} \times 10 = 46.86 \text{ marks.}$$

Case ②.

Q ③:

Income Group (x_i)	mid-values (m_i)	$d_i = \frac{m_i - A}{h}$	No. of firms	Average No. of workers	Total No. of workers (f_i)	$f_i d_i$
800-1000	900	-2	40	8	320	-440
1000-1200	1100	-1	32	12	384	-384
1200-1400	(1300) A	0	26	8	208	0
1400-1600	1500	1	28	8	224	224
1600-1800	1700	2	42	4	168	336
Total			168	40	1304	-464

$$A.M. = A + \frac{\sum_{i=1}^{5} f_i d_i}{n} \times h = 1300 - \frac{464}{1304} \times 200 = 1228.84.$$

Case-3

Q ④:

marks x_i	frequency (f_i)	$f_i x_i$
40	8	320
50	10	500
60	9	540
70	6	420
80	4	320
90	3	270
	40	2370

Total marks of all the students $= 50 \times 51.6 = 2580$

Total marks of 40 students who passed $= \sum f_i x_i = 2370$

marks of the remaining 10 students $= 2580 - 2370 = 210$

Hence average marks of 10 students who failed are $= \frac{210}{10} = 21$ marks.

Q:- The following tables gives the marks obtained by ten students in a class test:

Sol^n

Roll No. –	1	2	3	4	5	6	7	8	9	10
marks. –	2	8	3	10	9	4	3	7	2	6

Calculate average marks secured by students.

Sol^n:

Roll No.	marks (x_i)	(Deviation from assumed mean, $A=5$) (d_i)
1	2	-3
2	8	+3
3	3	-2
4	10	+5
5	9	+4
6	4	-1
7	3	-2
8	7	+2
9	2	-3
10	6	+1
Total	54	+4

By Direct method.

$$A.M = \frac{\Sigma x_i}{N} = \frac{54}{10} = 5.4 \text{ marks.}$$

By Deviation method

$$A.M = 5 + \frac{\Sigma d_i}{10} = 5 + \frac{4}{10} = 5.4 \text{ marks.}$$

Q:- The following table gives the marks obtained by twenty students in an examination:

marks –	1	2	3	4	5	6	7
No. of stu. –	1	2	4	3	6	3	1

Calculate average marks secured by students.

Sol:

marks (x_i)	No. of students (f_i)	Total marks ($x_i f_i$)	Dev. from assumed mean $A=4$, d_i	product of Dev & freq. ($f_i d_i$)
1	1	1	-3	-3
2	2	4	-2	-4
3	4	12	-1	-4
4	3	12	0	0
5	6	30	+1	+6
6	3	18	+2	+6
7	1	7	+3	+3
Total	20	84		+4

$$\text{average} = \frac{\sum x_i f_i}{N} = \frac{84}{20} = 4.2 \text{ marks.}$$

$$\text{average} = A + \frac{\sum f_i d_i}{n} = 4 + \frac{+4}{20} = 4 + 0.2 = 4.2 \text{ marks.}$$

(classified data)

Q:- The following table gives the marks obtained by twenty students in an examination.

marks	No. of students.
0 - 10	1
10 - 20	2
20 - 30	6
30 - 40	4
40 - 50	3
50 - 60	1
60 - 70	3

Calculate average marks secured by students.

marks (m)	No. of stu. (f)	midvalue (m')	Total marks (mf)	ass. m. (x = 35) dx	fdx
0-10	1	5	5	-30	-30
10-20	2	15	30	-20	-40
20-30	6	25	150	-10	-60
30-40	4	35	140	0	0
40-50	3	45	135	+10	+30
50-60	1	55	55	+20	+20
60-70	3	65	195	+30	+90
	20		710		+10

$$\text{average} = \frac{\Sigma mf}{n} = \frac{710}{20} = \underline{35.5} \text{ marks.}$$

$$\text{average} = \bar{x} + \frac{\Sigma fdx}{n} = 35 + \frac{10}{20} = 35 + 0.5 = \underline{35.5} \text{ ma}$$

Soln

(2) $\bar{x} = \frac{1}{N}\sum_{i=1}^{5} x_i = \frac{1}{5}[15 + 20 + 10 + 35 + 32] = 22.4$.

Thus, the arithmetic mean of the profit earned by these Companies during a year was Rs 22.4 Lakh.

Geometric mean: (Growth rate or decling rate)

Geometric mean = $\sqrt[n]{\text{product of all the } n \text{ values}}$

$G.M_1 = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdots x_n} = (x_1 x_2 x_3 \cdots x_n)^{\frac{1}{n}}$

* (G.M. of a set of n observations is the the n^{th} root of their product.)

Q:- The annual rate of growth of output of a company in the last five years.

year	2010	2011	2012	2013	2014
Growth Rate (%)	5.0	7.5	2.5	5.0	10.0
Output at the end of the year.	105	112.87	115.69	121.47	133.61

$G.M. = \sqrt[5]{5 \times 7.5 \times 2.5 \times 5 \times 10} = \sqrt[5]{4687.5}$

$= 5.9\ \%$ (average growth)

Calculation of G.M:

$G.M = (x_1 x_2 x_3 \cdots x_n)^{1/n}$

Taking log both side,

$\log(GM) = \log(x_1 x_2 x_3 \cdots x_n)^{\frac{1}{n}}$

$= \frac{1}{n} \log(x_1 x_2 x_3 \cdots x_n)$

$= \frac{1}{n}(\log x_1 + \log x_2 + \cdots + \log x_n)$.

$\log(GM) = \frac{1}{n} \sum_{i=1}^{n} (\log x_i)$

$\boxed{G.M. = \text{antilog}\left\{ \frac{1}{n} \sum_{i=1}^{n} \log x_i \right\}}$

If the observations $x_1, x_2, \ldots x_n$ ~~occur~~ $f_1, f_2, f_3 \ldots$ times respectively and the total of frequencies is $\boxed{n = \sum f_i}$,

$$G.M = (x_1^{f_1} x_2^{f_2} x_3^{f_3} \ldots x_n^{f_n})^{1/n}$$

Taking log both side

$$\log(GM) = \frac{1}{n}\{f_1 \log x_1 + f_2 \log x_2 + \ldots + f_n \log x_n\}.$$

$$= \frac{1}{n} \sum_{i=1}^{n} f_i \log x_i$$

$$\boxed{GM = \text{antilog}\left\{\frac{1}{n}\sum_{i=1}^{n} f_i \log x_i\right\}}$$

Q:- The rate of increase in population of a country during the last three decades is 5 percent, 8 percent and 12 percent. find the average rate of growth during the last three decades.

Sol^:-

Decade	Rate of increase in population (%)	Population at the end of Decade (x) taking preceding Decade as 100	$\log_{10} x$
		105	2.0212
1	5		
		108	2.0334
2	8		
		112	2.0492
3	12		

$$G.M. = \text{Antilog}\left\{\frac{1}{n}\sum_{i=1}^{n} \log x\right\} = \text{Antilog}\left\{\frac{1}{3}(6.1038)\right\}$$

$$= \text{Antilog}(2.0346) = 108.2$$

Hence the average rate of increase in population over the last three decades is $108.2 - 100 = 8.2\%$

Q:- A given machine is assumed to ~~depreciate~~ depreciate 40% in value in the first year, 25% in the 2nd year and 10% per year for the next three years, each % being calculated on the diminishing value.

#What is the average depreciation recorded on the diminishing value for the period of five years?

Sol^n:

Rate of Depreciation (%)	No. of years (f_i)	$\log_{10} x_i$	$f_i \log_{10} x_i$
40	1	1.6021	1.6021
25	1	1.3979	1.3979
10	3	1.0000	3.0000
Total.	5		= 6.0000

$$G.M. = \text{Antilog}\left\{\frac{1}{n}\sum_{i=1}^{n} f_i \log x_i\right\} = \text{Antilog}\left\{\frac{1}{5}\cdot(6.0000\right.$$

$$= \text{Antilog}(1.2) = 15.85$$

Hence the average rate of depreciation for first five years is 15.85%

Combined Geometric mean

The Combined geometric mean of observation formed by the geometric means of different sets of data is defined as:

$$\log(GM) = \frac{\sum_{i=1}^{n} n_i \log G_i}{\sum_{i=1}^{n} n_i}$$

where G_i is the geometric mean of the i^{th} data set having n_i number of observations.

Question: Three sets of data contain 8, 7, and 5 observations and their geometric means are 8.52, 10.12 and 7.75 respectively, find the Combined geometric mean of 20 observations.

$$G.M = \text{Antilog}\left[\frac{n_1 \log G_1 + n_2 \log G_2 + n_3 \log G_3}{n_1 + n_2 + n_3}\right]$$

$$= \text{Antilog}\left[\frac{8\log(8.52) + 7\log(10.12) + 5\log(7.75)}{8+7+5}\right]$$

$$= \text{Antilog}\left[\frac{8 \times 0.9304 + 7 \times 1.0051 + 5 \times 0.8893}{20}\right]$$

$$= \text{Antilog}\left(\frac{18.9254}{20}\right) = \text{Antilog}(0.94627)$$

$$= 8.835$$

Hence, the combined G.M. of 20 observations is 8.835

Harmonic mean:

The harmonic mean (H.M.) of a set of observations is the reciprocal of the arithmatic mean i.e.

$$\frac{1}{HM} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}$$

$$HM = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)} \quad \text{(for ungrouped data)}$$

if $f_1, f_2, f_3, \ldots f_n$ are the frequencies of observations $x_1, x_2, x_3, \ldots x_n$, then the harmonic mean is defined

$$HM = \frac{n}{\sum_{i=1}^{n} f_i\left(\frac{1}{x_i}\right)} \quad \text{(for grouped data.)}$$

where $n = \sum_{i=1}^{n} f_i$

Q:- ~~find the value of shares, if, changing after every one month, therefore, the required average price per share is the harmo.~~

Q:- find the harmonic mean of the following distribution of data.

Dividend yield (%)	2–6	6–10	10–14
Number of companies	10	12	18

Sol^n:-

Class Intervals (Dividendyield)	mid-value (m_i)	No. of Companies freq. (f_i)	Reciprocal $\frac{1}{m_i}$	$f_i\left(\frac{1}{m_i}\right)$
2-6	4	10	$\frac{1}{4}$	2.5
6-10	8	12	$\frac{1}{8}$	1.5
10-14	12	18	$\frac{1}{12}$	1.5
Total		$n=40$		5.5

The harmonic mean (HM) $= \frac{n}{\sum_{i=1}^{3} f_i \frac{1}{m_i}} = \frac{40}{5.5} = 7.27$

Hence the average dividend yield of 40 companies is 7.27 %

:- A investor buys Rs 20000 worth of shares of a Company, each month. During the first 3 months he bought the shares at a price of Rs.120, Rs160 and Rs. 210. After 3 months what is the average price paid by him for the shares.?

Since the value of shares is changing after every one month, therefore the required average price of share is the harmonic mean of the prices paid in 1st three months.

$$HM = \frac{3}{\frac{1}{120}+\frac{1}{160}+\frac{1}{210}} = \frac{3}{0.008+0.006+0.004} =$$

$$= \frac{3}{0.018} = \underline{\text{Rs. } 166.66}$$

Median

median is the value of the exact middle or central value of the items arranged in ascending order or descending order in a given series.

for Unclassified Data

In this case, the data is arranged in either ascending or descending order of magnitude.

(i) if No. of observation (n) is an odd numbers,

$$\text{median}(M) = \text{the size of } \left(\frac{n+1}{2}\right)^{th} \text{ item or observation in the data array}$$

(ii) if No. of observation is an even numbers.

$$\text{median}(M) = \frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observations}}{2}$$

example ⇒ Calculate the median of the following data that relates to the service time (in minutes) per customer for 7 customers at a railway reservation counter: 3.5, 4.5, 3, 3.8, 5.0, 5.5, 4.

Sol?. The data are arranged in ascending order

observation in the data array	:	1	2	3	4	5	6	7
service time (in minutes)	:	3	3.5	3.8	4	4.5	5	5.5

The median(M) = value of $\left(\frac{n+1}{2}\right)^{th}$ observation in the data array.

$$= \left\{\frac{(7+1)}{2}\right\}^{th} = 4^{th} \text{ observation}$$

$$= 4$$

Thus, the median service time is 4 minutes per customer.

Q:- Calculate the median of the following data that relate to the number of patients examined per hour in a (OPD) in a hospital:

10, 12, 15, 20, 13, 24, 17, 18.

Sol:- The data are arranged in ascending order

Observations in the data array: 1, 2, 3, 4, 5, 6, 7, 8

Patients examined per hrs: 10, 12, 13, 15, 17, 18, 20, 24

Since No. of observation are even.

$\left(\frac{n}{2}\right)^{th} = \left(\frac{8}{2}\right) = 4^{th}$ observation

$\left(\frac{n}{2}+1\right)^{th} = \left(\frac{8}{2}+1\right)^{th} = 5^{th}$ observation

median = average of 4^{th} & 5^{th} observation.

$= \frac{(4^{th} + 5^{th}) \text{ observation}}{2} = \frac{15+17}{2}$

$= \frac{32}{2} = 16$

Thus, median No. of patients examined per hr in OPD in a hospital are 16.

Discrete frequency Distribution:-

(1) Arrange the data in ascending or descending order
(2) obtain the sum of frequency $\Sigma f_i = N$
(3) obtain Cumulative frequency (cf)
(4) Find the median $M = \left(\frac{N+1}{2}\right)^{th}$ observation.
(5) the value for which the cumulative freq. includes $\left(\frac{n+1}{2}\right)^{th}$ value will be Taken as median

Calculate median for the following data.

value –	2,	3	4	5	6	7	8	9	10	11	12	13	14	15
frequency:	2	3	8	10	12	16	10	8	6	5	6	4	3	1

Sol:

value (x)	freq. (f_i)	Cumulative freq.
2	2	2
3	3	5
4	8	13
5	10	23
6	12	35
7	16	51
8	10	61
9	8	69
10	6	75
11	5	80
12	6	86
13	4	90
14	3	93
15	1	94

$n = 94$

median $= \left(\frac{n+1}{2}\right)^{th}$ observations.

$= \frac{94+1}{2} = \frac{95}{2} = 47.5$ th value

Since the median number lies in the cumulative frequency 51, corresponding observation is 7.

Hence, median = 7

Continuous frequency distribution

① Make the classes in exclusive form.
② find the total frequency $n = \Sigma f_i$ & median Number $\left(\frac{n}{2}\right)$.
③ Calculate cumulative frequency (c.f).
④ with help of median Number find the median class.
— The class in which median number falls is called the median class.
⑤ median $M = L_1 + \frac{\left(\frac{n}{2}\right) - cf}{f} (L_2 - L_1)$

where M = median
L_1 = lower limit of the median class.
L_2 = upper limit of the median class
f = frequency of the median class.
cf = cumulative freq. of preceding the median class.
n = sum of freq.

Q:- find the median of the following data.

marks obtained	0-5	5-10	10-15	15-20	20-25	25-30	30-35
no. of students	4	6	10	16	12	8	4

Sol:

marks (x_i)	no of stu. (f_i)	cf	
0-5	4	4	
5-10	6	10	
10-15	10	20	
15-20	16	(36)	median class
20-25	12	48	
25-30	8	56	
30-35	4	60	
	$\Sigma f = 60$		

median Number $= \left(\frac{n}{2}\right)^{th} = \left(\frac{60}{2}\right)$
$= 30^{th}$ No.
median Number lies in H. cf 36
class-Interval (15-20)
$L_1 = 15$, $L_2 = 20$, $f = 16$, $cf = 20$, $n = 60$.

median $M = 15 + \frac{\left(\frac{60}{2} - 20\right)}{16} \times (20 - 15)$
$= 15 + \frac{10 \times 5}{16} = 15 + 3.125$
$= 18.125$

mode

- The mode is the number which appears more times than any number in a given set.
- The value of the variable for which the frequency is maximum is called mode & denoted by Z

Q:- for individual series:

3, 4, 2, 1, 7, 6, 6, 7, 5, 6, 8, 9, 5

Sol:- in this data 6 is repeated maximum times

Hence, mode is 6

Q:- for discrete frequency distribution

find the mode of the given data

Height (in cm)	150	160	170	180	190	200	210
No. of person	2	4	8	10	6	5	3

Sol: in this data, maximum frequency is 10

Hence, corresponding observation is mode 180

$$\boxed{Z = 180}$$

Q:- for continuous series:-

$$\text{mode} = Z = L_1 + \left[\frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}}\right] \times (L_2 - L_1)$$

($f_m \rightarrow$ maximum frequency)

where L_2 = upper limit

L_1 = lower boundary of the mode class interval

f_m = Highest frequency of given distribution

f_{m-1} = frequency of f_m,

f_{m+1} = above freq. of f_m,

Q:- Calculate the mode of the given data

Class interval:	0–10	10–20	20–30	30–40	40–50	50–60
Frequency:	6	9	10	16	12	8

Sol?. The maximum frequency is 16 &
hence class is 30–40

$L_1 = 30$, $L_2 = 40$, $h = L_2 - L_1 = 10$

$f_m = 16$, $f_{m-1} = 10$, $f_{m+1} = 12$,

$$Z = L_1 + \left[\frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}}\right] \times h$$

$$= 30 + \left[\frac{(16-10)}{2\times16-10-12}\right] \times 10 = \underline{36}$$

mode

i.e.

$$Z = L_1 + \left[\frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}}\right](L_2 - L_1)$$

where L_1 = the lower limit of the modal class
L_2 = the upper limit of the modal class
f_m = the frequency of the modal class
f_{m-1} = the frequency of the class preceding the modal class.
f_{m+1} = the frequency of the class succeeding the modal class.

Relation b/w mean, median & mode

In a symmetric distribution,

$$\boxed{AM = Median = mode}$$

$$\boxed{a = M = Z}$$

In a non-symmetric distribution.

$$\boxed{a - M = \frac{1}{3}(a - Z)} \Rightarrow \boxed{Z = 3M - 2a}$$