(Please write your Exam Roll No.) Exam Roll No. ....

END TERM EXAMINATION

THIRD SEMESTER [B.TECH] JANUARY 2024

Paper Code: ES-201 **Subject: Computational Methods**

Time: 3 Hours **Maximum Marks: 75**

Note: Attempt five questions in all including Q.No.1 which is compulsory. Select one question from each unit. Assume missing data if any. Scientific calculator is allowed.

Q1 Attempt all questions:

a) The height of an observation tower was estimated to be 47m, whereas its actual height was 45m. Calculate the percentage relative error in the measurement. **(1.5)**

b) Derive the formula for evaluating $\sqrt{12}$ by Newton's Raphson method. **(1.5)**

c) Differentiate between partial and complete pivoting in solving linear system of algebraic equations. **(1.5)**

d) Estimate the missing term in the following table: **(1.5)**

x:	0	1	2	3	4
$f(x)$:	1	3	9	—	81

e) Define cubic spline function and state the conditions required for cubic spline interpolation. **(1.5)**

f) Find $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ using Simpson's 1/3rd rule for $n = 6$. **(2.5)**

g) Solve the IVP $\frac{dy}{dx} = xe^y, y(0) = 0$ using Picard's method and estimate $y(0.2)$. **(3)**

h) Determine which of the following equations are elliptic, parabolic, and hyperbolic. **(2)**

(i) $f_{xx} + 6f_{xy} + 9f_{yy} = 0$

(ii) $f_{xx} - 2f_{xy} + 2f_{yy} = 2x + 5y$

UNIT-I

Q2 a) Find the root of the equation $x = e^{-x}$, correct to three decimal places by Secant method by performing six iterations. **(8)**

b) Use Newton—Rapshon method to obtain a root of $\sin x = 1 - x$ to three decimal places. **(7)**

Q3 a) Perform four iterations of Golden section search method to minimize $f(x) = x^4 - 14x^3 + 60x^2 - 70x$, $x \in (0,2)$ with $\varepsilon = 10^{-3}$. **(7.5)**

b) Use steepest descent method for 3 iterations on $f(x_1, x_2, x_3) = (x_1 - 4)^4 + (x_2 - 3)^2 + 4(x_3 + 5)^4$ with initial point $x^{(0)} = [4, 2, -1]^T$ **(7.5)**

UNIT-II

Q4 a) Prove that $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{(1 + \delta^2/4)}$. **(7)**

b) The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface: **(8)**

P.T.O.

$x =$ height:	100	150	200	250	300	350	400
$y =$ distance:	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the value of y when $x = 410$ using Newton's backward interpolation formula.

Q5 a) Evaluate the integral $\int_0^6 \frac{e^x}{1+x} dx$ using Simpson's $1/3^{rd}$ and Simpson's $3/8^{th}$ rule respectively. Compare it with the exact value. (7)

b) Evaluate the integral $\int_0^{0.5} \left(\frac{x}{\sin x}\right) dx$ using Romberg's method, correct to three decimal places. (8)

UNIT-III

Q6 a) Investigate the values of λ and μ so that the system of equations (7)

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

have (i) no solution, (ii) unique solution, (iii) an infinite number of solutions.

b) Solve the system of equations (8)

$$\begin{cases} x_1 + 10x_2 - x_3 = 3 \\ 2x_1 + 3x_2 + 20x_3 = 7 \\ 10x_1 - x_2 + 2x_3 = 4 \end{cases}$$

Using the Gauss elimination with partial pivoting.

Q7 a) Solve the system of equations using Dolittle factorisation method (9)

$$\begin{cases} 3x + 2y + 7z = 4 \\ 2x + 3y + z = 5 \\ 3x + 4y + z = 7 \end{cases}$$

b) Determine the numerically dominant eigenvalue and eigenvector of the matrix $A = \begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}$ using Power method. Take the initial vector $X^{(0)} = [1,1,1]^T$. (6)

UNIT-IV

Q8 a) Employ Taylor's method to obtain the approximate value of y at $x = 0.2$ for the differential equation $dy/dx = 2y + 3e^x$, $y(0) = 0$. Compare the numerical solution obtained with the exact solution. (7)

P.T.O.

b) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1$ at $x=0.2, 0.4$. **(8)**

Q9 a) Solve the initial value problem $y\frac{dy}{dx}=x,\ y(0)=1$, using Euler's method in $0\le x\le 0.8$, with $h=0.2$. Compare the results with the exact solution at $x=0.8$. **(8)**

b) Solve the partial differential equation **(7)**

$u_{xx}+u_{yy}=x+y+1,\ 0\le x\le 1,\ 0\le y\le 1,$

$u=0$ on the boundary

Numerically up to three iterations with $h=1/3$. Obtain the results correct to three decimal places.

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