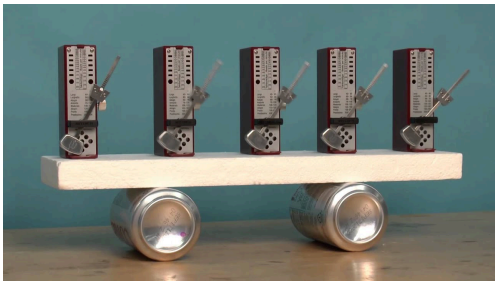
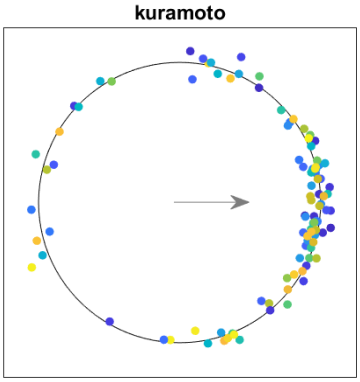
Collective Synchronization: The Kuramoto Model

Number of students: 1
Topic: Nonlinear dynamics, computational physics
Your Goal: Create a simulation of the Kuramoto model in Python, and investigate the conditions under which spontaneous synchronization occurs.



Background:
Collective synchronization is a remarkable effect observed in many diverse areas of our world. Synchronization can appear between living things, such as in the flocking behavior of birds and fish, the synchronized flashing of some types of fireflies, and even the synchronized pulsing of the cells of your heart to pump blood. But synchronization doesn't just occur between living things: it can also occur between pendulums, in the motion of planetary bodies, and in chemical reactions.



To understand the many phenomena of collective synchronization, a mathematical model called the Kuramoto model has been studied. In the Kuramoto model, a collection of individual oscillators is considered, where each oscillator wants to oscillate at its own natural frequency. Each oscillator is often represented as a point on a circle, and oscillations are shown by the point moving around the circle. However, each oscillator is also influenced by the motion of the other oscillators, being pushed and pulled in a specific way. Due to these interactions, the Kuramoto model exhibits collective behavior. Although it is a relatively simple model without the diverse nuances of real-life spontaneous synchronization, study of the Kuramoto model has still revealed much insight into spontaneous synchronization which can be generalized to many areas, and research related to the Kuramoto model is ongoing.

In this challenge question, you will use Python to create a computational simulation of the Kuramoto model. Most of the challenge question will be spent writing Python code in a guided Jupyter notebook environment. The first objective of the challenge question is to observe spontaneous synchronization in the results of your simulation. If you achieve this, you can then begin exploring the conditions under which synchronization occurs in the Kuramoto model, changing for example the coupling strength between the oscillators, or the underlying interaction network. Such investigation may lead you to encounters with topics such as phase transitions, network science, and universality.

Recommended resources:

- Veritasium: The Surprising Secret of Synchronization
- Wikipedia: Kuramoto Model
- Synchronization of oscillators: an ideal introduction to phase transitions