NUMERICAL METHODS LABORATORY(MA29202) & NUMERICAL TECHNIQUES LABORATORY(MA39110)

Assignment-8 Explicit Methods for Parabolic Equations [1]

1. Solve the following heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0$$

subject to the initial and boundary conditions

$$u(x,0) = \sin \pi x, \quad 0 \leq x \leq 1 \text{ and } u(0,t) = u(1,t) = 0, \quad t \geq 0.$$

For numerical simulations, take $\Delta x = 1/50$ and $\Delta t = \mathcal{F}\frac{(\Delta x)^2}{2}$, where $\mathcal{F} = 0.9$ is a safety factor. Obtained the numerical results after 300, 400 and 500 time steps, and plot these solutions together.

2. Solve the following heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0$$

subject to the following initial and boundary conditions

$$u(x,0) = \begin{cases} 2x, & 0 \leq x \leq 1/2, \\ 2(1-x), & 1/2 \leq x \leq 1, \end{cases} \quad u(0,t) = u(1,t) = 0, \quad t \geq 0 \tag{1}$$

For numerical simulations, take $\Delta x = 1/50$ and $\Delta t = \mathcal{F}\frac{(\Delta x)^2}{2}$, where $\mathcal{F} = 0.9$ is a safety factor. Obtained the numerical results after 50, 100 and 200 time steps, and plot these solutions together.

Explicit Finite Difference Scheme:

Replace the time derivative by a first-order forward difference and the spatial derivative by a central difference:

$$\frac{\left(u_i^{n+1} - u_i^n\right)}{\Delta t} = \frac{\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)}{(\Delta x)^2}$$

This can be rearranged to give the explicit FD scheme,

$$u_i^{n+1} = u_i^n + \frac{\Delta t}{(\Delta x)^2}\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right).$$

Note that the presence of u_{i+1}^n and u_{i-1}^n indicates the need for left and right ghost values.

[1] Sent on: `March 27, 2024`.