4. Frequency Analysis of signals

problems

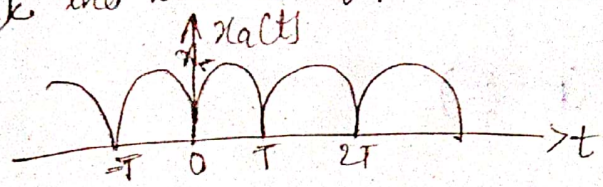
1) consider the full-wave rectified sinusoid.

(a) determine its spectrum $x_a(F)$

(b) compute the power of the signal

(c) plot the power spectral density

d) check the validity of Parseval's relation for this signal.



A) $x_a(t)$ is periodic, it can be represented by Fourier series

$$x_a(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi kt/T}$$

$$C_k = \frac{1}{T}\int_0^T A \sin(\pi t/T)\, e^{-j2\pi kt/T}\, dt$$

$$= \frac{A}{2jT}\int_0^T \left[e^{j\pi t/T} - e^{-j\pi t/T}\right] e^{-j2\pi kt/T}\, dt$$

$$= \frac{A}{2jT}\left[\frac{e^{j\pi(1-2k)t/T}}{j\pi(1-2k)\frac{1}{T}} - \frac{e^{-j\pi(1+2k)t/T}}{-j\pi(1+2k)\frac{1}{T}}\right]_0^T$$

$$= \frac{A}{2jT}\left[\frac{e^{j\pi(1-2k)}-1}{j\pi(1-2k)\frac{1}{T}} + \frac{e^{-j\pi(1+2k)}-1}{j\pi(1+2k)\frac{1}{T}}\right]$$

$$= \frac{-A}{2\pi}\left[\frac{-1-1}{(1-2k)} + \frac{-1-1}{(1+2k)}\right]$$

$$= \frac{-A}{2\pi}\left[-2\left[\frac{1}{1-2k} + \frac{1}{1+2k}\right]\right]$$

$$= \frac{A}{\pi}\left[\frac{2}{1-4k^2}\right] = \frac{2A}{\pi(1-4k^2)}$$

$$x_a(F) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi Ft}\, dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_k e^{j2\pi F_0 t}\, e^{-j2\pi Ft}\, dt$$

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$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} C_k e^{-j2\pi(f-kf_0)t}\,dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} C_k e^{-j2\pi(f-\frac{k}{T})t}\,dt$$

$$= \sum_{k=-\infty}^{\infty} C_k \int_{-\infty}^{\infty} e^{-j2\pi(f-\frac{k}{T})t}\,dt$$

$$= \sum_{k=-\infty}^{\infty} C_k\, \delta\left(f-\frac{k}{T}\right)$$

(b) power of the signal.

$$P = \frac{1}{T}\int_0^T x_a^2(t)\,dt$$

$$= \frac{1}{T}\int_0^T \left(A\sin\frac{\pi t}{T}\right)^2 dt$$

$$= \frac{A^2}{T}\int_0^T \sin^2\frac{\pi}{T}t\,dt$$

$$= \frac{A^2}{T}\int_0^T \frac{1-\cos 2\left(\frac{\pi}{T}\right)t}{2}\,dt$$

$$= \frac{A^2}{T}\int_0^T \left(\frac{1}{2} - \frac{\cos 2\left(\frac{\pi}{T}\right)t}{2}\right)dt$$

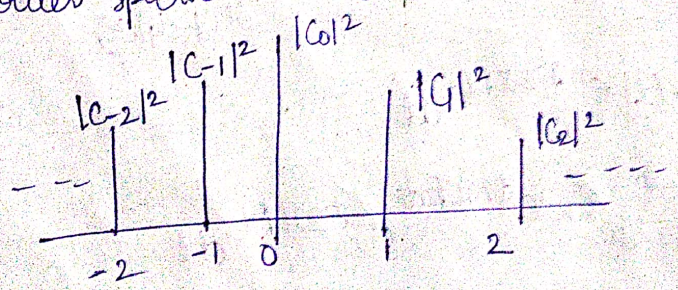
$$= \frac{A^2}{T}\left[\frac{T}{2}\right] - \frac{A^2}{T}\int_0^T \frac{\cos 2\left(\frac{\pi}{T}\right)t}{2}\,dt$$

$$= \frac{A^2}{2} - \frac{A^2}{2T}\left[\frac{\sin 2\left(\frac{\pi}{T}\right)t}{2\left(\frac{\pi}{T}\right)}\right]_0^T$$

$$= \frac{A^2}{2} - \frac{A^2}{2T}\left[\frac{\sin 2\frac{\pi}{T}T}{2\frac{\pi}{T}} - \frac{\sin 2\left(\frac{\pi}{T}\right)0}{2\left(\frac{\pi}{T}\right)}\right]$$

$$= \frac{A^2}{2}$$

(c) power spectral density.



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d) parseval's relation:-

$$P = \sum_{k=-\infty}^{\infty} |C_k|^2$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{2A}{\pi(1-4k^2)}\right)^2$$

$$= \frac{4A^2}{\pi^2} \sum_{k=-\infty}^{\infty} \frac{1}{(4k^2-1)^2}$$

$$= \frac{4A^2}{\pi^2} \left[\frac{1}{(4k^2-1)^2} \Big|_{k=0} + 2 \sum_{k=1}^{\infty} \frac{1}{(4k^2-1)^2} \right]$$

$$= \frac{4A^2}{\pi^2} \left[1 + \frac{2}{3^2} + \frac{2}{15^2} + \cdots \right]$$

$$= \frac{4A^2}{\pi^2} [1.231]$$

$$= 0.5A^2 = \frac{A^2}{2}$$

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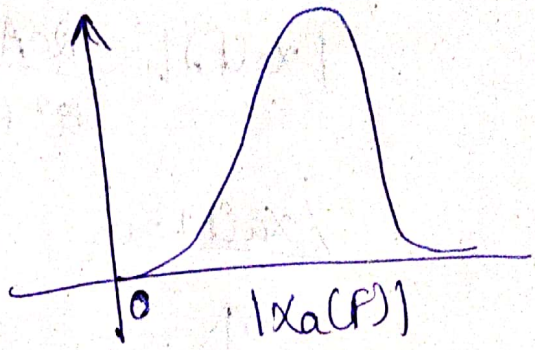
$= 0.5A^2 = \frac{A}{2}$,

2) compute and sketch the magnitude & phase spectra for the following signals $(a>0)$.

(a) $x_a(t) = \begin{cases} Ae^{-at} & , t \geq 0 \\ 0 & , t < 0 \end{cases}$

(b) $x_a(t) = Ae^{-a|t|}$



(A)(a) $x_a(t) = \begin{cases} Ae^{-at} & , t \geq 0 \\ 0 & , t < 0 \end{cases}$

$$X_a(F) = \int_0^\infty Ae^{-at} e^{-j2\pi Ft}\, dt$$

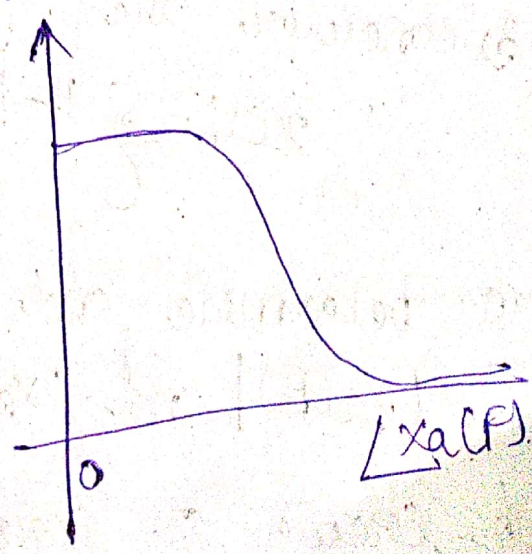
$$= A \int_0^\infty e^{-(a+2j\pi jF)t}\, dt$$

$$= A \left[\frac{e^{-(a+2\pi jF)t}}{-(a+2\pi jF)} \right]_0^\infty$$

$$= A \frac{1}{a+2j\pi F}$$

$$|X_a(F)| = \frac{A}{\sqrt{a^2+4\pi^2F^2}}$$

$$\angle X_a(F) = -\tan^{-1}\left(\frac{2\pi F}{a}\right)$$



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(b) $x_a(t) = A e^{-a|t|}$

$$X_a(F) = \int_{-\infty}^{\infty} A e^{-a|t|} e^{-j2\pi F t} dt$$

$$= \int_{-\infty}^{0} A e^{at} e^{-j2\pi F t} dt + \int_{0}^{\infty} A e^{-at} e^{-j2\pi F t} dt$$

$$= \int_{0}^{\infty} A e^{-at} e^{+j2\pi F t} dt + \int_{0}^{\infty} A e^{-at} e^{-j2\pi F t} dt$$

$$= A \int_{0}^{\infty} e^{-(-j2\pi F + a)t} dt + A \int_{0}^{\infty} e^{-(a+j2\pi F)t} dt$$

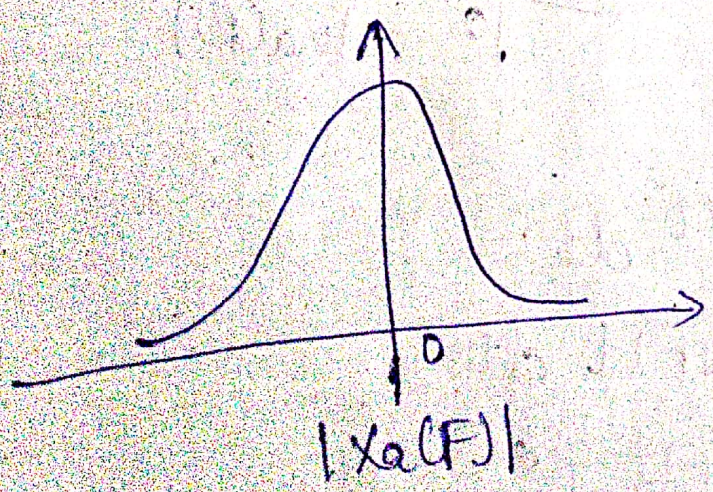
$$= A \left[\frac{e^{-(a-j2\pi F)t}}{-(a-j2\pi F)} \right]_0^{\infty} + A \left[\frac{e^{-(a+j2\pi F)t}}{-(a+j2\pi F)} \right]_0^{\infty}$$

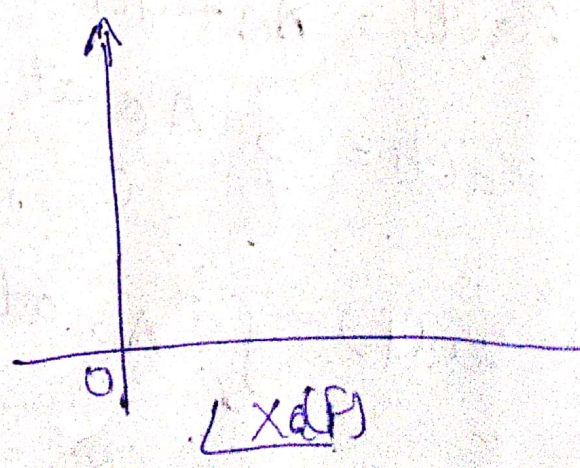
$$= \frac{A}{a-j2\pi F} + \frac{A}{a+j2\pi F}$$

$$= \frac{2aA}{a^2+(2\pi F)^2}$$

$$|X_a(F)| = \frac{2aA}{a^2+(2\pi F)^2}$$

$$\angle X_a(F) = 0$$





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3) Consider the signal

$$x(t) = \begin{cases} 1 - \frac{|t|}{T} & , |t| \le T \\ 0 & , \text{elsewhere.} \end{cases}$$

(a) Determine and sketch its magnitude and phase spectra $|X_a(F)|$ and $\angle X_a(F)$, respectively.

(b) Create a periodic signal $x_p(t)$ with fundamental period $T_P \ge 2T$, so that $x(t) = x_p(t)$ for $|t| \le T_P/2$.

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What are the Fourier coefficients c_k for the sigd. $x_p(t)$?

(c) Using the results in parts (a) and (b), show that

$$c_k = (1/TP)\, X_a(k/TP)$$

A) a) Let $y(t) = \begin{cases} \frac{1}{T}, & -T < t \le 0 \\ -1/T, & 0 < t \le T \end{cases}$

$$Y(F) = \int_{-T}^{0} \frac{1}{T} e^{-j2\pi F t}\,dt + \int_{0}^{T} \frac{-1}{T} e^{-j2\pi F t}\,dt$$

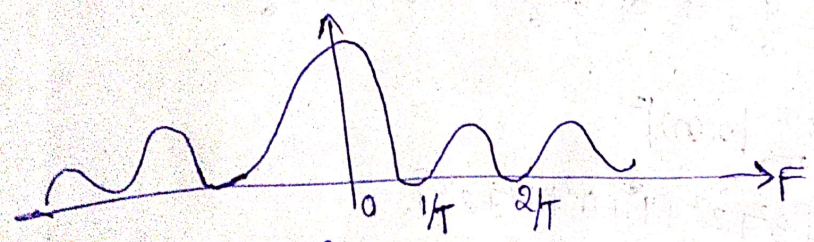
$$= \frac{-2\sin^2 \pi F T}{j\pi F T}$$

Then $X_a(F) = \frac{1}{j2\pi F} Y(F)$

$$= T\left(\frac{\sin \pi F T}{\pi F T}\right)^2$$

$$|X_a(F)| = T\left(\frac{\sin \pi F T}{\pi F T}\right)$$

$$\angle X_a(F) = 0$$



(b) $$c_k = \frac{1}{TP}\int_{-TP/2}^{TP/2} x(t)\, e^{-j2\pi k t/TP}\,dt$$

$$= \frac{1}{TP}\left[\int_{-T}^{0}\left(1+\frac{t}{T}\right)e^{-j2\pi k t/TP}\,dt + \int_{0}^{T}\left(1-\frac{t}{T}\right)e^{-j2\pi k t/TP}\,dt\right]$$

$$= \frac{T}{TP}\left[\frac{\sin(\pi k T/TP)}{kT/TP}\right]^2$$

(c) $$\frac{1}{TP}\, X_a\left(\frac{k}{TP}\right)$$

$$\frac{1}{TP}\, T\left[\frac{\sin \pi \frac{k}{TP} T}{\pi \frac{k}{TP} T}\right]^2$$

$$\frac{T}{TP}\left(\frac{\sin \pi k T/TP}{\pi k T/TP}\right)^2 = c_k$$

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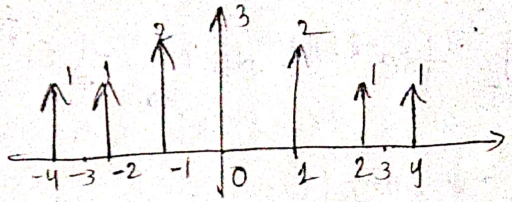
4) Consider the following periodic signal:

$$x(n) = \{\ldots, 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1, \ldots\}$$

a) Sketch the signal $x(n)$ and its magnitude and phase spectra.

(b) Using the results in part (a), verify Parseval's relation by computing the power in the time and frequency domains.

A) (a)



$$x(n) = \{\ldots, 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1, \ldots\}$$

$$N=6, \quad C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

$$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-j2\pi kn/6}$$

$$= \frac{1}{6}\left[3 + 2e^{-j\pi 2k/6} + e^{-j2\pi k/3} + 0 + e^{-j4\pi k/3} + 2e^{-j10\pi k/6}\right]$$

$$= \frac{1}{6}\left[3 + 4\cos\frac{\pi k}{3} + 2\cos\frac{2\pi k}{3}\right]$$

$$C_0 = \frac{9}{6},\ C_1 = \frac{4}{6},\ C_2 = 0,\ C_3 = \frac{1}{6},\ C_4 = 0,\ C_5 = \frac{4}{6}$$

(b) $$P_t = \frac{1}{6}\sum_{n=0}^{5} |x(n)|^2$$

$$= \frac{1}{6}\left[3^2 + 2^2 + 1^2 + 0^2 + 1^2 + 2^2\right]$$

$$= \frac{19}{16}$$

$$P_f = \sum_{n=0}^{5} |C(n)|^2$$

$$= \left[\left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0^2 + \left(\frac{1}{6}\right)^2 + 0^2 + \left(\frac{4}{6}\right)^2\right]$$

$$= \frac{19}{16}$$

5) Consider the signal

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

(a) Determine and sketch its power density spectrum

(b) Evaluate the power of the signal.

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A) (a) $x(n) = 2 + 2\cos\frac{3\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$

$= 2 + 2\left[\frac{e^{j3\pi n/4} + e^{-j3\pi n/4}}{2}\right] + \frac{e^{j\pi n/2} + e^{-j\pi n/2}}{2} + \frac{1}{2}\left[\frac{e^{j3\pi n/4} + e^{-j3\pi n/4}}{2}\right]$

$= 2 + e^{j3\pi n/4} + e^{-j3\pi n/4} + \frac{1}{2}e^{j\pi n/2} + \frac{1}{2}e^{-j\pi n/2} + \frac{1}{4}e^{j3\pi n/4} + \frac{1}{4}e^{-j3\pi n/4}$

$N = 8$

$C_k = \frac{1}{8}\sum_{n=0}^{7} x(n)\, e^{-j2\pi kn/4}$

$x(n) = \{\frac{11}{2}, 2 + \frac{3}{4}\sqrt{2}, 1, 2 - \frac{\sqrt{3}}{4}, \frac{1}{2}, 2 - \frac{3}{4}\sqrt{2}, 1, 2 + \frac{3}{4}\sqrt{2}\}$

$C_0 = 2,\ C_1 = C_7 = 1,\ C_2 = C_6 = \frac{1}{2},\ C_3 = C_5 = \frac{1}{4},\ C_4 = 0$

(b) $P_t = \sum_{n=0}^{7} |C_n|^2$

$= [2^2 + 2(1^2) + 2(\frac{1}{2})^2 + 2(\frac{1}{4})^2]$

$= \frac{53}{8}$

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$= \frac{53}{8}$

6) Determine & sketch the magnitude & phase spectra of the following periodic signals.

(a) $x(n) = 4 \sin \frac{\pi(n-2)}{3}$ (b) $x(n) = \cos \frac{2\pi}{3} n + \sin \frac{2\pi}{3} n$.

(c) $x(n) = \cos \frac{2\pi}{3} n \sin \frac{2\pi}{5} n$ (d) $x(n) = \{\cdots -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2\}$..

(e) $x(n) = \{\cdots -1, 2, \underset{\uparrow}{1}, 2, -1, 0, -1, 2, 1, 2, \cdots\}$

(f) $x(n) = \{\cdots 0, 0, \underset{\uparrow}{1}, 1, 0, 0, 0, 1, 1, 0, 0, \cdots\}$

(g) $x(n) = 1, -\infty < n < \infty$ (h) $x(n) = (-1)^n, -\infty < n < \infty$.

A) (a) $x(n) = 4 \sin \frac{\pi(n-2)}{3}$

$= 4 \sin \frac{2\pi(n-2)}{6}$

$c(k) = \frac{1}{6} \sum_{n=0}^{5} x(n) e^{-j2\pi kn/6}$

$= \frac{4}{6} \sum_{n=0}^{5} \sin \frac{2\pi(n-2)}{6} e^{-j2\pi kn/6}$

$= \frac{1}{\sqrt{3}} \left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{-j\pi k/3} + e^{-j2\pi k/3} \right]$

$= \frac{1}{\sqrt{3}} (-2j) \left[\sin \frac{2\pi k}{6} + \sin \frac{\pi k}{3} \right] e^{-j2\pi k/3}$

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$C_0 = 0,\ C_1 = -2je^{-j2\pi/3},\ C_2 = C_3 = C_4 = 0,\ C_5 = 2je^{j}$

$|C_1| = |C_5| = 2,\ |C_0| = |C_2| = |C_3| = |C_4| = 0$

$\angle C_1 = \pi + \frac{\pi}{2} - \frac{2\pi}{3} = \frac{5\pi}{6}$

$\angle C_5 = -\frac{5\pi}{6}$

$\angle C_0 = \angle C_2 = \angle C_3 = \angle C_4 = 0$

(b) $x(n) = \cos\frac{2\pi n}{3} + \sin\frac{2\pi n}{5} \Rightarrow N = 15$

$C_k = C_{1k} + C_{2k}$

where C_{1k} is the DTFS coefficients of $\cos\frac{2\pi n}{3}$ & C_{2k} is the DTFS coefficients of $\sin\frac{2\pi n}{5}$. But

$$\cos\frac{2\pi n}{3} = \frac{1}{2}\left(e^{j\frac{2\pi n}{3}} + e^{-j2\pi n/3}\right)$$

Hence

$$C_{1k} = \begin{cases} 1/2, & k = 5, 10 \\ 0, & \text{otherwise} \end{cases}$$

similarly

$$\sin\frac{2\pi n}{5} = \frac{1}{2j}\left(e^{j\frac{2\pi n}{5}} - e^{-j\frac{2\pi n}{5}}\right)$$

Hence

$$C_{2k} = \begin{cases} \frac{1}{2j}, & k = 3 \\ -\frac{1}{2j}, & k = 12 \\ 0, & \text{otherwise} \end{cases}$$

therefore

$$C_k = C_{1k} + C_{2k} = \begin{cases} \frac{1}{2j}, & k = 3 \\ 1/2, & k = 5 \\ 1/2, & k = 10 \\ -1/2j, & k = 12 \\ 0, & \text{otherwise} \end{cases}$$

(c) $x(n) = \cos\frac{2\pi}{3}n \sin\frac{2\pi}{5}n$

$$= \frac{1}{2}\sin\frac{16\pi n}{15} - \frac{1}{2}\sin\frac{4\pi n}{15}$$

$N = 15$

$$C_n = \begin{cases} \frac{-1}{2j}, & k = 2, 7 \\ \frac{1}{2j}, & k = 8, 13 \\ 0, & \text{otherwise} \end{cases}$$

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d) $N=5$

$$C_k = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-j2\pi nk/3}$$

$$= \frac{1}{5}\left[e^{-j\frac{2\pi k}{5}} + 2e^{-j4\pi k/5} - 2e^{-j6\pi k/5} - e^{-j\frac{8\pi k}{5}}\right]$$

$$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right)\right]$$

$C_0 = 0$, $C_1 = \frac{2j}{5}\left[-\sin\left(\frac{2\pi}{5}\right) + 2\sin\left(\frac{4\pi}{5}\right)\right]$

$C_2 = \frac{2j}{5}\left[\sin\left(\frac{4\pi}{5}\right) + 2\sin\left(\frac{2\pi}{5}\right)\right]$ $\quad C_3 = -C_2$, $C_4 = -C_1$

(e) $N=6$

$$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-j\frac{2\pi nk}{6}}$$

$$= \frac{1}{6}\left[1 + 2e^{-j\frac{\pi k}{3}} - e^{-j\frac{2\pi k}{3}} - e^{-j4\pi k/3} + 2e^{-j5\frac{\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1 + 4\cos\left(\frac{\pi k}{3}\right) - 2\cos\left(\frac{2\pi k}{3}\right)\right]$$

$C_0 = \frac{1}{2}$, $C_1 = \frac{2}{3}$, $C_2 = 0$, $C_3 = \frac{-5}{6}$, $C_4 = 0$, $C_5 = \frac{2}{3}$.

(f) $N=5$

$$C_k = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-j\frac{2\pi nk}{5}}$$

$$= \frac{1}{5}\left[1 + e^{-j2\pi k/5}\right]$$

$$= \frac{2}{5}\cos\left(\frac{\pi k}{5}\right) e^{-j\pi k/5}$$

$C_0 = \frac{2}{5}$ $\quad C_1 = \frac{2}{5}\cos\left(\frac{\pi}{5}\right) e^{-j\pi/5}$

$C_2 = \frac{2}{5}\cos\left(\frac{2\pi}{5}\right) e^{-j2\pi/5}$ $\quad C_3 = \frac{2}{5}\cos\left(\frac{3\pi}{5}\right) e^{-j3\pi/5}$

$C_4 = \frac{2}{5}\cos\left(\frac{4\pi}{5}\right) e^{-j4\pi/5}$

(g) $N=1$

$C_k = x(0) = 1$ $\quad (0'k)$

$C_0 = 1$

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(b) $N=2$

$$C_k = \frac{1}{2}\sum_{n=0}^{1} x(n)e^{-jn\pi k}$$

$$= \frac{1}{2}(1-e^{-j\pi k})$$

$C_0=0,\ C_1=1$

7) Determine the periodic signals $x(n)$, with fundamental period $N=8$, if their Fourier coefficients are given by

(a) $C_k = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$

(b) $C_k = \begin{cases} \sin k\pi/3, & 0\le k\le 6 \\ 0, & k=7 \end{cases}$

(c) $\{C_k\} = \{\ldots, 0, \frac{1}{4}, \frac{1}{2}, 1, 2, 1, \frac{1}{2}, \frac{1}{4}, 0, \ldots\}$

A) (a) $x(n) = \sum_{k=0}^{7} C_k e^{j\frac{2\pi nk}{8}}$

Note that if $C_k = e^{j\frac{2\pi nk}{8}}$, then

$$\sum_{k=0}^{7} e^{j\frac{2\pi pk}{8}} e^{j\frac{2\pi nk}{8}} = \sum_{k=0}^{7} e^{j2\pi(p+n)k/8}$$

$= 8,\ p=-n$

$= 0,\ p\neq -n$

since $C_k = \frac{1}{2}\left[e^{j2\pi k/8} + e^{-j2\pi k/8}\right] + \frac{1}{2j}\left[e^{j6\pi k/8} - e^{-j6\pi k/8}\right]$

$\therefore x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\delta(n+3) + 4j\delta(n-3),\ -3\le n\le 5$

(b) $C_0=0,\ C_1=\frac{\sqrt{3}}{2},\ C_2=\frac{\sqrt{3}}{2},\ C_3=0,\ C_4=-\frac{\sqrt{3}}{2},\ C_5=-\frac{\sqrt{3}}{2},\ C_6=C_7=0$

$$x(n) = \sum_{k=0}^{7} C_k e^{j2\pi nk/8}$$

$$= \frac{\sqrt{3}}{2}\left[e^{j\pi n/4} + e^{j2\pi n/4} - e^{j4\pi n/4} - e^{j5\pi n/4}\right]$$

$$= \sqrt{3}\left[\sin\frac{\pi n}{2} + \sin\frac{\pi n}{4}\right] e^{j\pi(3n-2)/4}$$

(c) $x(n) = \sum_{k=-3}^{4} C_k e^{j2\pi kn/8}$

$$= 2 + e^{j\pi n/4} + e^{-j\pi n/4} + \frac{1}{2}e^{j\pi n/2} + \frac{1}{2}e^{-j\pi n/2} + \frac{1}{4}e^{j3\pi n/4} + \frac{1}{4}e^{-j3\pi n/4}$$

$$= 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{4} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

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Q) Compute the Fourier Transform of the following signals.

(a) $x(n) = u(n) - u(n-6)$ (b) $x(n) = 2^n u(-n)$

(c) $x(n) = (\frac{1}{4})^n u(n+4)$ (d) $x(n) = (\alpha^n \sin(\omega_0 n)) u(n)$, $|\alpha| < 1$

(e) $x(n) = |\alpha|^n \sin \omega_0 n$, $|\alpha| < 1$

(f) $x(n) = \begin{cases} 2 - (\frac{1}{2})^n, & |n| \le 4 \\ 0, & \text{elsewhere} \end{cases}$

(g) $x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

(h) $x(n) = \begin{cases} A(2M + 1 - |n|), & |n| \le M \\ 0, & |n| > M \end{cases}$

Sketch the magnitude & phase spectra for parts (a), (f) & (g).

A) (a) $x(n) = u(n) - u(n-6)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{5} e^{-j\omega n} = \frac{1 - e^{-j6\omega}}{1 - e^{-j\omega}}$$

(b) $x(n) = 2^n u(-n)$

$$X(\omega) = \sum_{n=-\infty}^{0} 2^n e^{-j\omega n} = \sum_{m=0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^n$$

$$= \frac{2}{2 - e^{j\omega}}$$

(c) $x(n) = (\frac{1}{4})^n u(n+4)$

$$X(\omega) = \sum_{n=-4}^{\infty} (\tfrac{1}{4})^n e^{-j\omega n}$$

$$= \left(\sum_{m=0}^{\infty} (\tfrac{1}{4})^m e^{-j\omega m}\right) 4^4 e^{j4\omega}$$

$$= \frac{4^4 e^{j4\omega}}{1 - \frac{1}{4} e^{-j\omega}}$$

(d) $x(n) = \alpha^n \sin \omega_0 n \, u(n)$, $|\alpha| < 1$

$$X(\omega) = \sum_{n=0}^{\infty} \alpha^n \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}\right] e^{-j\omega n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} \left[\alpha e^{-j(\omega - \omega_0)}\right]^n - \frac{1}{2j} \sum_{n=0}^{\infty} \left[\alpha e^{-j(\omega + \omega_0)}\right]^n$$

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$$= \frac{1}{2j}\left[\frac{1}{1-\alpha e^{-j(\omega-\omega_0)}} - \frac{1}{1-\alpha e^{-j(\omega+\omega_0)}}\right]$$

$$= \frac{\alpha \sin\omega_0 \, e^{-j\omega}}{1-2\alpha\cos\omega_0 e^{-j\omega}+\alpha^2 e^{-j2\omega}}$$

(e) $x(n) = |\alpha|^n \sin\omega_0 n, \ |\alpha| < 1$

Note that $\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |\alpha|^n |\sin\omega_0 n|$

suppose that $\omega_0 = \frac{\pi}{2}$, so that $|\sin\omega_0 n| = 1$

$$\sum_{n=-\infty}^{\infty} |\alpha|^n = \sum_{n=-\infty}^{\infty} |x(n)| \to \infty$$

Fourier transform does not exist

(f) $x(n) = \begin{cases} 2-(1/2)^n, & |n| \le 4 \\ 0, & \text{otherwise} \end{cases}$

$$X(\omega) = \sum_{n=-4}^{4} x(n) e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} [2-(1/2)^n] e^{-j\omega n}$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}}$$

$$= -\frac{1}{2}[-4e^{j4\omega}+4e^{-j4\omega}-3e^{j3\omega}+e^{-j3\omega}-2e^{j2\omega}+2e^{-j2\omega} - e^{j\omega}+e^{-j\omega}].$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} + j[4\sin 4\omega + 3\sin 3\omega + 2\sin 2\omega + \sin\omega]$$

(g) $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$$= -2e^{j2\omega} - e^{j\omega} + e^{j\omega} + 2e^{-j2\omega}$$

$$= -2j[2\sin 2\omega + \sin\omega].$$

(h) $x(n) = \begin{cases} A(2M+1-|n|), & |n| \le M \\ 0, & |n| > M \end{cases}$

$$X(\omega) = \sum_{n=-M}^{M} x(n) e^{-j\omega n}$$

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$$= A\sum_{n=-M}^{M}(2M+1-|n|)e^{j\omega n}$$

$$= (2M+1)A + A\sum_{k=1}^{M}(2M+1-k)(e^{j\omega k}+e^{-j\omega k})$$

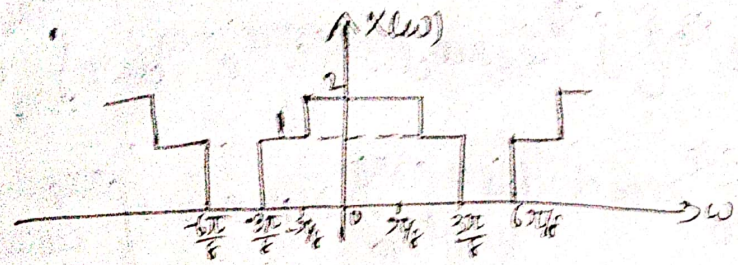
$$= (2M+1)A + 2A\sum_{k=1}^{M}(2M+1-k)\cos\omega k$$

10) Determine the signals having the following fourier transform

(a) $X(\omega) = \begin{cases} 0 & ; 0 < |\omega| < \omega_0 \\ 1 & ; \omega_0 < |\omega| < \pi \end{cases}$ (b) $X(\omega) = \cos^2\omega$

(c) $X(\omega) = \begin{cases} 1 & ; \omega - \pi/2 < |\omega| < \omega + \pi/2 \\ 0 & ; \text{elsewhere} \end{cases}$

(d)



A) a) $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_0} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_0}^{\pi} e^{j\omega n}\, d\omega$$

$$x(\omega) = \frac{1}{2\pi}(\pi-\omega_0) + \frac{1}{2\pi}(\pi-\omega_0) = \frac{\pi-\omega_0}{\pi}$$

For $n \neq 0$, $\int_{-\pi}^{-\omega_0} e^{j\omega n}\, d\omega = \frac{1}{jn} e^{j\omega n}\Big|_{-\pi}^{-\omega_0}$

$$= \frac{1}{jn}(e^{-j\omega_0 n} - e^{-j\pi n})$$

$$\int_{\omega_0}^{\pi} e^{j\omega n}\, d\omega = \frac{1}{jn}\left(e^{j\omega n}\Big|_{\omega_0}^{\pi}\right)$$

$$= \frac{1}{jn}(e^{j\pi n} - e^{j\omega_0 n})$$

Hence $x(n) = -\frac{\sin n\omega_0}{n\pi}$; $n \neq 0$

(b) $X(\omega) = \cos^2(\omega)$

$$= \left(\tfrac{1}{2}(e^{j\omega}) + \tfrac{1}{2}(e^{-j\omega})\right)^2$$

$$= \tfrac{1}{4}(e^{j2\omega} + 2 + e^{-j2\omega})$$

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$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{8\pi}\left[2\pi\delta(n+2) + 4\pi\delta(n) + 2\pi\delta(n-2)\right]$$

$$= \frac{1}{4}\left[\delta(n+2) + 2\delta(n) + \delta(n-2)\right]$$

(c) $$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \delta\omega/2} e^{j\omega n}\, d\omega$$

$$= \frac{2}{\pi}\,\delta(\omega)\left(\frac{\sin(n\delta\omega/2)}{n\delta\omega/2}\right) e^{jn\omega_0}$$

(d) $$x(n) = \frac{1}{2\pi}\, Re\left\{\int_0^{\pi/8} 2e^{j\omega n}\, d\omega + \int_{\pi/8}^{3\pi/8} e^{j\omega n}\, d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n}\, d\omega\right.$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/8} 2\cos n\omega\, d\omega + \int_{\pi/8}^{3\pi/8} \cos n\omega\, d\omega + \int_{6\pi/8}^{7\pi/8} \cos\omega n\, d\omega\right.$$

$$\left. + \int_{7\pi/8}^{\pi} 2\cos n\omega\, d\omega\right]$$

$$= \frac{1}{n\pi}\left[\sin\frac{7\pi n}{8} + \sin\frac{6\pi n}{8} - \sin\frac{3\pi n}{8} - \sin\frac{\pi n}{8}\right]$$

11) Consider the signal

$x(n) = \{1, 0, -1, \underset{\uparrow}{2}, 3\}$ with F.T $X(\omega) = X_R(\omega) + j(X_I(\omega))$. Determine & sketch the signal $y(n)$ with Fourier transform $Y(\omega) = X_R(\omega) + X_R(\omega)e^{j2\omega}$.

A) $$x_e(n) = \frac{x(n) + x(-n)}{2} = \{1/2, 0, 1, \underset{\uparrow}{2}, 1, 0, 1/2\}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2} = \{1/2, 0, -2, 0, 2, 0, 1/2\}$$

Then $$X_R(\omega) = \sum_{n=-3}^{3} x_e(n)\, e^{-j\omega n}$$

$$jX_I(\omega) = \sum_{n=-3}^{3} x_o(n)\, e^{j\omega n}$$

Now $Y(\omega) = X_R(\omega) + X_R(\omega)\, e^{j2\omega}$

$$y(n) = F^{-1}\{X_R(\omega)\} + F^{-1}\{X_R(\omega)\, e^{j2\omega}\}$$

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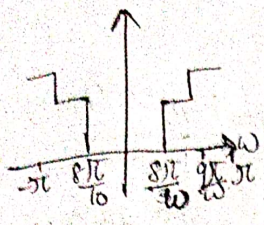
$= -j\,x_o(n) + x_e(n+2)$

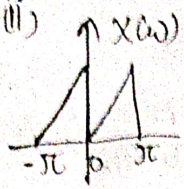
$= \{ 1/2, j0, 1, -j/2, 2, 1, +j/2, \underset{\uparrow}{0}, 1/2, -2j, 0, 1/2 \}$

12) Determine the signal $x(n)$ if its F.T. is

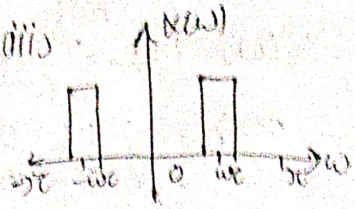
(i)



(ii)



(iii)



A) (i) $x(n) = \frac{1}{2\pi}\left[\int_{8\pi/10}^{9\pi/10} e^{j\omega n}d\omega + \int_{-9\pi/\omega}^{-8\pi/\omega} e^{j\omega n}d\omega + \int_{9\pi/\omega}^{\pi} e^{j\omega n}d\omega + 2\int_{-\pi}^{-9\pi/10} e^{j\omega n}d\omega\right]$

$= \frac{1}{2\pi}\left[\frac{1}{jn}\left(e^{j9\pi n/\omega} - e^{-j9\pi n/10} - e^{j8\pi n/\omega} + e^{j8n\pi/\omega}\right) + \frac{2}{jn}\left(-e^{j9\pi n/10} + e^{-j9\pi n/10} + e^{j\pi n} - e^{-j\pi n}\right)\right]$

$= \frac{1}{n\pi}\left[\sin \pi n - \sin 8\pi n/10 - \sin 9\pi n/\omega\right].$

$= -\frac{1}{n\pi}\left[\sin 4\pi n/5 + \sin 9\pi n/\omega\right].$

(ii) $x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} X(\omega)\, e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{0}^{\pi} X(\omega)\, e^{j\omega n}d\omega$

$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right) e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{0}^{\pi}\frac{\omega}{\pi}\, e^{j\omega n}d\omega$

$= \frac{1}{2\pi}\left[\frac{\omega}{jn\pi}e^{j\omega n}\Big|_{-\pi}^{\pi} + \frac{e^{j\omega n}}{jn}\Big|_{-\pi}^{0}\right].$

$= \frac{1}{jn}\sin\frac{jn}{2}\, e^{-jn\pi/2}$

(iii) $x(n) = \frac{1}{2\pi}\int_{\omega c-\omega/2}^{\omega c+\omega/2} 2e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{-\omega c-\omega/2}^{-\omega c+\omega/2} 2e^{j\omega n}d\omega$

$= \frac{1}{\pi}\left[\frac{1}{jn\pi}e^{j\omega n}\Big|_{\omega c-\omega/2}^{\omega c+\omega/2} + \frac{e^{j\omega n}}{jn}\Big|_{-\omega c-\omega/2}^{-\omega c+\omega/2}\right]$

$= \frac{2}{jn\pi}\left[\frac{e^{j(\omega c+\omega/2)n} - e^{j(\omega c-\omega/2)n} + e^{-j(\omega x-\omega/2)n} - e^{-j(\omega c-\omega/2)n}}{2j}\right]$

$= \frac{2}{jn\pi}\left[\sin(\omega c+\omega/2)n - \sin(\omega c-\omega/2)n\right]$

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13) Given the fourier transform of signal

$x(n) = \begin{cases} 1 & ; -M \le n \le M \\ 0 & ; \text{otherwise} \end{cases}$ was shown to be

$X(\omega) = 1 + 2\sum_{n=1}^{M} \cos\omega n$ Show that to be

$X(\omega) = 1 + 2\sum_{n=1}^{M} \cos\omega n$

$x_1(n) = \begin{cases} 1 & ; 0 \le n \le M \\ 0 & ; \text{otherwise} \end{cases}$; $X_1(\omega) = \dfrac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}$

$x_2(n) = \begin{cases} 1 & ; -M \le n \le -1 \\ 0 & ; \text{otherwise} \end{cases}$; $X_2(\omega) = \dfrac{e^{j\omega} - e^{j\omega(M+1)}}{1 - e^{j\omega}}$

Ans) $X_1(\omega) = \sum_{n=0}^{M} e^{-j\omega n}$

$= \dfrac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}$

$X_2(\omega) = \sum_{n=-M}^{-1} e^{-j\omega n} = \sum_{n=1}^{M} e^{j\omega n}$

$= \dfrac{1 - e^{j\omega M}}{1 - e^{j\omega}} e^{j\omega}$

$X(\omega) = X_1(\omega) + X_2(\omega)$

$= \dfrac{1 + e^{j\omega} - e^{j\omega} - 1 - e^{-j\omega(M+1)} - e^{j\omega(M+1)} + e^{j\omega M} + e^{-j\omega M}}{2 - e^{j\omega} - e^{-j\omega}}$

$= \dfrac{2\cos\omega M - 2\cos\omega(M+1)}{2 - 2\cos\omega}$

$= \dfrac{2\sin(\omega M + \omega/2)\cos\omega/2}{2\sin^2(\omega/2)}$

$= \dfrac{\sin(M + 1/2)\omega}{\sin(\omega/2)}$

14) consider the signal $x(n) = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$ with F.T $X(\omega)$. compute the following quantities without explicity computing $X(\omega)$

(a) $X(0)$ (b) $\angle X(\omega)$ (c) $\int_{-\pi}^{\pi} X(\omega)$ (d) $X(\pi)$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

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A) (a) $X(0) = \sum_n x(n) = -1$

(b) $\angle X(\omega) = \pi$ for all ω

(c) $x(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) d\omega$

$= \int_{-\pi}^{\pi} X(\omega) = 2\pi x(0) = -6\pi$

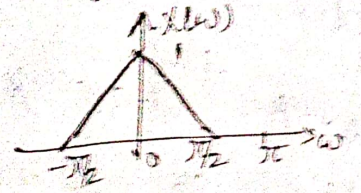
(d) $X(\pi) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\pi} = \sum_n (-1)^n x(n) = -3-4-2 = -9$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = 2\pi \sum_n |x(n)|^2 = (2\pi)(19) = 38\pi$

5) The centre of gravity of a signal $x(n)$ is defined as $c = \sum_{n=-\infty}^{\infty} n x(n) / \sum_{n=-\infty}^{\infty} x(n)$ and provides a measure of a time delay of sig.

(a) Express c in terms of $X(\omega)$

(b) Compute c for sig $x(n)$ whose FT is given as



A) (a) $X(\omega) = \sum_n x(n) e^{-j\omega n}$

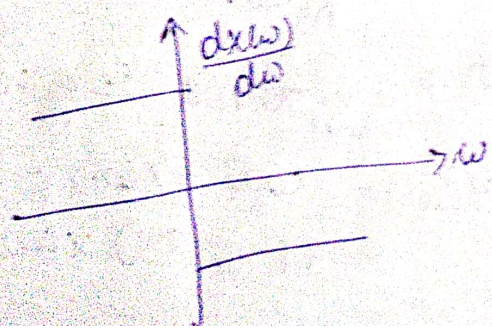
$x(0) = \sum_n x(n)$

$\left.\frac{dX(\omega)}{d\omega}\right|_{\omega=0} = -j \sum_n n x(n) e^{-j\omega n}\Big|_{\omega=0}$

$= -j \sum_n n x(n)$

$c = \frac{j \left.\frac{dX(\omega)}{d\omega}\right|_{\omega=0}}{X(0)}$

(b) $X(0) = 1$ $\quad c = \frac{0}{1} = 0$



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16) Consider the FT pair

$$a^n u(n) \xleftrightarrow{F} \frac{1}{1-ae^{-j\omega}}, \quad |a|<1$$

use the differentiation in frequency theorem and induction to show that.

$$x(n) = \frac{(n+l-1)!}{n!(l-1)!} a^n u(n) \xleftrightarrow{F} X(\omega) = \frac{1}{(1-ae^{-j\omega})^l}$$

A) $x(n) = a^n u(n) \longrightarrow \frac{1}{1-ae^{-j\omega}}$

$$x_k(n) = \frac{(n+k-1)!}{n!(k-1)!} a^n u(n) \rightleftharpoons \frac{1}{(1-ae^{-j\omega})^k}$$

Then $x_{k+1}(n) = \frac{(n+k)!}{n!k!} a^n u(n)$

$$= \frac{n+k}{k} x_k(n)$$

$$X_{k+1}(\omega) = \frac{1}{k} \sum_n n x_k(n) e^{-j\omega n} + \sum_n x_k(n) e^{-j\omega n}$$

$$= \frac{1}{k} j \frac{dX_k(\omega)}{d\omega} + X_k(\omega)$$

$$= \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^{k+1}} + \frac{1}{(1-ae^{-j\omega})^k}$$

17) Let $x(n)$ be an arbitory s/g, not necessarily real valued, with FT $X(\omega)$. Express the FT of following s/g's in terms of $X(\omega)$

(a) $x^*(n)$ (b) $x^*(-n)$ (c) $y(n) = x(n) - x(n-1)$

(d) $y(n) = \sum_{k=-\infty}^{n} x(k)$ (e) $y(n) = x(2n)$

(f) $y(n) = \begin{cases} x(n/2) & ; n \text{ even} \\ 0 & ; n \text{ odd} \end{cases}$

(a) $\sum_n x^*(n) e^{-j\omega n} = \left(\sum_n x(n) e^{-j(-\omega)n}\right)^* = X^*(-\omega)$

(b) $\sum_n x^*(-n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x^*(n) e^{j\omega n} = X^*(\omega)$

(c) $\sum_n y(n) e^{-j\omega n} = \sum_n x(n) e^{-j\omega n} - \sum_n x(n-1) e^{-j\omega n}$

$$Y(\omega) = X(\omega) + X(\omega) e^{-j\omega}$$

$$= (1-e^{-j\omega}) X(\omega)$$

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(d) $y(n) = \sum_{k=-\infty}^{n} x(k)$

$= y(n) - y(n-1)$

$= x(n)$

$X(\omega) = (1-e^{-j\omega})\, Y(\omega)$

$Y(\omega) = \dfrac{X(\omega)}{1-e^{-j\omega}}$

(e) $Y(\omega) = \sum_n x(2n) e^{-j\omega n}$

$= \sum_n x(n) e^{-j\frac{\omega}{2} n}$

$= X(\omega/2)$

(f) $Y(\omega) = \sum_n x(n/2) e^{-j\omega n}$

$= \sum_n x(n) e^{-j2\omega n}$

$= X(2\omega)$

18) Determine & sketch the F.T.s $X_1(\omega)$ & $X_2(\omega)$ & $X(\omega)$ of following s/g's.

(a) $x_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

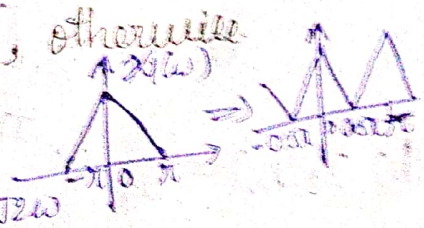
(b) $x_2(n) = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

(c) $x_3(n) = \{1, 0, 0, 1, 0, 0, \underset{\uparrow}{1}, 0, 0, 1, 0, 0, 1\}$

(d) Is there any relation between $X_1(\omega)$, $X_2(\omega)$, $X_3(\omega)$? what is its physical meaning?

(e) show that if $x_k(n) = \begin{cases} x(n/k), & \text{if } n/k \text{ integer} \\ 0, & \text{otherwise} \end{cases}$

then $X_k(\omega) = X(k\omega)$



A) (a) $X_1(\omega) = \sum_n x(n) e^{-j\omega n}$

$= e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$

$= 1 + 2\cos\omega + 2\cos 2\omega$

(b) $X_2(\omega) = \sum_n x_2(n) e^{-j\omega n}$

$= e^{j4\omega} + e^{j2\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$

$= 1 + 2\cos 2\omega + 2\cos 4\omega$

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(c) $X_3(\omega) = \sum_n x_3(n) e^{-j\omega n}$

$= e^{j6\omega} + e^{j3\omega} + 1 + e^{-j3\omega} + e^{-j6\omega}$

$= 1 + 2\cos 3\omega + 2\cos 6\omega$

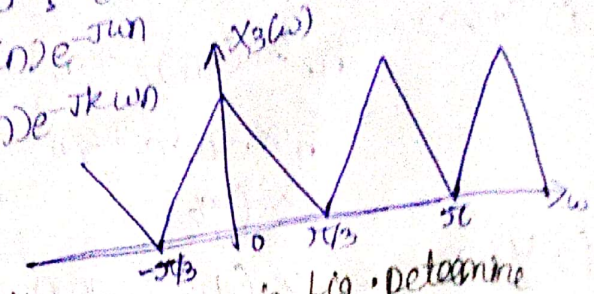
d) $X_2(\omega) = X_1(2\omega)$ & $X_3(\omega) = X_1(3\omega)$

(e) If $x_k(n) = \begin{cases} x(n/k), & n/k \text{ an integer} \\ 0, & \text{otherwise} \end{cases}$

Then $X_k(\omega) = \sum x_k(n) e^{-j\omega n}$

$= \sum_n x(n) e^{-jk\omega n}$

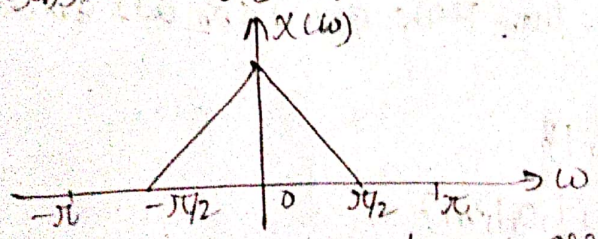
$= X(k\omega)$



19) Let $x(n)$ be a s/g with F.T as in fig. Determine and sketch F.T of s/g's

(a) $x_1(n) = x(n)\cos(\pi n/4)$ (b) $x_2(n) = x(n)\sin(\pi n/2)$

(c) $x_3(n) = x(n)\cos(\pi n/2)$ (d) $x_4(n) = x(n)\cos \pi n$



Note that these s/g sequences are obtained by amplitude modulation of a carrier $\cos\omega n$ (or) $\sin\omega n$ by the sequence $x(n)$.

A) (a) $x_1(n) = \frac{1}{2}(e^{j\pi n/4} + e^{-j\pi n/4})x(n)$

$X_1(\omega) = \frac{1}{2}[X(\omega - \pi/4) + X(\omega + \pi/4)]$

(b) $x_2(n) = \frac{1}{2j}(e^{j\pi n/2} + e^{-j\pi n/2})x(n)$

$X_2(\omega) = \frac{1}{2j}[X(\omega - \pi/2) + X(\omega + \pi/2)]$

(c) $x_3(n) = \frac{1}{2}(e^{j\pi n/2} + e^{-j\pi n/2})x(n)$

$X_3(\omega) = \frac{1}{2}[X(\omega - \pi/2) + X(\omega + \pi/2)]$

(d) $x_4(n) = \frac{1}{2}(e^{j\pi n} + e^{-j\pi n})x(n)$

$X_4(\omega) = \frac{1}{2}[X(\omega - \pi) + X(\omega + \pi)]$

$= X(\omega - \pi)$

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2a) Consider an aperiodic s/g $x(n)$ with F.T $X(\omega)$. Show that the F.S coefficients c_k^y of the periodic s/g $y(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$ are given by

$$c_k^y = \frac{1}{N} X\left(\frac{2\pi}{N}k\right), \quad k = 0, 1, \ldots N-1$$

A) $$c_k^y = \frac{1}{N} \sum_{n=0}^{N-1} y(n)\, e^{-j2\pi kn/N}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} \left[\sum_{l=-\infty}^{\infty} x(n-lN)\right] e^{-j2\pi kn/N}$$

$$= \frac{1}{N} \sum_{l=-\infty}^{\infty} \sum_{m=-lN}^{N-1-lN} x(m)\, e^{-j2\pi k(m+lN)/N}$$

But $$\sum_{l=-\infty}^{\infty} \sum_{m=-lN}^{N-1-lN} x(m)\, e^{-j\omega(m+lN)} = X(\omega)$$

$$\therefore c_k^y = \frac{1}{N} X\left(\frac{2\pi k}{N}\right).$$

2) prove that

$$X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin \omega_c n}{\pi n} e^{-j\omega n}$$ may be expressed

as $$X_N(\omega) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{\sin[(2N+1)(\omega-\theta)/2]}{\sin[(\omega-\theta)/2]}\, d\theta$$

A) Let $$x_N(n) = \frac{\sin \omega_c n}{\pi n}, \quad -N \le n \le N$$

$$= x(n)\, w(n)$$

where $$x(n) = \frac{\sin \omega_c n}{\pi n}, \quad -\infty \le n \le \infty$$

$$w(n) = 1, \quad -N \le n \le N$$
$$\quad\;\; 0, \quad \text{otherwise}$$

Then $$\frac{\sin \omega_c n}{\pi n} \xleftrightarrow{F} X(\omega)$$
$$= 1, \quad |\omega| \le \omega_c$$
$$= 0, \quad \text{otherwise}$$

$$X_N(\omega) = X(\omega) * W(\omega)$$

$$= \int_{-\pi}^{\pi} X(\theta)\, W(\omega - \theta)\, d\theta$$

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$$= \int_{-\omega_c}^{\omega_c} \frac{\sin((2N+1)(\omega-\theta)/2)}{\sin(\omega-\theta)/2} d\theta$$

Q) A sig $x(n)$ has the following F.T $X(\omega) = \frac{1}{1-ae^{-j\omega}}$. Determine the F.T of following sig's

(a) $x(2n+1)$ (b) $e^{j\pi n/2} x(n+2)$ (c) $x(-2n)$

(d) $x(n) \cos(0.3\pi n)$ (e) $x(n) * x(n-1)$ (f) $x(n) * x(-n)$

A) (a) $X_1(\omega) = \sum_n x(2n+1) e^{-j\omega n}$

$$= \sum_k x(k) e^{-j\omega k/2} \cdot e^{j\omega/2}$$

$$= X(\omega/2) e^{j\omega/2}$$

$$= \frac{e^{j\omega/2}}{1-ae^{j\omega/2}}$$

(b) $X_2(\omega) = \sum_n x(n+2) e^{j\pi n/2} e^{-j\omega n}$

$$= -\sum_k x(k) e^{-jk(\omega + j\pi/2)} e^{j2\omega}$$

$$= -X(\omega + j\pi/2) e^{j2\omega}$$

(c) $X_3(\omega) = \sum_n x(-2n) e^{-j\omega n}$

$$= -\sum_k x(k) e^{-jk\omega/2}$$

$$= X(-\omega/2)$$

(d) $X_4(\omega) = \sum_n \frac{1}{2}(e^{j0.3\pi n} + e^{-j0.3\pi n}) x(n) e^{-j\omega n}$

$$= \frac{1}{2}\sum_n x(n)[e^{-j(\omega-0.3\pi)n} + e^{-j(\omega+0.3\pi)n}]$$

$$= \frac{1}{2}[X(\omega-0.3\pi) + X(\omega+0.3\pi)]$$

(e) $X_5(\omega) = X(\omega)[X(\omega) e^{-j\omega}] = X^2(\omega) e^{-j\omega}$

(f) $X_6(\omega) = X(\omega) X(-\omega)$

$$= \frac{1}{(1-ae^{-j\omega})(1-ae^{j\omega})}$$

$$= \frac{1}{(1-2a\cos\omega+a^2)}$$

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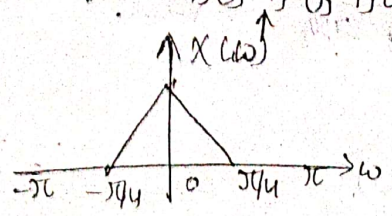
23) From a discrete-time s/g $x(n)$ with F.T $X(\omega)$, determine and sketch the F.T of following s/gs

(a) $y_1(n) = \begin{cases} x(n) & ; n \text{ even} \\ 0 & ; n \text{ odd} \end{cases}$

(b) $y_2(n) = x(2n)$

(c) $y_3(n) = \begin{cases} x(n/2) & ; n \text{ even} \\ 0 & ; n \text{ odd} \end{cases}$ Note that $y_1(n) = x(n)s(n)$

where $s(n) = \{\ldots 0, 1, 0, 1, 0, 1, 0, 1, \ldots\}$

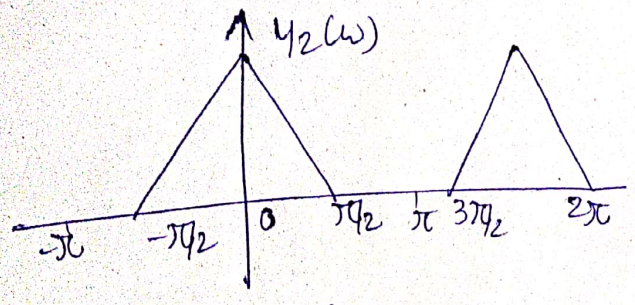


A) (a) $Y_1(\omega) = \sum_n y_1(n) e^{-j\omega n} = \sum_{n \text{ even}} x(n) e^{-j\omega n}$

The F.T $Y_1(\omega)$ can easily be obtained by combining result of (b) & (c)

(b) $y_2(n) = x(2n)$

$$Y_2(\omega) = \sum_n y_2(n) e^{-j\omega n}$$
$$= \sum_n x(2n) e^{-j\omega n}$$
$$= \sum_m x(m) e^{-j\omega m/2}$$
$$= X(\omega/2)$$



(c) $y_3(n) = \begin{cases} x(n/2) & ; n \text{ even} \\ 0 & ; \text{otherwise} \end{cases}$

$$Y_3(\omega) = \sum_n y_3(n) e^{-j\omega n}$$
$$= \sum_{n \text{ even}} x(n/2) e^{-j\omega n}$$
$$= \sum_m x(m) e^{-j2\omega m} = X(2\omega)$$

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