MTL732 Assignment

Ravi Pushkar 2018MT60790
Satendra Singh Parashar 2018MT60793

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Let the returns on n companies be $K_1, K_2, K_3, ..., K_n$ and let the corresponding weights be $w_1, w_2, w_3, ..., w_n$ and the corresponding mean be $\mu_1, \mu_2, \mu_3, ..., \mu_n$, respectively. Also, let the variance-covariance matrix be $C = [C_{ij}]$.

Suppose short selling is not allowed. A feasible portfolio is defined by

$$\sum_{i=1}^{n} w_i = 1 \tag{1}$$

$$w_i \geq 0, \quad i = 1, 2, ..., n \tag{2}$$

Return of the portfolio $K_v = w^T K$, where $K = (K_1, K_2, ..., K_m)$, $w = (w_1, w_2, ..., w_n)$.

We say that $-K_v$ is the loss. So, the investor will maximize the return such that $loss \leq d$
Thus, the problem simplifies to finding

$$\begin{gathered} \max_w (w^T K) \\ \text{such that } -K_v \leq d \quad or - w^T K \leq d \\ \textstyle\sum_{i=1}^n w_i = 1 \\ w_i \geq 0, \quad i = 1, 2, 3, ..., n \end{gathered}$$

This is a stochastic optimization problem.

$$\begin{gathered} \max_w (E[w^T K] \text{ such that } P(-w^T K \leq d) \geq \alpha \\ \textstyle\sum_{i=1}^n w_i = 1 \\ w_i \geq 0 \quad i = 1, 2, 3, ..., n \end{gathered}$$

The above is called chance constraints.
$P(w| - w^T K \leq d) \geq \alpha$

Now, Let K be a multivariate normal distribution. Suppose $K \sim N(\mu, \sigma)$ whose pdf is $\frac{1}{\sqrt{2\pi}^n C^{\frac{1}{2}}} exp(\frac{-1}{2}(x - \mu)^T C^{-1}(x - \mu))$

$$-w^T K \sim N(-w^T \mu, w^T C w) \tag{3}$$

$-w^T K \sim N(-w^T \mu, w^T C w)$
Here, we have assume that $|C| \neq 0$ as C is positive definite matrix.

$$P(\frac{-w^T K + w^T \mu}{\sqrt{w^T C w}} \leq \frac{d + w^T \mu}{\sqrt{w^T C w}}) \geq \alpha \tag{4}$$

$$P(z \leq \frac{d + w^T\mu}{\sqrt{w^T C w}}) \geq \alpha \tag{5}$$

$$\phi(\frac{d + w^T\mu}{\sqrt{w^T C w}}) \geq \alpha \tag{6}$$

$$\frac{d + w^T\mu}{\sqrt{w^T C w}} \geq \phi^{-1}(\alpha) \tag{7}$$

$$d \geq -w^T\mu + \phi^{-1}(\alpha)\sqrt{w^T C w} \tag{8}$$

As we have,

$$\sqrt{w^T C w} = ||C^{\frac{1}{2}} w||_2 \tag{9}$$

and,

$$d \geq -w^T\mu + \phi^{-1}(\alpha)||C^{\frac{1}{2}} w||_2 \tag{10}$$

So, the optimization problem is

$$\begin{gathered} max_w(w^T\mu) \\ \text{subject to } \textstyle\sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0, \quad i = 0, 1, 2, ..., n \\ -w^T\mu + \phi^{-1}(\alpha)||C^{\frac{1}{2}} w||_2 \leq d \end{gathered}$$

Clearly, this is a convex optimization problem, and the function in total is convex for $\alpha \in [0.5, 1)$ as the function $\phi^{-1}(\alpha)$ is positive in this range and $-w^T\mu$, and $||C^{\frac{1}{2}} w||_2$

are both convex functions. And their positive linear combination (in which the coefficients are positive) will be a convex function.