

1. (i) Number of comparisons:

To derive the formula for the total number of comparisons made by bubble sort for an array of size n , we can sum the number of comparisons made in each pass. In each pass, one less comparison is made than the previous pass. Therefore, the total number of comparisons is the sum of the first $n-1$ positive integers, which can be calculated using the formula:

$$1+2+3+\dots+(n-1) =$$

$$\frac{n(n-1)}{2}$$

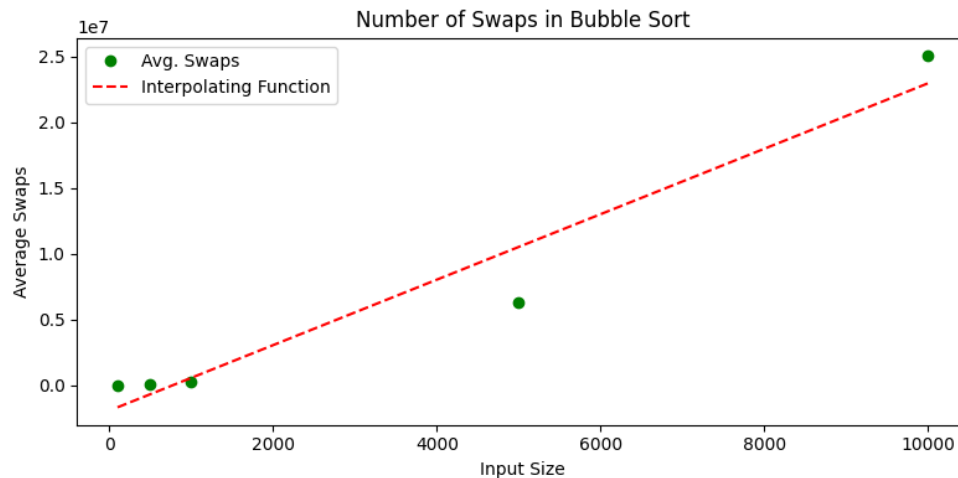
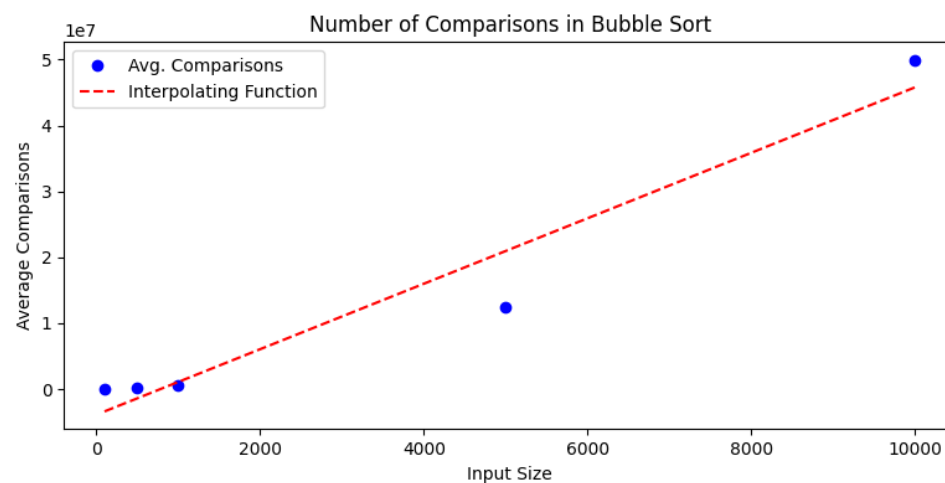
(ii) Average-case number of swaps:

In the average case, the number of swaps is estimated as half of the number of comparisons:

Average Swap

$$= \frac{\text{Total Comparisons}}{2} = \frac{n(n-1)}{4}$$

4.



The plots of average comparisons and swaps against input sizes both show a linear increase as the input size grows. In bubble sort, each comparison and each swap contributes to the overall

time complexity. Since both the number of comparisons and swaps grow linearly with the input size, the combined effect results in a quadratic time complexity, denoted as $O(n^2)$. Therefore, the linear trends observed in both plots align with the expected quadratic complexity of bubble sort.