Applied Geophysics – GEO 5660/6660
Quiz # 1, January 28th

Duration: 30 min
Relevant expressions on next page!

1. Velocity of P-waves in a certain limestone layer is 3.6 km/s. What are the wavelengths of (a) 10 Hz, and (b) 100 Hz P-waves traveling through this layer? (c) If its Poisson's Ratio, *v*, equals 0.18, what is the S-wave velocity through this layer? Express your answer to the nearest 100th (*km/s*).

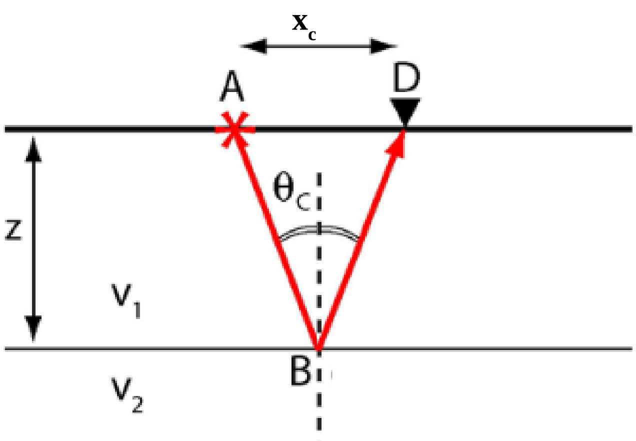
2. (a) In HW #1, we noted that the maximum theoretical value of Poisson's Ratio, *v*, was attained by ideal incompressible fluids. What would the S-wave velocity be in such a material? What can you say about its shear modulus, μ?

(b) In the Earth, shear wave velocity increases with depth, and so does density. What can be (qualitatively) said about the rate of increase in shear modulus with depth compared to that of the density?

3. In class, we derived the energy density for a monochromatic plane wave as: $\hat{e} = \rho c \omega^2 A^2/2$ (units: W/m^2), where ρ is the density, A the wave amplitude, c the wave velocity (V_p or V_s), and ω, the angular frequency ($2\pi/T$; T = period). We define (ρc) as the impedance, Z. A plane P-wave travelling through a gneissic basement (ρ = 2,500 kg/m^3; V_p = 3.2 km/s) enters a shallow sedimentary basin consisting mostly of unconsolidated sediments (ρ = 2,000 kg/m^3; V_p = 1.6 km/s).

Assuming no energy losses due to scattering at the interface between the basement and the sediments (that is, no reflections and no refracted S-wave conversions), by what factor does the P-wave amplitude increase within the basin? ***Hint:*** *Invoke conservation of energy.*

4. In the following seismic experiment, head-waves first appear at a cross-over distance, $AD = x_c$ = 200 m. V_1 = 1.5 km/s; V_2 = 4 km/s. What is the depth of the layer, z? Round to the nearest 10 m.



RELEVANT EXPRESSIONS

I. Velocities of P- and S-waves, V_P, V_S, in terms of the Lamé constants, λ, & μ (shear modulus), ρ, the density, K, the bulk modulus, and ν, the Poisson's ratio:

(a) $V_P = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}$

(b) $V_S = \sqrt{\frac{\mu}{\rho}}$

(c) $\nu = \frac{\lambda}{2(\lambda + \mu)}$

(d) $\frac{V_P}{V_S} = \sqrt{\frac{2(1-\nu)}{1-2\nu}}$

II. Snell's Law:

$$\frac{V_1}{V_2} = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

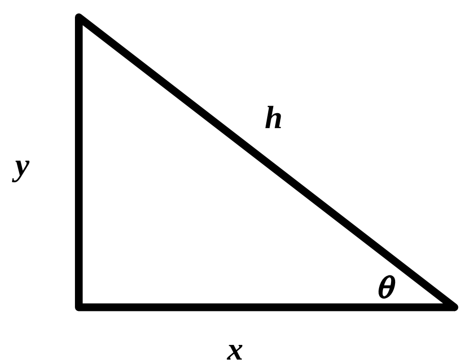
III. Basic Trigonometric relations:

$\sin(\theta) = \frac{y}{h}$;

$\cos(\theta) = \frac{x}{h}$;

$\tan(\theta) = \frac{y}{x}$;

$h = \sqrt{x^2 + y^2}$



Quiz #1

Ram Kanda
01/27/2020

1. $V_p = 3600\ m/s$ (a) $\nu = 10\,Hz \Rightarrow \lambda = \frac{V_p}{\nu} = 360\,m$

(b) $\nu = 100\,Hz \Rightarrow \lambda = \frac{V_p}{\nu} = 36\,m$

(c) From I(d) $\frac{V_P}{V_S} = \sqrt{\frac{2(1-\nu)}{(1-2\nu)}}$; $\nu = 0.18$

$$\therefore V_S = V_P\sqrt{\frac{1-2\nu}{2(1-\nu)}} = 3600\sqrt{\frac{(1-0.36)}{2(1-0.18)}}$$

$$= 3600\sqrt{\frac{\cancel{(0.64)}\ 0.32}{\cancel{2}\,(0.82)}}$$

$$\approx 3600 \times 0.625$$

$$\simeq 2250\ m/s$$

$$\therefore \boxed{V_S = 2.25\ km/s}$$

Maximum Value of $\nu \approx 0.5$ [for incompressible fluids]

2. (a) From I(d): $\lim_{\nu \to 0.5} \frac{V_P}{V_S} = \infty \Rightarrow \underline{V_S = 0}$ & by $\underline{I(b): \mu = 0}$

(b) $V_S = \sqrt{\mu/\rho}$: $\partial\mu/\partial z > \partial\rho/\partial z$ or Shear Modulus increases at a faster rate than density with increasing depth

3. Given no scattering (reflections or S-conversions)

Energy conservation for the wave gives:

$$\rho_g V_{P,g} A_g^2 = \rho_s V_{P,s} A_s^2$$

$$\therefore \frac{A_s}{A_g} = \sqrt{\frac{\rho_g V_{P,g}}{\rho_s V_{P,s}}} = \sqrt{\frac{\overset{5}{\cancel{2500}} \times \overset{4}{\cancel{3.2}}}{\underset{2}{\overset{4}{\cancel{2000}}} \times 1.6}} = \sqrt{2.5}$$

$$\therefore \frac{A_s}{A_g} = 1.581\ldots \sim 1.6$$

Plane wave gets amplified by a factor of 1.6 or 160%

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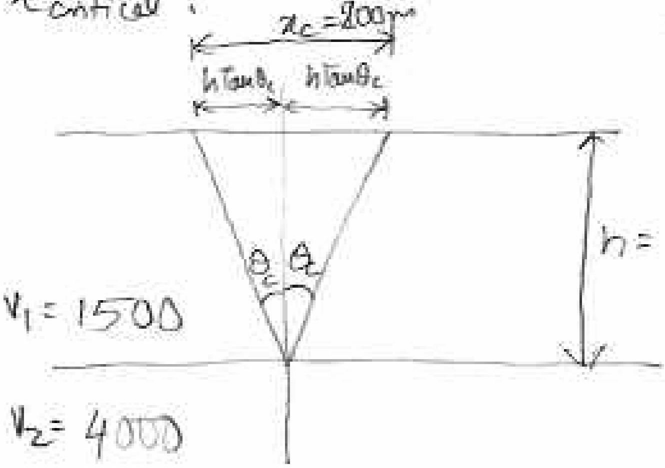
4. $x_{critical}$:



$$x_c = 2h \tan\theta_c \quad \cdots \text{(I)}$$

where, $\theta_c = \sin^{-1}\left(\frac{V_1}{V_2}\right) \cdots \text{(II)}$

$$\therefore \theta_c = \tan^{-1}\left(\frac{V_1}{\sqrt{V_2^2 - V_1^2}}\right) \cdots \text{(III)}$$

from (I) & (III), $x_c = 2h \tan\left(\tan^{-1}\left(\frac{V_1}{\sqrt{V_2^2 - V_1^2}}\right)\right)$

$$\Rightarrow x_c = \frac{2hV_1}{\sqrt{V_2^2 - V_1^2}}$$

$$\therefore h = \left(\frac{x_c}{2}\right)\sqrt{\left(\frac{V_2}{V_1}\right)^2 - 1}$$

$$\simeq 100\ (2.472)\ m$$

$$\therefore \boxed{h \approx 247\,m \sim 250\,m}$$