

Assignment 1

Ravi Kumar - SM21Mtech12010

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Chapter II, Examples II,Q.22(i)

1. Find the conditions that the four points $\begin{pmatrix} x1 \\ y1 \end{pmatrix}, \begin{pmatrix} x2 \\ y2 \end{pmatrix}, \begin{pmatrix} x3 \\ y3 \end{pmatrix}, \begin{pmatrix} x4 \\ y4 \end{pmatrix}$ may be the vertices of a square.

Solution :

The given points are:

$$\mathbf{P} = \begin{pmatrix} x1 \\ y1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} x2 \\ y2 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} x3 \\ y3 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} x4 \\ y4 \end{pmatrix},$$

Condition for the given four points be the vertices of a square are :-

- 1) If distance between all the four sides are equal and
- 2) distance between two diagonals are equal.

Now If we have two vectors, say,

$$\mathbf{U} = \begin{pmatrix} u1 \\ u2 \end{pmatrix}, \mathbf{V} = \begin{pmatrix} v1 \\ v2 \end{pmatrix}$$

then distance can be calculated using norm of a vector, i.e.,

$$\|\mathbf{U} - \mathbf{V}\| = \sqrt{(u1 - v1)^2 + (u2 - v2)^2}$$

Here,

$$d1 = \|\mathbf{P} - \mathbf{Q}\| = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

$$d2 = \|\mathbf{Q} - \mathbf{R}\| = \sqrt{(x3 - x2)^2 + (y3 - y2)^2}$$

$$d3 = \|\mathbf{R} - \mathbf{S}\| = \sqrt{(x4 - x3)^2 + (y4 - y3)^2}$$

$$d4 = \|\mathbf{S} - \mathbf{P}\| = \sqrt{(x1 - x4)^2 + (y1 - y4)^2}$$

$$diagonal1 = \sqrt{d1^2 + d2^2}$$

$$diagonal2 = \sqrt{d2^2 + d3^2}$$

Now if

- 1) $d1 = d2 = d3 = d4$
- 2) $diagonal1 = diagonal2$

Then, we can say that the given point are the vertices of a square.