

SM5083

Assignment 1

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1. CHAPTER II EXAMPLE II Q22(i)

Find the conditions that the four points

$$\begin{pmatrix} x1 \\ y1 \end{pmatrix}, \begin{pmatrix} x2 \\ y2 \end{pmatrix}, \begin{pmatrix} x3 \\ y3 \end{pmatrix}, \begin{pmatrix} x4 \\ y4 \end{pmatrix}$$

may be the vertices of a square.

Solution:

Given (1)

$$\mathbf{A} = \begin{pmatrix} x1 \\ y1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x2 \\ y2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x3 \\ y3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} x4 \\ y4 \end{pmatrix} \quad (2)$$

Condition for the given four points be the vertices of a square are ;-

- 1) If distance between all the four sides are equal and
- 2) distance between two diagonals are equal. Now

If we have two vectors, say,

$$\mathbf{U} = \begin{pmatrix} x1 \\ y1 \end{pmatrix} \text{ and } \mathbf{V} = \begin{pmatrix} x2 \\ y2 \end{pmatrix} \quad (3)$$

then distance can be calculated using norm of a vector, i.e.,

$$\|\mathbf{U} - \mathbf{V}\| = \sqrt{(u1 - v1)^2 + (u2 - v2)^2} \quad (4)$$

Here, From equations (4)

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(x2 - x1)^2 + (y2 - y1)^2} \quad (5)$$

$$\|\mathbf{B} - \mathbf{C}\| = \sqrt{(x3 - x2)^2 + (y3 - y2)^2} \quad (6)$$

$$\|\mathbf{C} - \mathbf{D}\| = \sqrt{(x4 - x3)^2 + (y4 - y3)^2} \quad (7)$$

$$\|\mathbf{D} - \mathbf{A}\| = \sqrt{(x1 - x4)^2 + (y1 - y4)^2} \quad (8)$$

and then calculate distance of diagonal using

$$\text{diagonal} = \sqrt{2} * \text{sidelength} \quad (9)$$

Now from equations (5), (6), (7) and (9)

if,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{C} - \mathbf{D}\| = \|\mathbf{D} - \mathbf{A}\| \quad (10)$$

And from equation (9), calculate diagonal of square if,

$$\text{diagonal1} = \text{diagonal2} \quad (11)$$

Now from equation (10) and (11)

if both equation satisfy

Then, we can say that the given point are the vertices of a square.

Python code at:

https://github.com/ravi12010/SM5083_Assignment1/blob/main/Assignment_1.ipynb

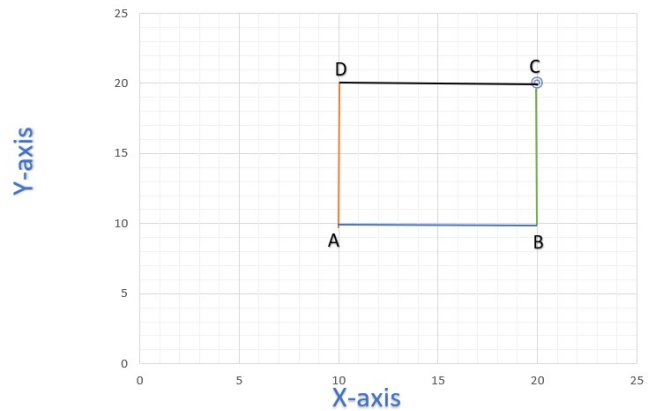


Fig. 0. The given points form a square