SM5083
Assignment 1

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1. Chapter II Example II Q22(i)

Find the conditions that the four points

$$\begin{pmatrix} x1 \\ y1 \end{pmatrix}, \begin{pmatrix} x2 \\ y2 \end{pmatrix}, \begin{pmatrix} x3 \\ y3 \end{pmatrix}, \begin{pmatrix} x4 \\ y4 \end{pmatrix}$$

may be the vertices of a square.

Solution:

$$\text{Given} \tag{1}$$

$$\mathbf{A} = \begin{pmatrix} x1 \\ y1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x2 \\ y2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x3 \\ y3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} x4 \\ y4 \end{pmatrix} \tag{2}$$

Condition for the given four points be the vertices of a square are ;-
1) If distance between all the four sides are equal and
2) distance between two diagonals are equal. Now

If we have two vectors, say,

$$\mathbf{U} = \begin{pmatrix} x1 \\ y1 \end{pmatrix} \quad \text{and } \mathbf{V} = \begin{pmatrix} x2 \\ y2 \end{pmatrix} \tag{3}$$

then distance can be calculated using norm of a vector, i.e.,

$$||\mathbf{U} - \mathbf{V}|| = \sqrt{(x2 - x1)^2 + (y2 - y1)^2} \tag{4}$$

Here, From equations (4)

$$||\mathbf{A} - \mathbf{B}|| = \sqrt{(x2 - x1)^2 + (y2 - y1)^2} \tag{5}$$

$$||\mathbf{B} - \mathbf{C}|| = \sqrt{(x3 - x2)^2 + (y3 - y2)^2} \tag{6}$$

$$||\mathbf{C} - \mathbf{D}|| = \sqrt{(x4 - x3)^2 + (y4 - y3)^2} \tag{7}$$

$$||\mathbf{D} - \mathbf{A}|| = \sqrt{(x1 - x4)^2 + (y1 - y4)^2} \tag{8}$$

and then calculate distance of diagonal using

$$\mathbf{diagonal} = \sqrt{2} * \mathbf{sidelength} \tag{9}$$

Now from equations (5), (6), (7) and (8) if,

$$||\mathbf{A} - \mathbf{B}|| = ||\mathbf{B} - \mathbf{C}|| = ||\mathbf{C} - \mathbf{D}|| = ||\mathbf{D} - \mathbf{A}|| \tag{10}$$

And from equation (9), calculate diagonal of square if,

$$\mathbf{diagonal1} = \mathbf{diagonal2} \tag{11}$$

Now from equation (10) and (11)
if both equation satisfy
Then, we can say that the given point are the vertices of a square.

Python code at:

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https://github.com/ravi12010/
    SM5083_Assignment1/blob/main/
    Assignment_1.ipynb
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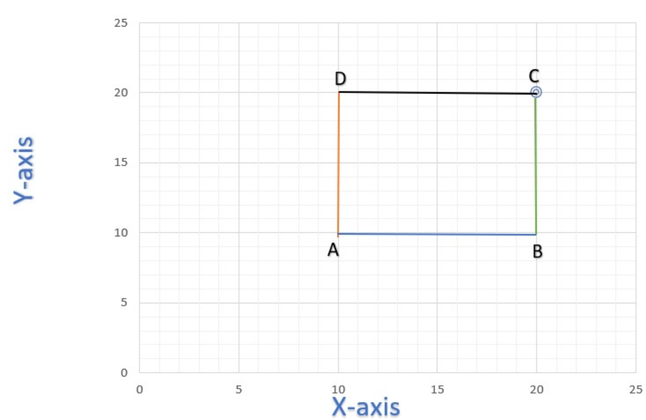

Fig. 0. The given points form a square