Assignment 1

Ravi Kumar - SM21Mtech12010

August 2021

Chapter II, Examples II,Q.22(i)

1.Find the conditions that the four points $\binom{x1}{y1}, \binom{x2}{y2}, \binom{x3}{y3}, \binom{x4}{y4}$ may be the vertices of a square.

Solution :

The given points are:

$$\mathbf{P} = \binom{x1}{y1}, \mathbf{Q} = \binom{x2}{y2}, \mathbf{R} = \binom{x3}{y3}, \mathbf{S} = \binom{x4}{y4},$$

Condition for the given four points be the vertices of a square are ;-
1) If distance between all the four sides are equal and
2) distance between two diagonals are equal.

Now If we have two vectors, say,

$$\mathbf{U} = \binom{u1}{u2}, \mathbf{V} = \binom{v1}{v2}$$

then distance can be calculated using norm of a vector, i.e.,

$$\|\mathbf{U} - \mathbf{V}\| = \sqrt{(u1 - v1)^2 + (u2 - v2)^2}$$

Here,

$$\begin{aligned} d1 &= \|\mathbf{P} - \mathbf{Q}\| = \sqrt{(x2 - x1)^2 + (y2 - y1)^2} \\ d2 &= \|\mathbf{Q} - \mathbf{R}\| = \sqrt{(x3 - x2)^2 + (y3 - y2)^2} \\ d3 &= \|\mathbf{R} - \mathbf{S}\| = \sqrt{(x4 - x3)^2 + (y4 - y3)^2} \\ d4 &= \|\mathbf{S} - \mathbf{P}\| = \sqrt{(x1 - x4)^2 + (y1 - y4)^2} \end{aligned}$$

and then calculate distance of diagonal using

$$diagonal = \sqrt{2} * sidelength$$

Now if
1) d1 = d2 = d3 = d4
2) diagonal1 = diagonal2

Then, we can say that the given point are the vertices of a square.