SM5083
Assignment 2

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1. CHAPTER III MISCELLANEOUS EXAMPLES VI Q9

If the lines $y = x\tan(\frac{11\pi}{24})$, $y = x\tan(\frac{19\pi}{24})$ be at right angles, Show that the angle between the axes is $(\frac{\pi}{4})$

Solution:

Given equation of two lines are (1)

$$\mathbf{y} = x\tan(\frac{11\pi}{24}) \tag{2}$$

$$\mathbf{y} = x\tan(\frac{19\pi}{24}) \tag{3}$$

Let the equations of the straight lines AB and CD are i.e.

$$\mathbf{y} = m_1\mathbf{x} + \mathbf{c_1} \quad \text{and } \mathbf{y} = m_2\mathbf{x} + \mathbf{c_2} \ \textit{respectively}, \tag{4}$$

intersect at a point P and make angles $\theta 1$ and $\theta 2$ respectively with the positive direction of x-axis
So the angle θ between the lines having slope m_1 and m_2 is given by

$$\tan\theta = \frac{(m_2 - m_1)}{1 + m_1 m_2} \tag{5}$$

Clearly, the slope of the line AB and CD are m_1 and m_2 respectively.
From equations (2) and (3)

$$\mathbf{m_1} = \tan(\frac{11\pi}{24}) \tag{6}$$

$$\mathbf{m_2} = \tan(\frac{19\pi}{24}) \tag{7}$$

Now, put the value of m_1 and m_2 in equations (5)

$$\tan\theta = \frac{\tan(\frac{19\pi}{24}) - \tan(\frac{11\pi}{24})}{1 + \tan(\frac{19\pi}{24})\tan(\frac{11\pi}{24})} \tag{8}$$

Using Formula

$$\tan(\mathbf{A} - \mathbf{B}) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)} \tag{9}$$

Then,

$$\tan\theta = \tan(\frac{19\pi}{24} - \frac{11\pi}{24}) \tag{10}$$

$$\tan\theta = \tan(\frac{8\pi}{24}) \tag{11}$$

$$\tan\theta = \tan(\frac{\pi}{3}) \tag{12}$$

So,

$$\theta = \frac{\pi}{3} \tag{13}$$

Hence, the angle between the axes is not $(\frac{\pi}{4})$

Python code at:

https://github.com/ravi12010/SM5083_Assignment2/blob/main/Assignment2.ipynb