SM5083
Assignment 1

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1. CHAPTER III MISCELLANEOUS EXAMPLES VI Q9

If the lines $y = x\tan(\frac{11\pi}{24})$, $y = x\tan(\frac{19\pi}{24})$ be at right angles, Show that the angle between the axes is $(\frac{\pi}{4})$

Solution:

Given equation of two lines are (1)

$$\mathbf{y} = x\tan(\frac{11\pi}{24}) \tag{2}$$

$$\mathbf{y} = x\tan(\frac{19\pi}{24}) \tag{3}$$

Let the equations of the straight lines AB and CD are i.e.

$$\mathbf{y} = m_1(\mathbf{x} + \mathbf{c_1}) \quad \text{and } \mathbf{y} = m_2(\mathbf{x} + \mathbf{c_2}) \ respectively, \tag{4}$$

intersect at a point P and make angles $\theta 1$ and $\theta 2$ respectively with the positive direction of x-axis
So the angle θ between the lines having slope m_1 and m_2 is given by

$$\tan\theta = \frac{(m_2 - m_1)}{1 + m_1 m_2} \tag{5}$$

Clearly, the slope of the line AB and CD are m_1 and m_2 respectively.
From equations (2) and (3)

$$\mathbf{m_1} = \tan(\frac{11\pi}{24}) \tag{6}$$

$$\mathbf{m_2} = \tan(\frac{19\pi}{24}) \tag{7}$$

Now, put the value of m_1 and m_2 in equations (5)

$$\tan\theta = \frac{\tan(\frac{19\pi}{24}) - \tan(\frac{11\pi}{24})}{1 + \tan(\frac{19\pi}{24})\tan(\frac{11\pi}{24})} \tag{8}$$

Using Formula

$$\tan(\mathbf{A} + \mathbf{B}) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)} \tag{9}$$

Then,

$$\tan\theta = \tan(\frac{19\pi}{24} + \frac{11\pi}{24}) \tag{10}$$

$$\tan\theta = \tan(\frac{30\pi}{24}) \tag{11}$$

$$\tan\theta = \tan(\frac{5\pi}{4}) \tag{12}$$

$$\tan\theta = \tan(\pi + \frac{\pi}{4}) \tag{13}$$

So,

$$\tan\theta = \tan(\frac{\pi}{4}) \tag{14}$$

Hence, the angle between the axes is $(\frac{\pi}{4})$