

# MATHEMATICS

January 4, 2024

1. Write the discriminant of the quadratic equation  $(x + 5)^2 = 2(5x - 3)$ .
2. Find after how many places of decimal the decimal form of the number  $\frac{27}{2^3 \cdot 5^4 \cdot 3^2}$  will terminate.
3. Express 429 as a product of its prime factors.
4. Find the sum of first 10 multiples of 6
5. Find the value ( $s$ ) of  $x$ , if the distance between the points  $A(0, 0)$  and  $B(x, -4)$  is 5 units.
6. Two concentric circles of radii  $a$  and  $b$  ( $a > b$ ) are given. Find the length of the chord of the larger circle which touches the smaller circle.
7. In Figure 1,  $PS = 3\text{ cm}$ ,  $QS = 4\text{ cm}$ ,  $\angle PRQ = \theta$ ,  $\angle PSQ = 90^\circ$ ,  $PQ \perp RQ$  and  $RQ = 9\text{ cm}$ . Evaluate  $\tan \theta$ .

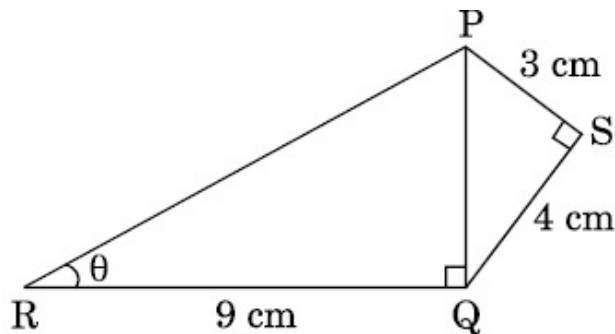


Figure 1: Triangle PSQR

8. If  $\tan \alpha = \frac{5}{12}$ , find the value of  $\sec \alpha$ .
9. Points  $A(3, 1)$ ,  $B(5, 1)$ ,  $C(a, b)$  and  $D(4, 3)$  are vertices of a parallelogram  $ABCD$ . Find the values of  $a$  and  $b$ .

10. Points  $P$  and  $Q$  trisect the line segment joining the points  $A(-2, 0)$  and  $B(0, 8)$  such that  $P$  is near to  $A$ . Find the coordinates of points  $P$  and  $Q$ .

11. Solve the following pair of linear equations :

$$3x - 5y = 4$$

$$2y + 7 = 9x$$

12. If HCF of 65 and 117 is expressible in the form  $65n - 117$ , then find the value of  $n$ .

13. On a morning walk, three persons step out together and their steps measure  $30\text{cm}$ ,  $36\text{cm}$  and  $40\text{cm}$  respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps ?

14. A die is thrown once. Find the probability of getting

(a) a composite number,

(b) a prime number.

15. Using completing the square method, show that the equation  $x^2 - 8x + 18 = 0$  has no solution.

16. Cards numbered 7 to 40 were put in a box. Poonam selects a card at random. What is the probability that Poonam selects a card which is a multiple of 7 ?

17. The perpendicular from  $A$  on side  $BC$  of a  $\triangle ABC$  meets  $BC$  at  $D$  such that  $DB = 3CD$ . Prove that  $2AB^2 = 2AC^2 + BC^2$ .

18.  $AD$  and  $PM$  are medians of triangles  $ABC$  and  $PQR$  respectively where  $\triangle ABC \sim \triangle PQR$ . Prove that  $\frac{AB}{PQ} = \frac{AD}{PM}$ .

19. Check whether  $g(x)$  is a factor of  $p(x)$  by dividing polynomial  $p(x)$  by polynomial  $g(x)$ , where  $p(x) = x^5 - 4x^3 + x^2 + 3x + 1$ ,  $g(x) = x^3 - 3x + 1$

20. Find the area of the triangle formed by joining the mid-points of the sides of the triangle  $ABC$ , whose vertices are  $A(0, -1)$ ,  $B(2, 1)$  and  $C(0, 3)$ .

21. Draw the graph of the equations  $x - y + 1 = 0$  and  $3x + 2y - 12 = 0$ . Using this graph, find the values of  $x$  and  $y$  which satisfy both the equations.

22. Prove that  $\sqrt{3}$  is an irrational number.

23. Find the largest number which on dividing 1251, 9377 and 15628 leaves remainders 1, 2 and 3 respectively.

24.  $A$ ,  $B$  and  $C$  are interior angles of a triangle  $ABC$ . Show that

$$(a) \sin\left(\frac{B+C}{2}\right) = \cos \frac{A}{2}$$

- (b) If  $\angle A = 90^\circ$ , then find the value of  $\tan\left(\frac{B+C}{2}\right)$
25. If  $\tan(A + B) = 1$  and  $\tan(A - B) = \frac{1}{\sqrt{3}}$ ,  $0^\circ < A + B < 90^\circ$ ,  $A > B$ , then find the values of  $A$  and  $B$ .
26. In Figure 2,  $PQ$  is a chord of length  $8\text{ cm}$  of a circle of radius  $5\text{ cm}$ . The tangents at  $P$  and  $Q$  intersect at a point  $T$ . Find the length  $TP$ .

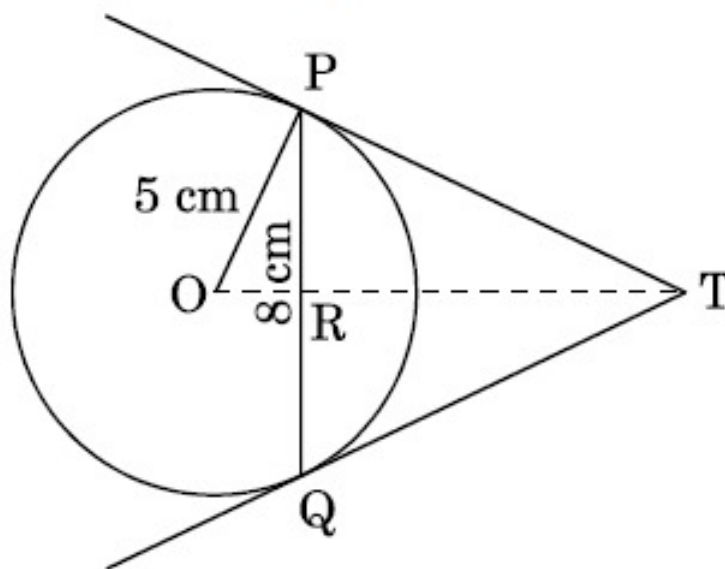


Figure 2: CIRCLE

27. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
28. Water in a canal,  $6\text{ m}$  wide and  $1.5\text{ m}$  deep, is flowing with a speed of  $10\text{ km/h}$ . How much area will it irrigate in 30 minutes if  $8\text{ cm}$  of standing water is needed?
29. A car has two wipers which do not overlap. Each wiper has a blade of length  $21\text{ cm}$  sweeping through an angle  $120^\circ$ . Find the total area cleaned at each sweep of the blades. (Take  $\pi = \frac{22}{7}$ )
30. A pole has to be erected at a point on the boundary of a circular park of diameter  $13\text{ m}$  in such a way that the difference of its distances from two diametrically opposite fixed gates  $A$  and  $B$  on the boundary is  $7\text{ m}$ . Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?
31. If  $m$  times the  $m^{\text{th}}$  term of an Arithmetic Progression is equal to  $n$  times its  $n^{\text{th}}$  term and  $m \neq n$ , show that the  $(m + n)^{\text{th}}$  term of the A.P is zero

32. The sum of the first three numbers in an Arithmetic Progression is 18. If the product of the first and the third term is 5 times the common difference, find the three numbers.
33. Construct a triangle  $ABC$  with side  $BC = 6cm$ ,  $AB = 5cm$  and  $\angle ABC = 60^\circ$ . Then construct another triangle whose sides are  $\frac{3}{4}$  of the corresponding sides of the triangle  $ABC$
34. In Figure 3, a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge  $6cm$  and the hemisphere fixed on the top has a diameter of  $4.2cm$ . Find
- (a) the total surface area of the block.
  - (b) the volume of the block formed. (Take  $\pi = \frac{22}{7}$ )

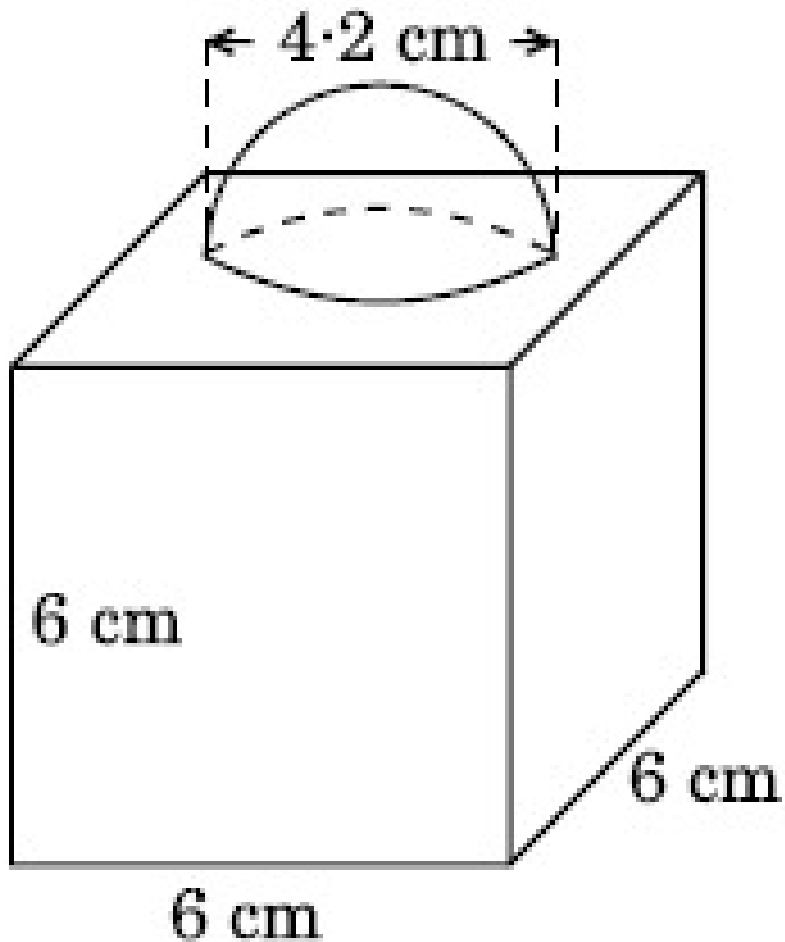


Figure 3: CUBE AND HEMISPHERE

35. A bucket open at the top is in the form of a frustum of a cone with a capacity of  $12308.8\text{cm}^3$ . The radii of the top and bottom circular ends are  $20\text{cm}$  and  $12\text{cm}$  respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (Use  $\pi = 3.17$ )
36. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.
37. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

38. If  $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$ , then prove that  $\tan \theta = 1$  or  $\tan \theta = \frac{1}{2}$
39. The shadow of a tower standing on a level ground is found to be  $40m$  longer when the Sun's altitude is  $30^\circ$  than when it was  $60^\circ$ . Find the height of the tower.  
(Given  $\sqrt{3} = 1.732$ )
40. Solve the following pair of linear equations :
- $$3x + 4y = 10 \quad (1)$$
- $$2x - 2y = 2 \quad (2)$$
41. A chord of a circle of radius  $14cm$  subtends an angle of  $60^\circ$  at the centre. Find the area of the corresponding minor segment of the circle. (Use  $\pi = \frac{22}{7}$  and  $\sqrt{3} = 1.732$ )
42. Find the value of  $k$  so that the area of triangle  $ABC$  with  $A(k + 1, 1)$ ,  $B(4, -3)$  and  $C(7, -k)$  is 6 square units.
43. If  $\frac{2}{3}$  and  $-3$  are the zeroes of the polynomial  $ax^2 + 7x + b$ , then find the values of  $a$  and  $b$ .
44. Construct a triangle, the lengths of whose sides are  $5cm$ ,  $6cm$  and  $7cm$ . Now construct another triangle whose sides are  $\frac{5}{7}$  times the corresponding sides of the first triangle.
45. Prove that :
- $$\frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta} = \sec \theta \csc \theta - 2 \sin \theta \cos \theta$$
46. A motorboat whose speed in still water is  $9km/h$ , goes  $15km$  downstream and comes back to the same spot, in a total time of  $3hours\ 45minutes$ . Find the speed of the stream.