MATHEMATICS

January 10, 2024

1 DISCRETE

1. Find after how many places of decimal the decimal form of the number $\frac{27}{2^3.5^4.3^2}$ will terminate.
2. Express 429 as a product of its prime factors.
3. If HCF of 65 and 117 is expressible in the form $65n - 117$, then find the value of n.
4. On a morning walk, three persons step out together and their steps measure $30cm$, $36cm$ and $40cm$ respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps ?
5. Prove that $\sqrt{3}$ is an irrational number.
6. Find the largest number which on dividing 1251, 9377 and 15628 leaves remainders 1, 2 and 3 respectively.
7. Find the sum of first 10 multiples of 6
8. If m times the m^{th} term of an Arithmetic Progression is equal to n times its n^{th} term and $m \neq n$, show that the $(m+n)^{th}$ term of the A.P is zero
9. The sum of the first three numbers in an Arithmetic Progression is 18. If the product of the first and the third term is 5 times the common difference, find the three numbers.

2 ALGEBRA

10. Write the discriminant of the quadratic equation $(x+5)^2 = 2(5x-3)$.
11. Using completing the square method, show that the equation $x^2 - 8x + 18 = 0$ has no solution.

12. Check whether $g(x)$ is a factor of $p(x)$ by dividing polynomial $p(x)$ by polynomial $g(x)$,where $p(x) = x^5 - 4x^3 + x^2 + 3x + 1$, $g(x) = x^3 - 3x + 1$

13. If $\frac{2}{3}$ and -3 are the zeroes of the polynomial $ax^2 + 7x + b$, then find the values of a and b.

14. If $\tan\alpha = \frac{5}{12}$, find the value of $\sec\alpha$.

15. A, B and C are interior angles of a triangle ABC. Show that

    (a) $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

    (b) If $\angle A = 90°$, then find the value of $\tan\left(\frac{B+C}{2}\right)$

16. If $\tan(A + B) = 1$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0° < A + B < 90°$, $A > B$, then find the values of A and B.

17. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $\tan\theta = \frac{1}{2}$

18. Prove that :

$$\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\csc\theta - 2\sin\theta\cos\theta$$

19. A motorboat whose speed in still water is $9km/h$, goes $15km$ downstream and comes back to the same spot, in a total time of $3hours\ 45minutes$. Find the speed of the stream.

3 MATRICES

20. Solve the following pair of linear equations :

$$3x - 5y = 4$$
$$2y + 7 = 9x$$

21. Solve the following pair of linear equations :

$$3x + 4y = 10$$
$$2x - 2y = 2$$

22. Find the value of k so that the area of triangle ABC with $A(k+1, 1)$, $B(4, -3)$ and $C(7, -k)$ is 6 square units.

4 VECTORS

23. Find the value (s) of x, if the distance between the points $A\,(0,0)$ and $B\,(x,-4)$ is 5 units.

24. Points $A\,(3,1)$, $B\,(5,1)$, $C\,(a,b)$ and $D\,(4,3)$ are vertices of a parallelogram $ABCD$. Find the values of a and b.

25. Points P and Q trisect the line segment joining the points $A\,(-2,0)$ and $B\,(0,8)$ such that P is near to A. Find the coordinates of points P and Q.

26. Find the area of the triangle formed by joining the mid-points of the sides of the triangle ABC, whose vertices are $A\,(0,-1)$, $B\,(2,1)$ and $C\,(0,3)$.

5 LINEAR FORMS

27. Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Using this graph, find the values of x and y which satisfy both the equations.

6 CONSTRUCTION

28. In Figure 1, $PS = 3cm$, $QS = 4cm$, $\angle PRQ = \theta$, $\angle PSQ = 90°$, $PQ \perp RQ$ and $RQ = 9cm$. Evaluate $\tan\theta$.

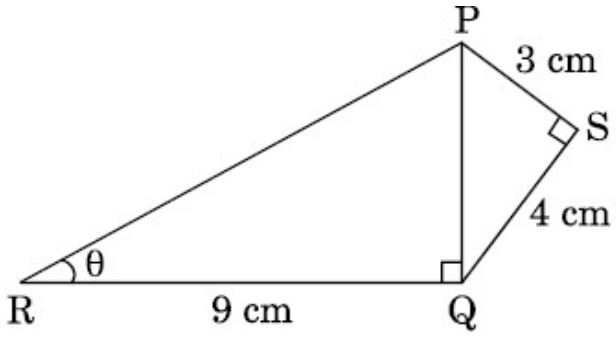

Figure 1: Triangle PSQR

29. Two concentric circles of radii a and b $(a > b)$ are given. Find the length of the chord of the larger circle which touches the smaller circle.

30. Construct a triangle, the lengths of whose sides are $5cm$, $6cm$ and $7cm$. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of the first triangle.

31. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

32. Construct a triangle ABC with side $BC = 6cm$, $AB = 5cm$ and $\angle ABC = 60°$. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC

33. The perpendicular from A on side BC of a $\triangle ABC$ meets BC at D such that $DB = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$.

34. AD and PM are medians of triangles ABC and PQR respectively where $\triangle ABC \sim \triangle PQR$. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

7 GEOMETRY

35. The shadow of a tower standing on a level ground is found to be $40m$ longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower. ($Given\ \sqrt{3} = 1.732$)

36. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

37. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

38. A pole has to be erected at a point on the boundary of a circular park of diameter $13m$ in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is $7m$. Is it possible to do so ? If yes, at what distances from the two gates should the pole be erected ?

39. Water in a canal, $6m$ wide and $1.5m$ deep, is flowing with a speed of $10km/h$. How much area will it irrigate in 30 minutes if $8cm$ of standing water is needed?

40. A car has two wipers which do not overlap. Each wiper has a blade of length $21cm$ sweeping through an angle 120°. Find the total area cleaned at each sweep of the blades. ($Take \pi = \frac{22}{7}$)

41. In Figure 2, a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge $6cm$ and the hemisphere fixed on the top has a diameter of $4.2cm$. Find

    (a) the total surface area of the block.

    (b) the volume of the block formed.(Take $\pi = \frac{22}{7}$)

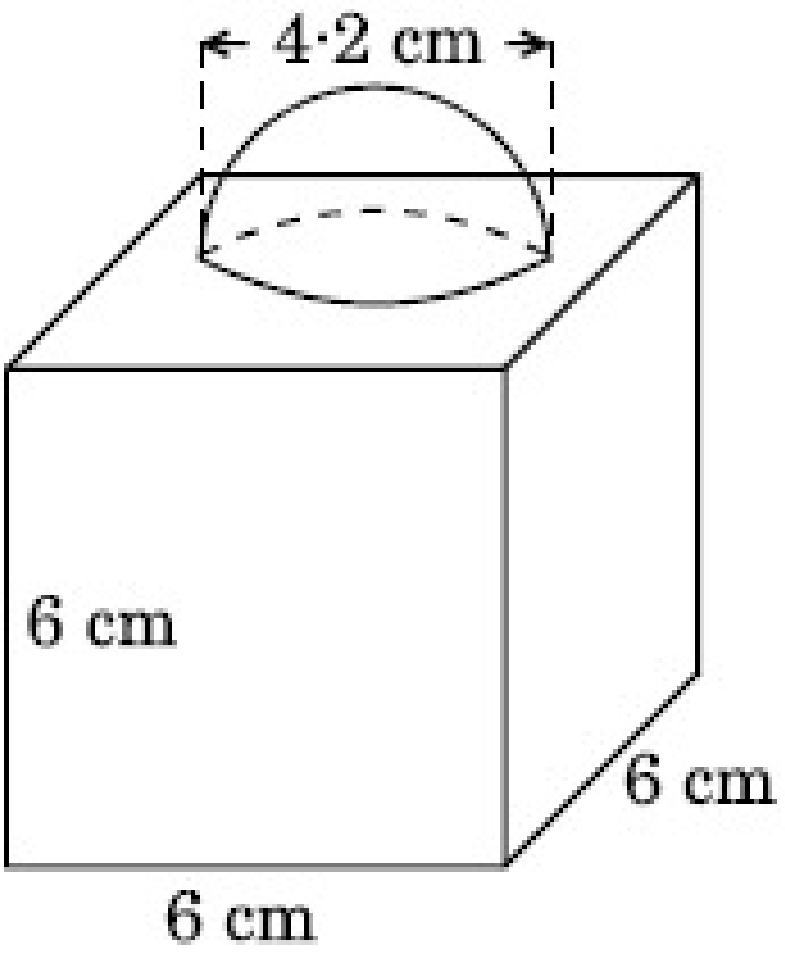

Figure 2: CUBE AND HEMISPHERE

42. A bucket open at the top is in the form of a frustum of a cone with a capacity of $12308.8cm^3$. The radii of the top and bottom circular ends are $20cm$ and $12cm$ respectively. Find the height of the bucket and the area of metal sheet used in making the bucket.($Use\pi = 3.17$)

8 CIRCLES

43. In Figure 3, PQ is a chord of length 8*cm* of a circle of radius 5*cm*. The tangents at P and Q intersect at a point T. Find the length TP.

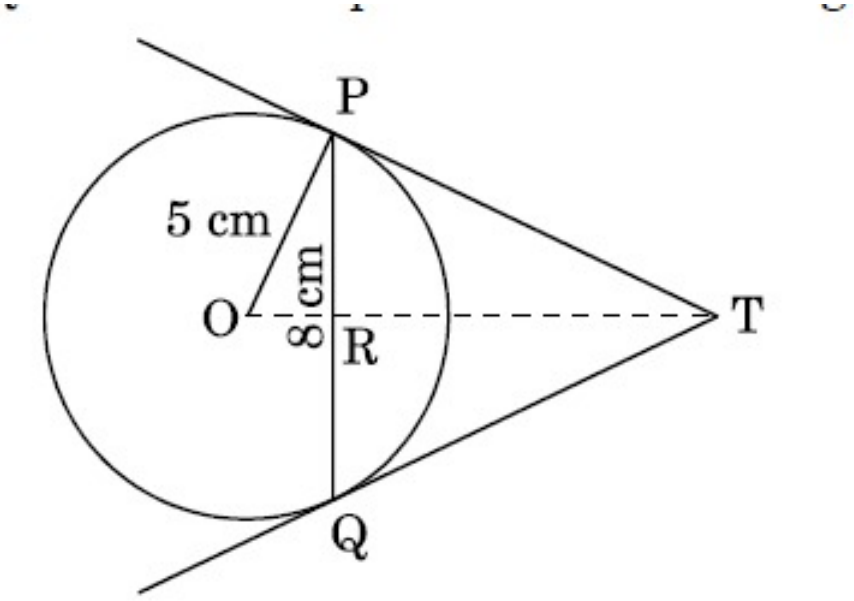

Figure 3: CIRCLE

44. A chord of a circle of radius 14*cm* subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle. (Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.732$)

9 PROBABILITY

45. A die is thrown once. Find the probability of getting

    (a) a composite number,

    (b) a prime number.

46. Cards numbered 7 to 40 were put in a box. Poonam selects a card at random. What is the probability that Poonam selects a card which is a multiple of 7 ?